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Aquifer response to stream-stage and recharge variations. I. Analytical step-response functions

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Abstract

Laplace transform step-response functions are presented for various homogeneous confined and leaky aquifer types and for anisotropic, homogeneous unconfined aquifers interacting with perennial streams. Flow is one-dimensional, perpendicular to the stream in the confined and leaky aquifers, and two-dimensional in a plane perpendicular to the stream in the water-table aquifers. The stream is assumed to penetrate the full thickness of the aquifer. The aquifers may be semi-infinite or finite in width and may or may not be bounded at the stream by a semipervious streambank. The solutions are presented in a unified manner so that mathematical relations among the various aquifer configurations are clearly demonstrated. The Laplace transform solutions are inverted numerically to obtain the real-time step-response functions for use in the convolution (or superposition) integral. To maintain linearity in the case of unconfined aquifers, fluctuations in the elevation of the water table are assumed to be small relative to the saturated thickness, and vertical flow into or out of the zone above the water table is assumed to occur instantaneously. Effects of hysteresis in the moisture distribution above the water table are therefore neglected. Graphical comparisons of the new solutions are made with known closed-form solutions. Published by Elsevier Science B.V.

Keywords: Stream/aquifer interaction; Mathematical models; Confined aquifers; Leaky aquifers; Unconfined aquifers; Seepage

1. Introduction

Increased demand for water associated with population growth has heightened public awareness of the importance of the proper management of limited water resources. With this awareness has come a recognition by the public that ground-water reservoirs and surface-water supplies are connected to one another, and that the use of one can affect the quantity and quality of the other. It is perhaps because of this that water-resource managers have taken considerable interest in quantification of the interaction of surface water and ground water. Analytical models are helpful tools in this endeavor.

One perceived difficulty in the use of analytical models is the fact that the necessary boundary conditions—stream stage and regional recharge or evapotranspiration—change continuously. While it is recognized that the effects of variable boundary conditions can be simulated with numerical models, it is not widely appreciated that these variations can also be effectively simulated by combining analytical

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Nomenclature

Symbol	Definition [dimension (L, length; T, time)]
a	Streambank leakance [L]
A	Dimensionless streambank leakance
b	Thickness of aquifer (or saturated thickness for water-table aquifer) [L]
b'	Thickness of aquitard (or saturated thickness for water-table aquitard) [L]
c	Instantaneous step change in water level of stream [L]
d	Thickness of semiprevious streambank material [L]
h	Head in aquifer [L]
h'	Head in aquitard [L]
h_D	Dimensionless step-response function for head in aquifer
$\bar{h}_D$	Dimensionless Laplace transform step-response function for head in aquifer
$\bar{h}_D^*$	Dimensionless Laplace transform average head in a partially penetrating observation well
$\hat{\bar{h}}_D$	Dimensionless Laplace transform average head in a fully penetrating observation well
h_i	Initial water level (or potentiometric surface) in stream–aquifer system [L]
h_0	Water level in stream after step change [L]
K	Hydraulic conductivity of confined and leaky aquifers [L/T]
K_D	Dimensionless ratio of vertical to horizontal hydraulic conductivity
K_s	Hydraulic conductivity of semipervious streambank material [L/T]
K_x, K_z	Horizontal and vertical hydraulic conductivity of water-table aquifers, respectively [L/T]
K'	Vertical hydraulic conductivity of aquitard [L/T]
m	Dimensionless grouping
n	Integer counter in infinite summations
p	Laplace transform variable [dimensionless]
q_n	Terms in the Laplace transform solutions for water-table aquifers [dimensionless]
q'	Volumetric flow rate to or from aquifer per unit volume of aquifer [1/T]
$\bar{q}_D$	Dimensionless Laplace transform leakage between aquifer and aquitard
Q	Seepage rate per unit length of stream [L^2/T]
Q_D	Dimensionless seepage between stream and aquifer
$\bar{Q}_D$	Dimensionless Laplace transform seepage between stream and aquifer
S	Storativity (storage coefficient) of aquifer [dimensionless]
S_s	Specific storage of aquifer [1/L]
S_s'	Specific storage of aquitard [1/L]
S_y	Specific yield of aquifer [dimensionless]
S_y'	Specific yield of aquitard [dimensionless]
t	Time [T]
t_D	Dimensionless time
t_{Dy}	Dimensionless time with respect to specific yield
T	Transmissivity of aquifer [L^2/T]
W	Term for aquifer width in Laplace transform solutions for confined and leaky aquifers [dimensionless]
W_n	Term for aquifer width in Laplace transform solutions for water-table aquifers [dimensionless]
x	Horizontal coordinate [L]
x_D	Dimensionless horizontal coordinate
x_L	Width of aquifer [L]

x_{LD}	Dimensionless width of aquifer
x_0	Distance from middle of stream–aquifer boundary (half-width of stream) [L]
x_{0D}	Dimensionless distance to streambank
z	Vertical coordinate [L]
z_1	Vertical coordinate of bottom of screened interval of observation well [L]
z_2	Vertical coordinate of top of screened interval of observation well [L]
z_D	Dimensionless vertical coordinate in aquifer
z_{D1}	Dimensionless vertical coordinate of bottom of screened interval of observation well
z_{D2}	Dimensionless vertical coordinate of top of screened interval of observation well
z_p	Vertical coordinate of observation piezometer opening [L]
Greek letters	
β_0	Dimensionless product of anisotropic ratio of vertical to horizontal hydraulic conductivity and square of dimensionless distance to streambank
ϵ_n	Roots of equations in Laplace transform solutions for water-table aquifers [dimensionless]
γ_1	Dimensionless ratio of aquitard to aquifer hydraulic conductivity
τ	Time variable of integration (delay time) [T]
σ	Dimensionless ratio of aquifer storativity to aquifer specific yield
σ_1	Dimensionless ratio of aquitard to aquifer storativity
σ'	Dimensionless ratio of aquifer storativity to aquitard specific yield

models of stream–aquifer systems with the method of convolution (superposition). The analytical approach is often the simplest and quickest way to obtain answers to questions posed by water-resource managers. Analytical models can also be instrumental in improving our understanding of physical processes occurring within a ground-water flow system. The analytical approach can be used to predict short-term water-table fluctuations in response to a passing flood wave, the flux of water between the aquifer and stream, cumulative bank storage, and stream base flow during periods of little or no precipitation. Analytical models can also be used to estimate aquifer hydraulic properties and recharge.

The literature is replete with analytical solutions for the interaction of confined, leaky, and water-table aquifers with an adjoining stream. A detailed but not fully comprehensive review of these solutions and their applications is provided by Barlow and Moench (1998) and will not be repeated here. Because they involve one-dimensional horizontal flow in the aquifer and one-dimensional vertical flow in the aquitard, solutions for confined and leaky aquifers are relatively simple mathematically. Nevertheless, they have been found to be quite practical and are often cited in the literature. Solutions for confined aquifers have even been used for unconfined aquifers, replacing the confined-aquifer storage coefficient (storativity) with specific yield. This latter approach has limitations, however, because it neglects vertical components of flow and improperly defines the behavior of the free surface.

In this paper, Laplace transform step-response functions are presented for several confined, leaky, and water-table aquifer configurations. The Laplace domain solutions are numerically inverted to the real-time domain with the Stehfest (1970) algorithm (see Moench and Ogata, 1984). Following Hall and Moench (1972), the stream is assumed to penetrate the full thickness of an aquifer, the aquifer may be semi-infinite or finite in width, and the stream channel may or may not be lined with materials that have hydraulic properties different from those of the aquifer (semi-pervious streambank).

The homogeneous aquifer models described in this paper involve one-dimensional flow (perpendicular to the stream) in confined and leaky aquifers, and two-dimensional flow (in a vertical plane perpendicular to the stream) in water-table aquifers. All aquifers are assumed to be bounded below by a horizontal, impermeable base. The leaky aquifers: (1) are overlain by aquitards that are bounded above by either an impermeable layer or a constant-head source bed; or (2) are overlain by a water-table aquitard. The water-table aquifers are overlain by a thick unsaturated zone from which water drains or imbibes instantaneously in response to a fluctuating water table. Moisture redistribution in the unsaturated zone is assumed to be unaffected by hysteresis. The several solutions presented in this paper differ from previously developed analytical approaches primarily in the wide range of aquifer types to which they can be applied.

Though developed for the condition of a sudden change of the water level in a stream relative to that of the aquifer, the step-response functions are equally applicable to the condition of a sudden change of the water level in an aquifer relative to that of the stream, caused, for example, by basin-wide recharge, irrigation, or evapotranspiration (see, for example Kraijenhoff van de Leur (1958), Rorabaugh (1960, 1964), Singh (1969), Singh and Stall (1971), Daniel (1976) and Rutledge (1993, 1997)). Because stream-stage fluctuations often occur simultaneously with recharge or evapotranspiration, it is important to consider the combined effect of such simultaneous stresses on the stream–aquifer interaction.

In the companion paper by Barlow et al. (2000), the various step-response functions are combined with the convolution method to demonstrate time-varying head, seepage at the streambank, and cumulative bank storage that occur as a result of a hypothetical sinusoidal stream-stage hydrograph. In addition, Barlow et al. (2000) apply the methodology to two field sites.

