An Investigation of Wellbore Storage and Skin Effect in Unsteady Liquid Flow: I. Analytical Treatment

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ABSTRACT

Due to the cost of extended pressure-drawdown or buildup well tests and the possibility of acquisition of additional information from well tests, the modern trend has been toward development of well-test analysis methods pertinent for short-time data. "Short-time" data may be defined as pressure information obtained prior to the usual straight-line portion of a well test. For some time there has been a general belief that the factors affecting short-time data are too complex for meaningful interpretations. Among these factors are wellbore storage, various skin effects such as perforations, partial penetration, fractures of various types, the effect of a finite formation thickness, and non-Darcy flow. A number of recent publications have dealt with short-time well-test analysis. The purpose of this paper is to present a fundamental study of the importance of wellbore storage with a skin effect to short-time transient flow. Results indicate that proper interpretations of short-time well-test data can be made under favorable circumstances.

Upon starting a test, well pressures appear controlled by wellbore storage entirely, and data cannot be interpreted to yield formation flow capacity or skin effect. Data can be interpreted to yield the wellbore storage constant, however. After an initial period, a transition from wellbore storage control to the usual straight line takes place. Data obtained during this period can be interpreted to obtain formation flow capacity and skin effect in certain cases. One important result is that the steady-state skin effect concept is invalid at very short times. Another important result is that the time required to reach the usual straight line is normally not affected significantly by a finite skin effect.

INTRODUCTION

Many practical factors favor short-duration well testing. These include loss of revenue during shut-in, costs involved in measuring drawdown or buildup data for extended periods, and limited availability of bottomhole-pressure bombs where it is necessary to survey large numbers of wells. On the other hand, reservoir engineers are well aware of the desirability of running long-duration tests. The result is usually a compromise, and not necessarily a satisfactory one. This situation is a common dilemma for the field engineer who must specify the details of special well tests and annual surveys, and interpret the results. For this reason, much effort has been given to the analysis of short-time tests. The term "short-time" is used herein to indicate either drawdown or buildup tests run for a period of time insufficient to reach the usual straight-line portion. Nevertheless, it is rare that either buildup or drawdown data taken before the traditional straight-line portion are ever used in analysis of oil or gas well performance. Well files often contain well-test data that were abandoned when it was realized that the straight line had not been reached. This situation is particularly odd when it is realized that early data *are* used commonly in other technologies which employ similar, or analogous, transient tests.

It is the objective of this study to investigate techniques which may be used to interpret information obtained from well tests at times prior to the normal straight-line period.

THEORY

The problem to be considered is the classic one of flow of a slightly compressible (small pressure gradients) fluid in an ideal radial flow system. That is, flow is perfectly radial to a well of radius r_w in an isotropic medium, and gravitational forces are neglected. We will consider that the medium is infinite in extent, since interest is focused on times short enough for outer boundary effects not to be felt at the well. The initial condition is taken as constant pressure, p_i, for radii greater than or equal

to r_w.

The inner boundary condition will be taken as production at constant surface rate from a wellbore of finite volume, and it will be assumed that a steady-state (zero storage capacity) skin effect exists at the sand face. This boundary condition without a skin effect was first introduced by van Everdingen and Hurst[1] and is sometimes called the wellbore storage, unloading, or afterflow problem. Later, van Everdingen[2] and Hurst[3] extended the problem to include a steady-state skin effect. Both van Everdingen and Hurst presented solutions to the problem for the special case of a line-source well. Solution, including a skin effect, was presented in the form of the real inversion integral of a Laplace transform solution, but only the long-time approximation has been published.

The inner boundary condition described is actually a special case of Jaeger's "general boundary condition" applied to unsteady heat-conduction problems.[4-6] In the "general boundary condition," Jaeger considered the cylindrical core (region between $r = 0$ and $r = r_w$) to contain a solid of perfect conductor or well-stirred fluid in which (1) heat could be generated at a constant rate per unit volume, (2) heat could be transferred to the surrounding cylindrical solid through a film resistance, and (3) a mass of the well-stirred fluid could be withdrawn at constant rate. The first edition of the book on heat conduction by Carslaw and Jaeger[6] contained a rigorous solution to a heat-conduction problem. It was analogous to the fluid-flow problem originally posed by van Everdingen and Hurst[1] for wellbore unloading without a skin effect for a finite radius well. It is interesting that the real inversion integral published by Carslaw and Jaeger has been evaluated in several publications in connection with problems other than the van Everdingen-Hurst problem.[7,8] Jaeger also evaluated the integral and presented useful long-time approximations.[6,9] In the second edition of their book on heat conduction, Carslaw and Jaeger[6] presented a review of the problem that is pertinent to the present study. We will present the fluid-flow analog briefly. The procedure is similar to the heat flow problem originally presented by Blackwell.[10]

The diffusivity equation for fluid flow in terms of dimensionless variables is

$$\frac{\partial^2 p_D}{\partial r_D^2} + \frac{1}{r_D} \frac{\partial p_D}{\partial r_D} = \frac{\partial p_D}{\partial t_D} . \quad (1)$$

The initial and outer boundary conditions are

$$p_D (r_D, 0) = 0 \quad \cdots \cdots \cdots \quad (2)$$

$$\lim_{r_D \to \infty} \left\{ p_D (r_D, t_D) \right\} = 0 ; \quad \cdots \quad (3)$$

while the inner boundary condition is

$$\bar{C} \frac{dp_{wD}}{dt_D} - \left(\frac{\partial p_D}{\partial r_D} \right)_{r_D=1} = 1 . \quad (4)$$

and

$$p_{wD} = \left[p_D - s \left(\frac{\partial p_D}{\partial r_D} \right) \right]_{r_D=1} , . \quad (5)$$

Eq. 4 states that the dimensionless wellbore unloading rate plus the dimensionless sand face flow rate must equal unity, the surface flow rate. The wellbore unloading or storage constant, $\bar{C}$, is that defined by van Everdingen and Hurst.[1] That is,

$$\bar{C} = \frac{C}{2 \pi \phi h c r_w^2} \quad \cdots \cdots \cdots \quad (6)$$

C represents the volume of wellbore fluid unloaded or stored, cc/atm. Storage may be by virtue of *either* compressibility *or* a changing liquid level.

Eq. 5 introduces a steady-state skin effect and, thus, a pressure drop at the sand face which is proportional to the sand-face flow rate. Note from Eq. 4 that

$$\frac{q_{sf}}{q} = - \left(\frac{\partial p_D}{\partial r_D} \right)_{r_D=1}$$

$$= 1 - \bar{C} \frac{dp_{wD}}{dt_D} , \quad \cdots \cdots \quad (7)$$

where q is the constant surface flow rate, and q_{sf} is the sand-face flow rate. Finally, the dimensionless flowing pressure, p_{wD}, is the same as $\bar{\bar{p}}_B(t_D)$ used previously by van Everdingen[2] and Ramey.[11] Thus, p_{wD} represents the pressure within