2. Mathematical development

This section describes the simplifying assumptions and boundary-value problems for each of the confined, leaky, and water-table aquifer configurations leading to the Laplace transform step-response functions. Detailed derivations of the Laplace transform solutions for all aquifer types are given by Barlow and Moench (1998, Attachment 1). The following assumptions apply to all aquifer configurations in this paper.

1. Each aquifer is homogeneous and of uniform thickness.
2. The lower boundary of each aquifer is horizontal and impermeable.
3. Hydraulic properties of the aquifers do not change with time.
4. The porous medium and fluid are slightly compressible.
5. Observation wells or piezometers are infinitesimal in diameter and respond instantly to pressure changes in the aquifer.
6. The water level in the stream is initially at the same elevation as the water level everywhere in the aquifer and aquitard.
7. The semipervious streambank material, if present, is homogeneous, isotropic, and has negligible capacity to store water.
8. The stream forms a vertical boundary to the aquifer and fully penetrates the aquifer.
9. The stream flows in a straight line (that is, without sinuoisity).

The assumption that the stream fully penetrates the aquifer is a common assumption made to simplify the mathematics. The assumption was discussed by Hantush (1965) who stated that, to be valid, the observation piezometers should be at least the distance of $1.5b$ away from the streambank (where b is the aquifer thickness). In this paper, the fitting parameter, a, for semipervious streambank material is used to loosely account for constricted flow at the streambank due to partial penetration and effects of other idealizations. Hantush (1965) describes the parameter, a, as the effective width of aquifer material required to cause the same head loss as the semipervious streambank itself. For a recent analysis of a finite-width stream that slightly penetrates a water-table aquifer the reader is referred to Zlotnik and Huang (1999).

2.1. Confined and leaky aquifers

Figs. 1–4 are diagrammatic cross-sections through the idealized semi-infinite confined and leaky aquifer configurations, with and without a semipervious streambank, for which analytical solutions are presented. Analogous figures could be drawn for finite-width aquifers by placing a vertical impermeable boundary at some distance $x = x_L$. The aquifers are bounded below by impermeable material and above by either impermeable material (confined case, Fig. 1) or by a poorly permeable aquitard

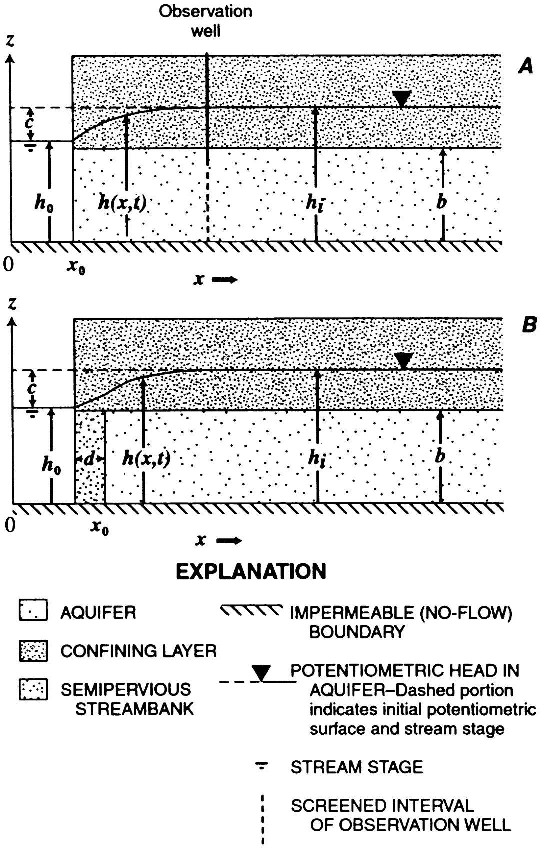

Fig. 1. Semi-infinite, confined aquifer: (A) without semipervious streambank material; and (B) with semipervious streambank material.

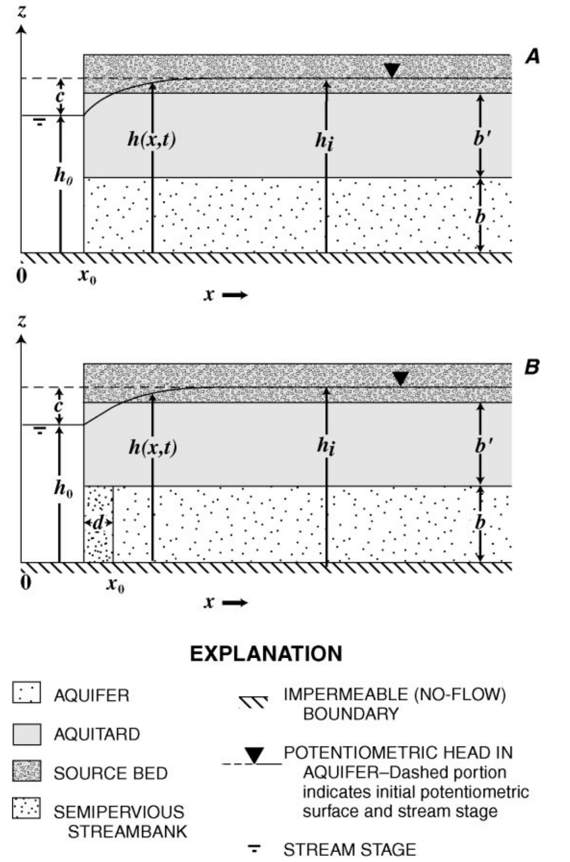

Fig. 2. Semi-infinite, leaky aquifer with constant head overlying the aquitard (case 1): (A) without semipervious streambank material; and (B) with semipervious streambank material.

(leaky case, Figs. 2–4). Laplace transform solutions for the step-response functions are presented for confined aquifers and for three types of leaky aquifers. The leaky aquifers differ from one another by the condition at the upper boundary of the aquitard as follows: (1) a source bed with a constant head overlying the aquitard (leaky aquifer case 1, Fig. 2); (2) an impermeable layer overlying the aquitard (leaky aquifer case 2, Fig. 3); and (3) an aquifer that is overlain by a water-table aquitard (leaky aquifer case 3, Fig. 4). The flow in each type of aquifer is horizontal and one-dimensional. The figures show the location of the origin of the coordinate system at the base of the aquifer and middle of the stream. To the general assumptions listed above, one must add the following assumptions regarding the aquitard.

1. The aquitard is homogeneous, isotropic, and of uniform thickness.
2. The hydraulic conductivity of the aquitard must be much smaller than the hydraulic conductivity of the underlying aquifer and flow in the aquitard is strictly vertical.
3. For case 3, that of a leaky aquifer overlain by a water-table aquitard, water in the zone above the free surface is released (or taken up) instantaneously in a vertical direction in response to a decline (or rise) in the elevation of the water table. In addition, the change in saturated thickness of the water-table aquitard due to stream-stage fluctuations or recharge is small compared with the initial saturated thickness of the aquitard. Finally, pressure changes caused by recharge are

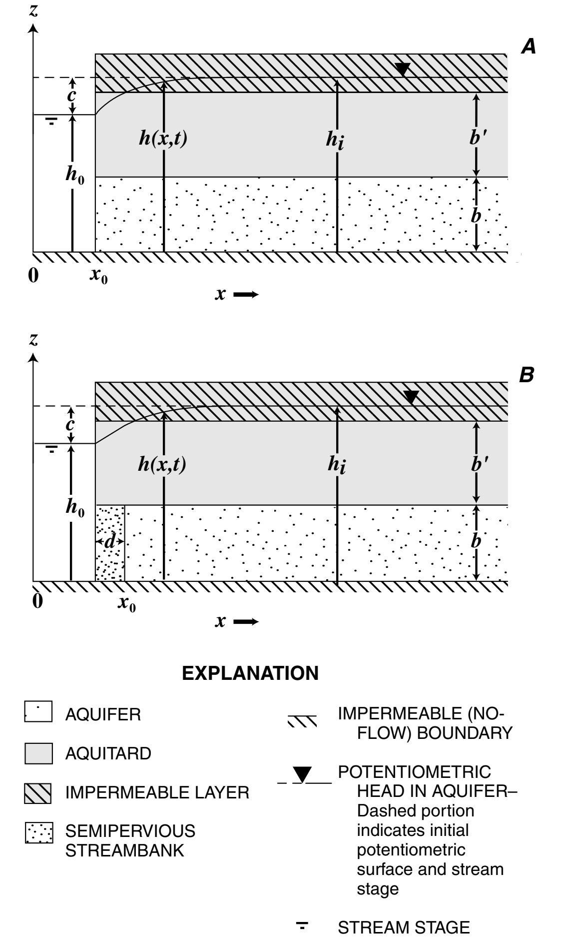

Fig. 3. Semi-infinite, leaky aquifer with impermeable layer overlying the aquitard (case 2): (A) without semipervious streambank material; and (B) with semipervious streambank material.

propagated nearly instantaneously by virtue of aquitard diffusivity to the underlying aquifer.

Validity of assumption 2 requires a large contrast in hydraulic conductivity between the aquifer and aquitard. Neuman and Witherspoon (1969) found that the errors introduced by this assumption are usually less than 5% if the hydraulic conductivity of the aquifer is 100 times the hydraulic conductivity of the aquitard.

2.1.1. Boundary-value problems

2.1.1.1. Aquifer. The governing partial differential equation describing one-dimensional, horizontal

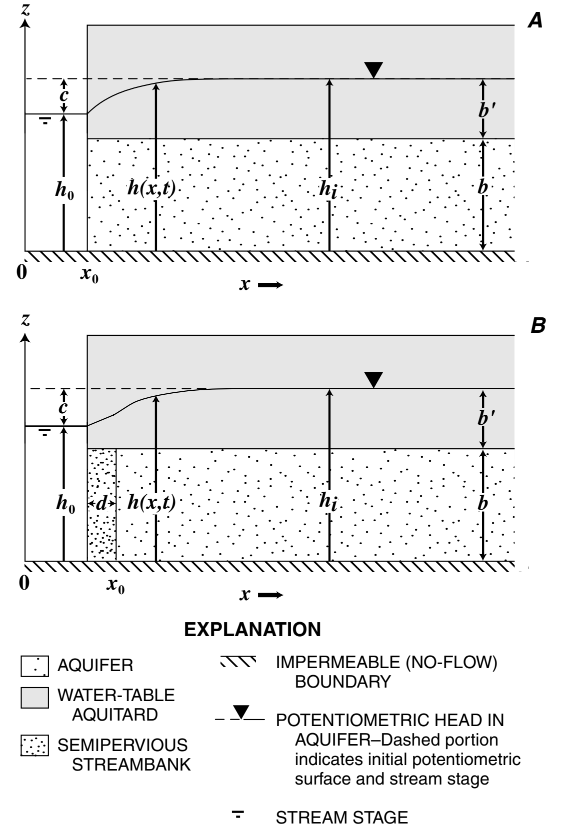

Fig. 4. Semi-infinite, leaky aquifer overlain by a water-table aquitard (case 3): (A) without semipervious streambank material; and (B) with semipervious streambank material.

ground-water flow in a confined or leaky aquifer is

$$\frac{\partial^2 h}{\partial x^2} = \frac{S_s}{K}\frac{\partial h}{\partial t} + q', \tag{1}$$

where h is the vertically averaged head in the aquifer; x the horizontal coordinate; S_s the specific storage of the aquifer; K the hydraulic conductivity of the confined or leaky aquifer; t the time; q' is the source term and equals $-(K'/Kb)((\partial h'/\partial z)_{z=b})$; K' is the vertical hydraulic conductivity of the aquitard; and h' is the head in the aquitard. For confined aquifers, $K' = 0$, hence $q' = 0$. The domain for Eq. (1) for semi-infinite aquifers is $x_0 \le x < \infty$ and for finite-width aquifers is $x_0 \le x \le x_L$, where x_L is the width of a finite-width aquifer. In Eq. (1), h is a function of x and t, and h' is a function of z and t.

Table 1
Dimensionless variables and variable groupings for confined and leaky and aquifers

Dimensionless variable or grouping	Definition
x_D	x/x_0
x_{LD}	x_L/x_0
x_{0D}	x_0/b
h_D	$(h_i - h)/c$
t_D	$Kt/S_s x_0^2$
A	$Kd/K_s x_0$
σ_1	$S'_s b'/S_s b$
σ'	$S_s b/S'_y$
γ_1	$x_0/b'\sqrt{K'b'/Kb}$
m	$\sigma_1 p/\gamma_1^2$

The initial condition for all boundary-value problems is

$$h(x,0) = h_i \tag{2}$$

where h_i is the initial water level (or potentiometric surface) in the stream–aquifer system.

Several boundary conditions are used for the confined and leaky aquifers; the particular set of boundary conditions used for each system depends on the conditions being modeled. For a semi-infinite aquifer, the boundary condition as x approaches infinity is

$$h(\infty, t) = h_i, \tag{3}$$

whereas for a finite-width aquifer, the boundary condition at $x = x_L$ is

$$\frac{\partial h}{\partial x}(x_L, t) = 0. \tag{4}$$

The boundary condition used at the stream–aquifer interface depends upon the presence or absence of semipervious streambank material. For conditions of no semipervious streambank material, a specified head is used at x_0

$$h(x_0, t) = h_0, \tag{5}$$

where h_0 is the water level in the stream after the instantaneous step change. For conditions in which semipervious streambank material is present, a head-dependent flux boundary condition is used at x_0

$$\frac{\partial h(x_0,t)}{\partial x} = -\frac{1}{a}[h_0 - h(x_0,t)], \tag{6}$$

where a is streambank leakance and $[h_0 - h(x_0,t)]$ is the change in head across the semipervious streambank material. Streambank leakance is defined as

$$a = \frac{Kd}{K_s}, \tag{7}$$

where d is the thickness of the semipervious streambank material and K_s is the hydraulic conductivity of the semipervious streambank material. The ratio K_s/d can and should be considered a single fluid-transfer parameter. The use of a is similar to the concept in well hydraulics of an infinitesimally thin well-bore skin at a pumped well.

2.1.1.2. Aquitard. For leaky aquifer conditions, a governing partial differential equation describing one-dimensional, vertical flow in the overlying aquitard must be solved with appropriate boundary conditions and coupled with Eq. (1) This equation is

$$\frac{\partial^2 h'}{\partial z^2} = \frac{S'_s}{K'}\frac{\partial h'}{\partial t}, \tag{8}$$

where S'_s is the specific storage of the aquitard. The domain for which Eq. (8) is applicable is $b \le z \le b + b'$.

The initial condition for head in the aquitard for all boundary-value problems is

$$h'(z,0) = h_i. \tag{9}$$

The boundary condition along the aquitard–aquifer boundary ($z = b$) is

$$h'(b,t) = h. \tag{10}$$

Alternative boundary conditions are used for the top of the aquitard ($z = b + b'$) that depend upon the presence and hydraulic conditions of the overlying bed. For the condition of constant head overlying the aquitard (case 1), the boundary condition at the top of the aquitard is

$$h'(b + b', t) = h_i. \tag{11}$$

For the condition of an impermeable layer overlying the aquitard (case 2) the boundary condition is

$$\frac{\partial h'}{\partial z}(b + b', t) = 0. \tag{12}$$

For the condition in which the overlying material is unsaturated, the aquitard is under water-table conditions (case 3, modeled after Cooley and Case (1973)).

In this case, the boundary condition at the water table is

$$\frac{\partial h'}{\partial z}(b+b',t) = -\frac{S'_y}{K'}\frac{\partial h'}{\partial t}(b+b',t), \tag{13}$$

where S'_y is the specific yield of the aquitard.

2.1.2. Laplace transform step-response functions

The dimensional boundary-value problems described by Eqs. (1)–(13) are made dimensionless by substituting the dimensionless variables and variable groupings shown in Table 1. The mathematical development is outlined in Attachment 1 in Barlow and Moench (1998). The Laplace transform step-response functions for all confined and leaky aquifer types can be written in the most general form as

$$\bar{h}_D = \frac{W\exp[-\sqrt{p+\bar{q}_D}(x_D-1)]}{p\{1+\sqrt{p+\bar{q}_D}A\tanh[\sqrt{p+\bar{q}_D}(x_{LD}-1)]\}}, \tag{14}$$

where $\bar{h}_D$ is the dimensionless Laplace transform step-response function at any point (x_D) in a vertical cross-section of the aquifer. The bar over the step-response function (h_D) represents the Laplace transform. The Laplace transform variable, p, is inversely related to dimensionless time t_D. For semi-infinite aquifers, x_{LD} goes to infinity and the hyperbolic tangent in Eq. (14) is unity. Parameter W is a function of the width of the aquifer perpendicular to the stream and is defined as

$$W = \frac{\exp[-2\sqrt{p+\bar{q}_D}(x_{LD}-x_D)]+1}{\exp[-2\sqrt{p+\bar{q}_D}(x_{LD}-1)]+1}.$$

W equals 1 for semi-infinite conditions. Parameter A is dimensionless streambank leakance

$$A = \frac{a}{x_0},$$

where a, streambank leakance, is defined by Eq. (7). For conditions in which there is no semipervious streambank material, $A = 0$.

Parameter (or source term) $\bar{q}_D$ accounts for leakage between the aquifer and overlying aquitard and takes on different forms depending upon the type of leaky aquifer. In fact, mathematical expressions for $\bar{q}_D$ can also be derived for completely different aquifer types (e.g. various double-porosity aquifer geometries such as those described by Moench (1984) for well hydraulics and applied to stream–aquifer interaction by Onder (1998)) that are beyond the scope of this paper. For a confined aquifer with no overlying aquitard

$$\bar{q}_D = 0;$$

for a leaky aquifer with constant head overlying the aquitard (case 1)

$$\bar{q}_D = \gamma_1^2\sqrt{m}\coth(\sqrt{m});$$

for a leaky aquifer with an impermeable layer overlying the aquitard (case 2)

$$\bar{q}_D = \gamma_1^2\sqrt{m}\tanh(\sqrt{m});$$

and for a leaky aquifer overlain by a water-table aquitard (case 3)

$$\bar{q}_D = \gamma_1^2\sqrt{m}\frac{[\sqrt{m}(\sigma'\gamma_1^2)\tanh(\sqrt{m})+p]}{[\sqrt{m}(\sigma'\gamma_1^2)+p\tanh(\sqrt{m})]}.$$

Parameters γ_1, m, σ' are defined in Table 1.

Eq. (14) is the general solution for all of the confined and leaky aquifer types. For example, for a semi-infinite, confined aquifer with no semipervious streambank material between the aquifer and stream, $W = 1$, $A = 0$, and $\bar{q}_D = 0$. Under these conditions, Eq. (14) becomes

$$\bar{h}_D = \frac{\exp[-\sqrt{p}(x_D-1)]}{p}, \tag{15}$$

which can be analytically inverted from the Laplace domain and written in the real-time domain as

$$h_D = \operatorname{erfc}\left[\frac{(x_D-1)}{(4t_D)^{1/2}}\right] \tag{16}$$

Eq. (16) is the form most often cited in the literature for the condition in which the origin of the coordinate system is at $x_0 = 0$ (Hall and Moench, 1972; Neuman, 1981).

The Laplace transform solution for seepage between the stream and aquifer can be determined by finding the gradient of the step-response solution at the stream–aquifer boundary (i.e. at $x_D = 1$). This gradient is found by differentiation of Eq. (14) with respect to x_D and evaluation of the resulting solution at $x_D = 1$

$$\bar{Q}_D = -\left.\frac{d\bar{h}_D}{dx_D}\right|_{x_D=1} \tag{17}$$

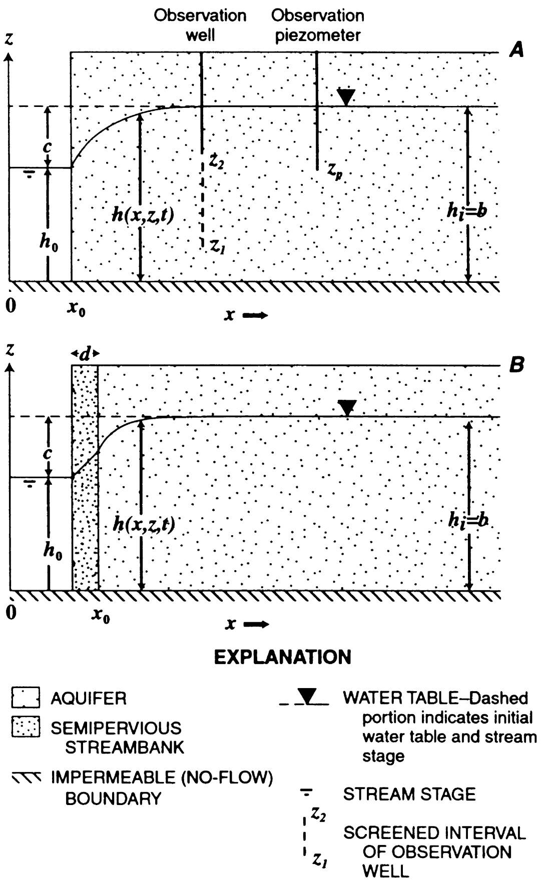

Fig. 5. Semi-infinite, water-table aquifer: (A) without semipervious streambank material; and (B) with semipervious streambank material.

where $\bar{Q}_D$ is dimensionless seepage in the Laplace domain. As described in Barlow and Moench (1998, Attachment 1), the gradient at the stream–aquifer boundary for the confined and leaky aquifers, based on Eq. (14), is

$$\bar{Q}_D = \frac{-\sqrt{p+\bar{q}_D}}{p\{1+\sqrt{p+\bar{q}_D}A\tanh[\sqrt{p+\bar{q}_D}(x_{LD}-1)]\}} \times \left\{\frac{\exp[-2\sqrt{p+\bar{q}_D}(x_{LD}-1)]-1}{\exp[-2\sqrt{p+\bar{q}_D}(x_{LD}-1)]+1}\right\}. \quad (18)$$

For a semi-infinite, confined aquifer with no semipervious streambank material between the aquifer and stream, $A = 0$, $\bar{q}_D = 0$, and the exponential terms in the brackets equal -1. Under these conditions, Eq. (18) becomes

$$\bar{Q}_D = \frac{\sqrt{p}}{p}, \quad (19)$$

which can be analytically inverted from the Laplace domain and written in the real-time domain as

$$Q_D = -\left(\frac{1}{(\pi t_D)^{1/2}}\right), \quad (20)$$

where Q_D is dimensionless seepage in the real-time domain. Eq. (20) is identical to that given by Hall and Moench (1972).

2.2. Water-table aquifers

Fig. 5 shows diagrammatic cross-sections through idealized semi-infinite water-table aquifers, with and without a semipervious streambank, for which analytical solutions are presented. An analogous figure could be drawn for a finite-width aquifer (with an impermeable boundary at $x = x_L$). Each aquifer is bounded by a stream with a depth that extends from the impermeable boundary underlying the aquifer ($z = 0$) to the water table at $z = b$. Except for the addition of the finite-width aquifer and the stream with a semipervious streambank, the solution presented here is equivalent to the solution presented by Neuman (1981). Ground-water flow is assumed to be two-dimensional in the x, z plane perpendicular to the stream for each of the water-table aquifers. As with the confined and leaky aquifers, the distance from the middle of the stream to the stream–aquifer boundary is x_0 (Fig. 5). To the general assumptions listed above, one must add the following assumptions regarding models for the water-table aquifers.

1. Each aquifer can be anisotropic, provided that the principal directions of the hydraulic conductivity tensor are parallel to the x, z coordinate axes.
2. Water is released (or taken up) instantaneously in a vertical direction from (or into) the zone above the water table in response to a decline (or rise) in the elevation of the water table.
3. The change in saturated thickness of the aquifer due to stream-stage fluctuations or recharge is small compared with the initial saturated thickness.
4. Seepage and ground-water head at the stream–aquifer boundary are independent of depth.

Because ground-water flow is assumed to be two-dimensional, heads can vary in both the x and z directions and are not necessarily uniform over the thickness of each aquifer as with the confined and leaky aquifers. Fig. 5A shows schematic drawings of a partially penetrating observation well and an observation piezometer at which ground-water-level measurements could be made. The head measured at the observation well is the average head that exists over the screened interval of the well. Because ground-water heads can vary over the vertical thickness of the aquifer, it is likely that heads measured in an observation piezometer and in a partially penetrating observation well located at the same distance from the stream would not be equivalent. The only condition under which the heads would be equivalent is that in which a uniform head distribution occurred over the full saturated thickness of the aquifer, such as might occur far from the stream where flow is essentially horizontal.

With regard to the zone above the water table where water is held under tension, assumption 2 implies that the equilibrium profile of soil moisture versus depth in the unsaturated and nearly saturated (capillary fringe) zones moves instantaneously in the vertical direction by an amount equal to the change in altitude of the water table. This assumption is commonly made in the analytical treatment of flow in water-table aquifers (e.g. Neuman, 1972, 1981) but may not lead to accurate representation of head variations in piezometers located near the water table (Moench, 1995). In fact, as suggested by an analysis of field data in the companion paper by Barlow et al. (2000), a water-table aquifer may respond very much like a confined aquifer if the unsaturated zone is thin and the specific yield is small. Assumption 2 also requires that there be no hysteresis in the relation between the soil water content and soil matric potential as the water table fluctuates in response to stream-stage variations.

2.2.1. Boundary-value problems

The governing partial differential equation describing two-dimensional, cross-sectional (x, z) flow in a water-table aquifer is

$$\frac{\partial^2 h}{\partial x^2} + \frac{K_z}{K_x}\frac{\partial^2 h}{\partial z^2} = \frac{S_s}{K_x}\frac{\partial h}{\partial t}, \tag{21}$$

where K_x and K_z are the horizontal and vertical hydraulic conductivities of the water-table aquifer, respectively. The x-domain for Eq. (21) for semi-infinite aquifers is $x_0 \le x < \infty$ and for finite-width aquifers is $x_0 \le x < x_L$. The z-domain for all water-table aquifers is $0 \le z \le b$. In Eq. (21), h is a function of x, z, and t.

The initial condition for all solutions is

$$h(x, z, 0) = h_i, \tag{22}$$

where h_i is the initial head in the aquifer.

Several boundary conditions are used for each of the water-table aquifers; the particular set of boundary conditions used for each system depends on the conditions being modeled. For a semi-infinite aquifer, the boundary condition as x approaches infinity is

$$h(\infty, z, t) = h_i, \tag{23}$$

whereas for a finite-width aquifer, the boundary condition at $x = x_L$ is

$$\frac{\partial h}{\partial x}(x_L, z, t) = 0. \tag{24}$$

The boundary condition used at the stream–aquifer interface depends upon the presence or absence of semipervious streambank material. For conditions in which there is no semipervious streambank material, a specified head is used at x_0.

$$h(x_0, z, t) = h_0, \tag{25}$$

where h_0 is the water level in the stream after the instantaneous step change. For conditions in which semipervious streambank material is present, a head-dependent flux boundary condition is used at x_0

$$\frac{\partial h}{\partial x}(x_0, z, t) = -\frac{1}{a}[h_0 - h(x_0, z, t)], \tag{26}$$

where a, streambank leakance, is defined in Eq. (7) and $[h_0 - h(x_0, t)]$ is the change in head across the semipervious streambank material.

The boundary condition at the water table ($z = b$) is

$$\frac{\partial h}{\partial z}(x, b, t) = -\frac{S_y}{K_z}\frac{\partial h}{\partial t}, \tag{27}$$

where S_y is the specific yield of the aquifer. Eq. (27) results, in part, from the assumption that drainage from the zone above the water table occurs instantaneously.

Table 2
Dimensionless variables and variable groupings for water-table aquifers

Dimensionless variable or grouping	Definition
x_D	x/x_0
x_{LD}	x_L/x_0
x_{0D}	x_0/b
z_D	z/b
z_{D1}	z_1/b
z_{D2}	z_2/b
h_D	$(h_i - h)/c$
t_D	$K_x t/S_s x_0^2 = Tt/Sx_0^2$
t_{Dy}	$Tt/S_y x_0^2$
A	$K_x d/K_s x_0$
σ	$S_s b/S_y$
K_D	K_z/K_x
β_0	$K_D x_{0D}^2$

It is the same free-surface condition used by Neuman (1972, 1981).

The boundary condition at the impermeable (no-flow) lower boundary ($z = 0$) is

$$\frac{\partial h}{\partial z}(x, 0, t) = 0. \tag{28}$$

2.2.2. Laplace transform step-response functions

The dimensional boundary-value problems described by Eqs. (21)–(28) are made dimensionless by substituting the dimensionless variables and variable groupings shown in Table 2. The Laplace transform step-response functions for all water-table aquifer types can be written in the most general form as

$$\bar{h}_D = 2\sum_{n=0}^{\infty} \frac{W_n \exp[-q_n(x_D - 1)] \sin(\epsilon_n) \cos(\epsilon_n z_D)}{\{1 + Aq_n \tanh[q_n(x_{LD} - 1)]\} p[\epsilon_n + 0.5 \sin(2\epsilon_n)]}, \tag{29}$$

where

$$q_n = (\epsilon_n^2 \beta_0 + p)^{1/2} \tag{30}$$

and ϵ_n are the roots of

$$\epsilon_n \tan(\epsilon_n) = \frac{p}{\sigma\beta_0}. \tag{31}$$

In Eq. (29), $\bar{h}_D$ is the Laplace transform step-response function at any point (x_D, z_D) of a water-table aquifer. For the semi-infinite aquifers, x_{LD} goes to infinity and the hyperbolic tangent in Eq. (29) in unity. Parameter W_n is a function of the width of the aquifer perpendicular to the stream and is defined as

$$W_n = \frac{\exp[-2q_n(x_{LD} - x_D)] + 1}{\exp[-2q_n(x_{LD} - 1)] + 1}.$$

W_n equals 1 for semi-infinite conditions. As with the confined and leaky aquifer types, parameter A (Table 2) is dimensionless streambank leakance. For conditions in which there is no semipervious streambank material, $A = 0$.

Eq. (29) is the Laplace transform solution for head at each point in a water-table aquifer, such as at an observation piezometer. For a partially penetrating observation well (Fig. 5A), the average head in the well ($\bar{h}_D^*$) is found by integrating Eq. (29) over the screened interval z_{D1} to z_{D2}. The result is

$$\bar{h}_D^* = \frac{2}{(z_{D2} - z_{D1})} \times \sum_{n=0}^{\infty} \frac{W_n \exp[-q_n(x_D - 1)] \sin(\epsilon_n)[\sin(\epsilon_n z_{D2}) - \sin(\epsilon_n z_{D1})]}{\{1 + Aq_n \tanh[q_n(x_{LD} - 1)]\} p\epsilon_n[\epsilon_n + 0.5 \sin(2\epsilon_n)]}. \tag{32}$$

By setting $z_{D1} = 0$ and $z_{D2} = 1$, one obtains the average head in a fully penetrating observation well ($\hat{\bar{h}}_D$)

$$\hat{\bar{h}}_D = 2 \times \sum_{n=0}^{\infty} \frac{W_n \exp[-q_n(x_D - 1)] \sin^2(\epsilon_n)}{\{1 + Aq_n \tanh[q_n(x_{LD} - 1)]\} p\epsilon_n[\epsilon_n + 0.5 \sin(2\epsilon_n)]}. \tag{33}$$

Eqs. (29)–(33) are general solutions for all of the water-table aquifer types. For example, for a semi-infinite, water-table aquifer with no semipervious streambank material, $W_n = 1$ and $A = 0$. Under these conditions, and the additional condition in which the head is measured in a fully penetrating observation well, Eq. (33) becomes

$$\hat{\bar{h}}_D = 2\sum_{n=0}^{\infty} \frac{\exp[-q_n(x_D - 1)] \sin^2(\epsilon_n)}{p\epsilon_n[\epsilon_n + 0.5 \sin(2\epsilon_n)]}. \tag{34}$$

Eq. (34) reduces to the Laplace transform step-response function for a confined aquifer (Eq. (15)) if

Table 3
Parameters and dimensions of the hypothetical confined and leaky aquifers

Parameter	Value
Aquifer	
Hydraulic conductivity (K)	200 ft d^{-1} (7.1×10^{-4} m s^{-1})
Specific storage (S_s)	1×10^{-5} ft^{-1} (3.3×10^{-5} m^{-1})
Thickness (b)	25 ft (7.6 m)
Width of aquifer[a] (x_L)	500 ft (152.4 m)
Distance from middle of stream to stream–aquifer boundary (x_0)	25 ft (7.6 m)
Aquitard[b]	
Vertical hydraulic conductivity (K')	2 ft d^{-1} (7.1×10^{-6} m s^{-1})
Specific storage (S_s')	1×10^{-4} ft^{-1} (3.3×10^{-4} m^{-1})
Specific yield[c] (S_y')	2.5×10^{-1}
Thickness or saturated thickness (b')	25 ft (7.6 m)

[a] For finite-width aquifers.
[b] For leaky aquifers.
[c] For leaky aquifers overlain by a water-table aquitard.

specific yield is zero (see Barlow and Moench (1998), Attachment 1).

The Laplace transform solution for seepage between the stream and aquifer can be determined by finding the gradient of the step-response function at the stream–aquifer boundary (i.e. at $x_D = 1$). This gradient is found by differentiation of Eq. (33) with respect to x_D and evaluation of the resulting solution at $x_D = 1$

$$\bar{Q}_D = -\left.\frac{d\hat{\bar{h}}_D}{dx_D}\right|_{x_D=1}, \qquad (35)$$

where $\bar{Q}_D$ is dimensionless seepage in the Laplace domain. The general solution for dimensionless seepage at the streambank is

$$\bar{Q}_D = -2\sum_{n=0}^{\infty}\frac{q_n \sin^2(\epsilon_n)}{\{1 + Aq_n \tanh[q_n(x_{LD}-1)]\}p\epsilon_n[\epsilon_n + 0.5\sin(2\epsilon_n)]}\left\{\frac{\exp[-2q_n(x_{LD}-1)] - 1}{\exp[-2q_n(x_{LD}-1)] + 1}\right\}. \qquad (36)$$

3. Evaluation of analytical solutions for step input

In this section, the analytical solutions are evaluated for hypothetical confined, leaky, and water-table aquifers for a unit-step increase (input) in the elevation of stream stage relative to that of piezometric head in the adjoining aquifer. The evaluation demonstrates the influence of aquifer type, aquifer extent, and aquifer and streambank hydraulic properties on ground-water heads and seepage rates. The solutions also are compared graphically to several previously published solutions.

Changes in ground-water heads are related to a unit-step increase by (Tables 1 and 2)

$$h_i - h(x,t) = -h_D c, \qquad (37)$$

where c is the step increase in water level of the stream relative to the water level in the aquifer, h_D is determined by numerical inversion of the chosen expression for dimensionless head, and the negative sign in introduced in Eq. (37) so that changes in ground-water heads are positive for a rise in stream stage. The unit-step increase is equal to 1.0 ft (0.3 m). English units are used in this and the companion paper and SI-unit equivalents are given in parentheses.

Dimensional seepage rates are determined from Eq. (17) or Eq. (35), Darcy's law, and the definition of h_D and x_D given in Table 1

$$Q(t) = \frac{K_x bc}{x_0} Q_D, \qquad (38)$$

where $Q(t)$ is seepage rate per unit stream length at time t, and Q_D is the dimensionless seepage in the real-time domain. For confined and leaky aquifers, K_x is replaced by K.

3.1. Confined and leaky aquifers

Parameters and dimensions of the hypothetical confined and leaky aquifers and overlying aquitards used in the evaluation are shown in Table 3. Changes in ground-water heads were calculated at a hypothetical observation well 100 ft (30.5 m) from the middle of the stream, which is 75 ft (22.9 m) from the stream–aquifer boundary.

Figs. 6 and 7 show changes in ground-water heads

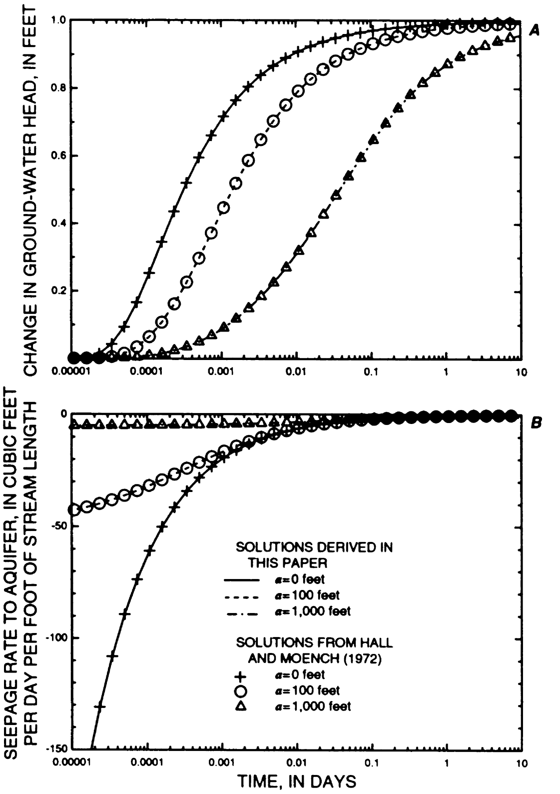

Fig. 6. (A) Change in ground-water head; and (B) seepage rate to aquifer, for 1-foot (0.3 m) increase in stream stage, semi-infinite confined aquifer with and without semiprevious streambank material. (1.0 ft = 3.1×10^{-1} m; 1.0 $ft^3\ d^{-1}\ ft^{-1} = 1.1\times10^{-6}\ m^3\ s^{-1}\ m^{-1}$).

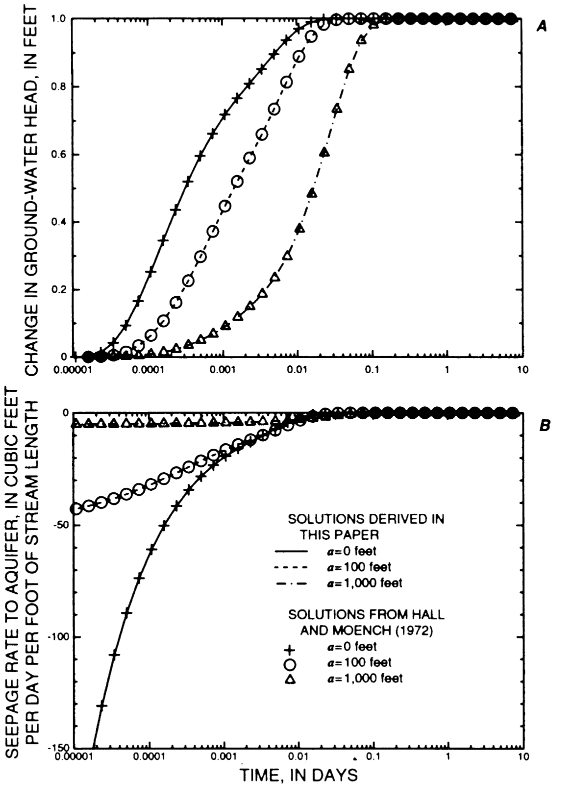

Fig. 7. (A) Change in ground-water head; and (B) seepage rate to aquifer, for 1-foot (0.3 m) increase in stream stage, finite-width confined aquifer with and without semipervious streambank material. (1.0 ft = 3.1×10^{-1} m; 1.0 $ft^3\ d^{-1}\ ft^{-1} = 1.1\times10^{-6}\ m^3\ s^{-1}\ m^{-1}$).

and seepage rates for a semi-infinite and finite-width confined aquifer, respectively, with and without semi-pervious streambank material. Heads and seepage rates were calculated by use of the Laplace transform step-response functions and by use of the real-time domain solutions given by Hall and Moench (1972) for the same parameters and dimensions shown in Table 3. Negative seepage rates indicate that water flows from the stream to the adjoining aquifer. Results for two streambed-leakance values are shown in the figures, $a = 100$ ft (30.5 m) and $a = 1000$ ft (304.8 m). Comparisons (in Figs. 6 and 7) between the inversion of the Laplace transform solutions and real-time domain solutions of Hall and Moench (1972) for both heads and seepage rates for all of the semi-infinite and finite-width aquifer conditions show no discernible difference.

Both sets of head solutions asymptotically approach the unit-step stream-stage increase of 1.0 ft (Figs. 6A and 7A). Initially, for $a = 0$, seepage rates from the stream to the adjoining aquifer are large (Figs. 6B and 7B). With increased time, ground-water heads near the stream approach the stream-stage level and, as a result, hydraulic gradients and seepage rates at the stream–aquifer boundary approach zero. The inclusion of a streambank leakance term delays the increase in ground-water heads at the observation well and reduces seepage rates to the aquifer at early-time periods. As the streambank leakance term is increased, seepage rates at the stream–aquifer interface are greatly diminished by the increased hydraulic resistance at the streambank.

The response of semi-infinite and finite-width

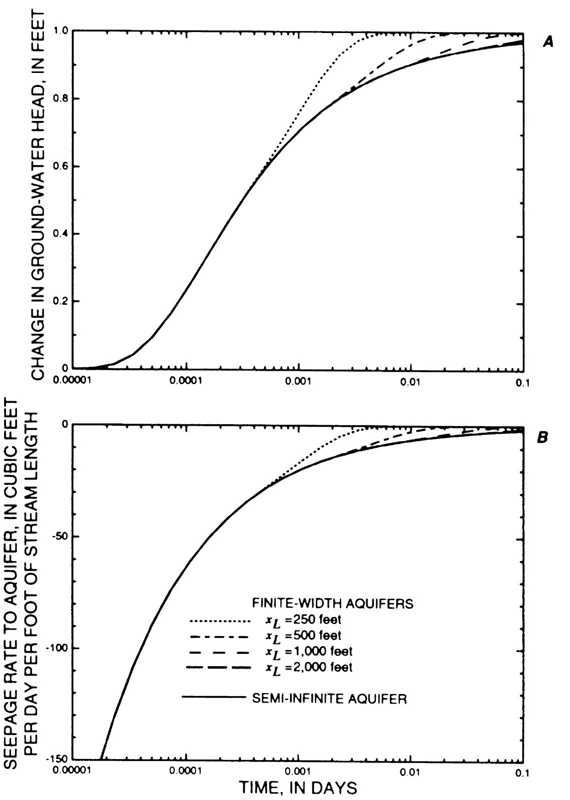

Fig. 8. (A) Change in ground-water head; and (B) seepage rate to aquifer, for 1-foot (0.3 m) increase in stream stage, finite-width and semi-infinite confined aquifers. (1.0 ft $= 3.1\times10^{-1}$ m; 1.0 ft^3 d^{-1} ft^{-1} $= 1.1\times10^{-6}$ m^3 s^{-1} m^{-1}).

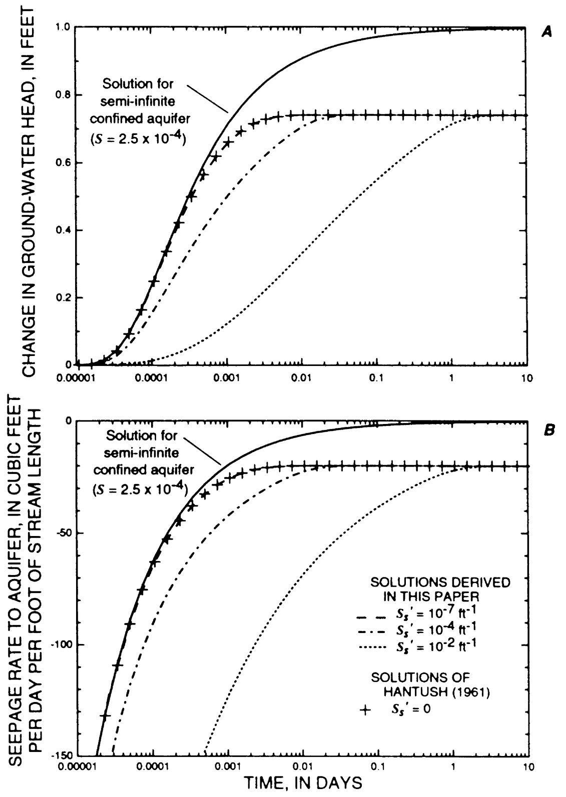

Fig. 9. (A) Change in ground-water head; and (B) seepage rate to aquifer, for 1-foot (0.3 m) increase in stream stage, semi-infinite leaky aquifer with constant head overlying the aquitard. (1.0 ft $= 3.1\times10^{-1}$ m; 1.0 ft^3 d^{-1} ft^{-1} $= 1.1\times10^{-6}$ m^3 s^{-1} m^{-1}).

confined aquifers without semipervious streambank material are compared for several values of aquifer width in Fig. 8. At early-time periods (less than about 4×10^{-4} days), the semi-infinite and finite-width aquifers respond similarly. At later times, the narrower aquifers (x_L small) cause ground-water heads to rise more quickly and seepage rates to approach zero more rapidly than do those for the wider aquifers (x_L large) because of the overall smaller storage capacity available in the narrower aquifers. As the width of the finite-width aquifer increases, the responses approach the responses for the semi-infinite aquifer solutions.

Solutions for a semi-infinite leaky aquifer with constant head overlying the aquitard (leaky aquifer case 1) without semipervious streambank material are shown in Fig. 9 for several values of the specific storage of the aquitard (S_s'). Also shown in the figure are the solutions for a semi-infinite confined aquifer with storativity (S) of 2.5×10^{-4}. Each of the leaky aquifer solutions asymptotically approaches a constant (steady-state) value of ground-water head that is smaller, and a constant rate of seepage that is larger, than the confined aquifer solutions. These result from the constant-head boundary condition that overlies the aquitard and provides an infinite source (or sink) of ground-water storage to the aquifer/aquitard system. The figure shows that the response of the leaky aquifer system is delayed relative to the confined aquifer, and that the delay is increased as the specific storage of the aquitard increases. The real-time domain solutions of Hantush (1961) for similar leaky aquifer conditions

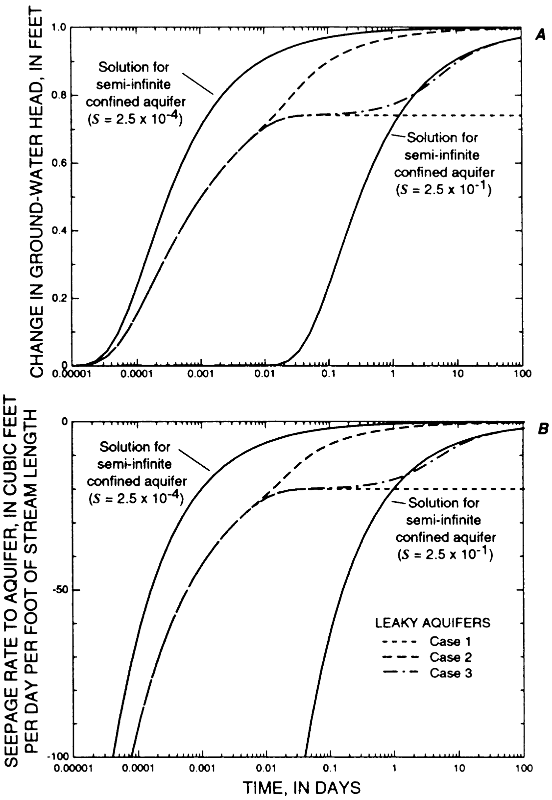

Fig. 10. (A) Change in ground-water head; and (B) seepage rate to aquifer, for 1-foot (0.3 m) increase in stream stage, semi-infinite leaky aquifers. (1.0 ft $= 3.1\times10^{-1}$ m; 1.0 $ft^3\ d^{-1}\ ft^{-1} = 1.1\times10^{-6}\ m^3\ s^{-1}\ m^{-1}$).

are also shown in Fig. 9. Hantush's solutions do not consider storage in the aquitard; consequently, those solutions are equivalent to the solutions presented in this paper only when the specific storage of the aquitard is unreasonably small, such as the value of $1.0\times10^{-7}\ ft^{-1}$ ($3.3\times10^{-7}\ m^{-1}$) shown in the figure.

Solutions for all three types of leaky aquifers (case 1, case 2, and case 3) without semipervious streambank material are compared in Fig. 10. Also shown in the figure are solutions for a semi-infinite confined aquifer with storativity of 2.5×10^{-4} and 2.5×10^{-1}. These two storativities are limiting values for the confined/leaky systems modeled here: the value 2.5×10^{-4} is that of the confined aquifer (no aquitard) and the value 2.5×10^{-1} equals the specific yield of the water-table aquitard. The leaky-aquifer head solutions quickly depart from the confined aquifer solution at early times (Fig. 10A). The solutions for the three leaky aquifer types yield identical drawdowns up to a time of about 0.01 days, when they begin to diverge from one another because of the influence of the upper boundary condition of the aquitard.

Table 4
Parameters and dimensions of the hypothetical water-table aquifer

Parameter	Value
Horizontal hydraulic conductivity (K_x)	200 ft d^{-1} ($7.1\times10^{-4}\ m\ s^{-1}$)
Vertical hydraulic conductivity (K_z)	40 ft d^{-1} ($1.4\times10^{-4}\ m\ s^{-1}$)
Specific storage (S_s)	$1\times10^{-5}\ ft^{-1}$ ($3.3\times10^{-5}\ m^{-1}$)
Specific yield (S_y)	2.5×10^{-1}
Saturated thickness (b)	25 ft (7.6 m)
Distance from middle of stream to stream–aquifer boundary (x_0)	25 ft (7.6 m)

At late time, the solutions for case 1 (aquitard overlain by constant-head boundary) asymptotically approach steady-state values of head and seepage (as also shown in Fig. 9) because of the constant-head boundary condition that overlies the aquitard. Solutions for case 2 (aquitard overlain by an impermeable boundary) asymptotically approach the confined aquifer solutions but are shifted in time relative to the confined aquifer solutions by a factor of $1 + (1/\sigma_1)$. The shift is analogous to that which occurs in flow to a well in leaky aquifers (see Moench, 1985, p. 1129). The leaky aquifer solutions for case 2 approach the confined aquifer solutions because the impermeable boundary condition at the top of the aquitard prevents any additional source (or sink) of leakage to the aquifer at late time.

Solutions for case 3 (water-table aquitard) are identical to those of case 1, up to a time of about 0.1 days, because the large storage capacity provided by the water-table boundary causes the system to respond as it would to a constant-head boundary overlying the aquitard. At late times, the solutions for case 3 lie between those of cases 1 and 2, because the rate of flow into storage at the water table slows. Eventually, head changes and seepage rates for the water-table aquitard system approach those of a confined aquifer with storativity equal to the specific yield of the aquitard (2.5×10^{-1}).

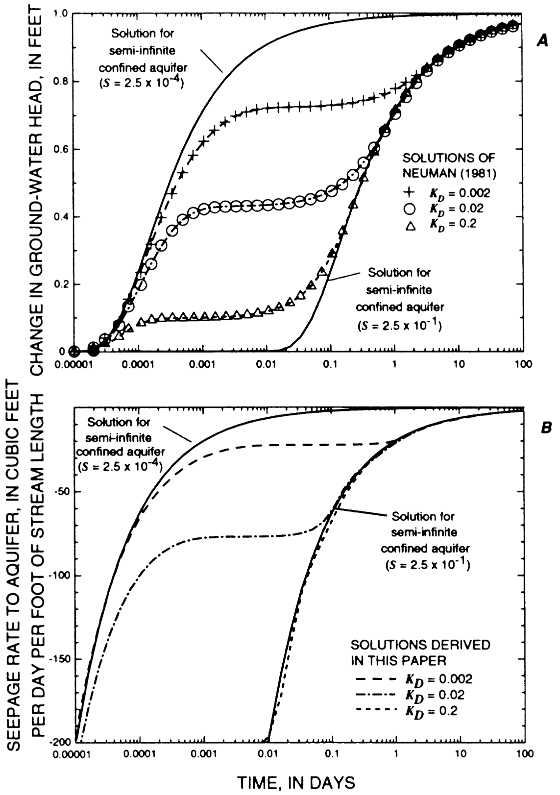

Fig. 11. (A) Change in ground-water head; and (B) seepage rate to aquifer, for 1-foot (0.3 m) increase in stream stage, semi-infinite water-table aquifer without semipervious streambank material. (1.0 ft $= 3.1 \times 10^{-1}$ m; 1.0 ft^3 d^{-1} ft^{-1} $= 1.1 \times 10^{-6}$ m^3 s^{-1} m^{-1}).

3.2. Water-table aquifers

Parameters and dimensions of the hypothetical water-table aquifer used in the evaluation are shown in Table 4. Changes in ground-water heads were calculated at a hypothetical observation well 100 ft from the middle of the stream, which is 75 ft from the stream aquifer boundary.

Fig. 11 shows changes in ground-water heads and seepage rates for a semi-infinite water-table aquifer without semipervious streambank material for three values of K_D (ratio of vertical to horizontal hydraulic conductivity), calculated using the Laplace transform step-response functions. Ground-water heads shown in the figure are the average head over the full saturated thickness of the aquifer at the hypothetical observation well. As with the confined and leaky solutions, negative seepage rates indicate that water flows from the stream to the adjoining aquifer, in response to the unit-step increase in stream stage. Also shown in the figure are the limiting solutions (see Table 4) for a semi-infinite confined aquifer with storativity of 2.5×10^{-4} and 2.5×10^{-1}.

Ground-water heads calculated using the real-time domain solution of Neuman (1981) for flow to a fully penetrating stream in a semi-infinite water-table aquifer also are shown in Fig. 11A. By making only minor modifications in the computer program DELAY2 that Neuman (1972) developed for the mathematically similar problem of flow to a well, ground-water heads were calculated from Neuman's (1981) solution for flow to a fully penetrating stream. Comparisons for the three values of K_D shown in Fig. 11A between the numerical inversion of the Laplace transform step-response function for ground-water head presented in this paper and Neuman's (1981) real-time solution show no discernible difference.

Ground-water heads in Fig. 11A for any particular value of K_D show the three characteristic segments of the response of water-table aquifers to a step change in the stream stage. Physical explanations for these three segments have been described by several authors for the case of ground-water flow to a pumped well (e.g. see discussions by Neuman (1972, 1974)), and the explanations are similar for the response of a water-table aquifer to stream-stage fluctuations. During the early-time segment, the aquifer responds as would a strictly confined aquifer with storativity equal to 2.5×10^{-4}. That is, water goes into elastic storage by expansion of the aquifer materials and compression of the pore water. Effects of vertical flow into the zone above the water table are not prevalent during the early-time segment when horizontal flow dominates. The length of time during which elastic-storage effects are prominent is increased as the ratio of vertical to horizontal hydraulic conductivity (K_D) is decreased. This is due to increased resistance to vertical flow in the aquifer, because of the smaller values of vertical hydraulic conductivity. Although not shown in Fig. 11, the length of time during which elastic-storage effects are prominent also decreases as the ratio of storativity to specific yield (σ, Table 2) decreases (Neuman, 1972).

During the intermediate-time segment, flow into

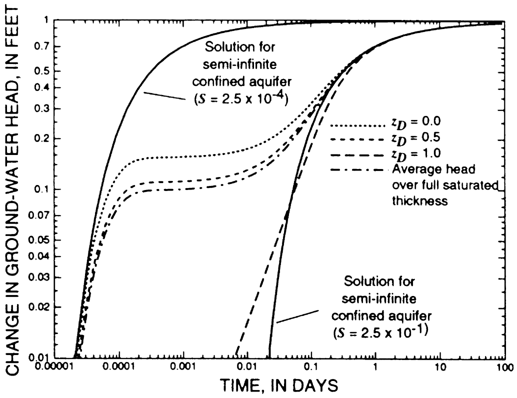

Fig. 12. Change in ground-water head for 1-foot (0.3 m) increase in stream stage at several vertical positions in a semi-infinite water-table aquifer for $K_D = 0.2$ (1.0 ft $= 3.1 \times 10^{-1}$ m; 1.0 $ft^3\ d^{-1}\ ft^{-1} = 1.1 \times 10^{-6}\ m^3\ s^{-1}\ m^{-1}$).

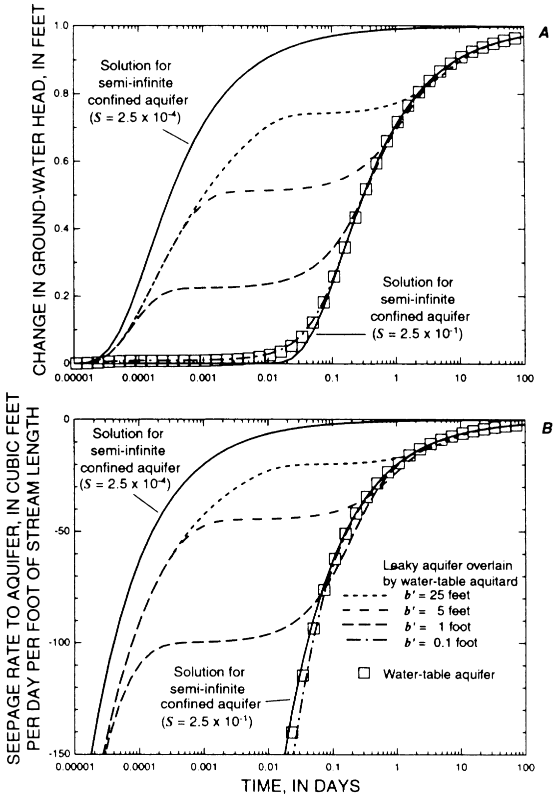

Fig. 13. (A) Change in ground-water head; and (B) seepage rate to aquifer, for 1-foot (0.3 m) increase in stream stage, semi-infinite water-table aquifer and leaky aquifer overlain by water-table aquitard. (1.0 ft $= 3.1 \times 10^{-1}$ m; 1.0 $ft^3\ d^{-1}\ ft^{-1} = 1.1 \times 10^{-6}\ m^3\ s^{-1}\ m^{-1}$).

the unsaturated zone becomes important and the rate of change of ground-water heads is slowed (Fig. 11A). The delayed response of the water table is similar to the response of the leaky aquifer systems shown in Fig. 10. Vertical-flow components are important during this segment as the water table rises. Finally, during the late-time segment, the aquifer again responds as would a strictly confined aquifer and ground-water heads converge on the solution for a confined aquifer with storativity equal to 2.5×10^{-1} (Fig. 11A), which equals the specific yield of the aquifer. Water goes into storage only by an increase in the elevation of the water table. Horizontal ground-water flow dominates during this time segment, as it did during the early-time segment.

Fig. 12 shows ground-water heads in piezometers located at three vertical positions in the aquifer and the average head over the full saturated thickness of the aquifer for $K_D = 0.2$. Vertical variations in ground-water heads over the saturated thickness of the aquifer in this instance result in upward flow into the zone above the water table. The results shown in Fig. 12 are similar to those presented by Neuman (1972, Fig. 4, p. 1037) for the case of ground-water flow to a well. Ground-water heads below the water table ($z_D < 1.0$) respond quickly to the change in head at the stream–aquifer boundary as a result of elastic storage of the aquifer. An equivalent head change at the water table ($z_D = 1.0$) is delayed relative to head changes deeper in the aquifer in response to saturation of the pores as the water table rises. The average head change over the thickness of the aquifer responds more quickly than that at the water table, but lags behind those for $z_D = 0.0$ and $z_D = 0.5$. At late time, because the stream is assumed to penetrate the full thickness of the aquifer, all of the curves approach the solution for the confined aquifer with storativity equal to 2.5×10^{-1}, which implies that heads are uniform over the thickness of the aquifer and that horizontal ground-water flow dominates. As noted by Neuman (1972), the convergence of the curves to the single, uniform solution is consistent with the Dupuit–Forchheimer theory of horizontal ground-water flow in a water-table aquifer. It is only after this point in time that the use of the confined aquifer solution with storativity equal to the specific yield of the aquifer is truly justified for a fully penetrating stream.

Fig. 13A shows a comparison of the response in a water-table aquifer to that of an aquifer overlain by a water-table aquitard. As noted by Boulton and Streltsova (1975) for the case of flow to a pumped well, the upper boundary condition in a water-table aquifer is the same as that in an aquifer overlain by a water-table aquitard. Hence, ground-water heads (and seepage rates) calculated for the two aquifer types should approach one another as the thickness of the water-table aquitard becomes zero. This is also true for stream–aquifer settings and is demonstrated by the results shown in Fig. 13, in which simulations are shown for several values of aquitard thickness (see Table 3 for the flow parameters) and a single simulation for the water-table aquifer (Table 4) in which $K_D = 1.0$. As shown in the figure, ground-water heads and seepage rates for the water-table aquitard condition approach those of the water-table aquifer as the thickness of the aquitard is reduced from 25 ft (7.6 mm) to 0.1 ft (0.03 m).

4. Summary

Laplace transform step-response functions are presented for several cases of transient, hydraulic interaction between a fully penetrating stream and a confined, leaky, or water-table aquifer. The various aquifers (confined, leaky, or unconfined) may be semi-infinite or finite in width and may or may not be connected with the stream through a semipervious streambank. The solutions are based on the governing differential equation of transient ground-water flow in a saturated, homogeneous, slightly compressible, and anisotropic (for water-table) aquifer. All solutions are based on the condition of an instantaneous step change in stream stage. They are equally applicable to the condition of an instantaneous regional rise or decline in the altitude of the water table or piezometric surface caused by area-wide recharge, irrigation, or evapotranspiration. The one-dimensional solutions for confined and leaky aquifers are presented in a format that combines all aquifer configurations in a single expression for which the appropriate source term ($\bar{q}_D$) is chosen. The source term parameter differs depending upon the upper boundary condition for the aquitard. The two-dimensional solutions for the response of a water-table aquifer are presented in a similar format.

Of primary interest are the expanded solutions for water-table aquifers and for leaky aquifers overlain by water-table aquitards. The general aspects of the response of water-table aquifers and water-table aquitards to changes in the water level of a bounding stream are similar to those that occur in response to the withdrawal or injection of ground water from a well pumping from a water-table aquifer or leaky aquifer overlain by a water-table aquitard; thus, conclusions drawn in this study for these aquifer types are similar to previous investigations in the field of well hydraulics.

Each of the stream–aquifer systems modeled in this paper derive from linear partial differential equations of ground-water flow and by linear boundary conditions. The assumed linearity of the systems allows for use of the convolution integral, which is the subject of the companion paper by Barlow et al. (2000).

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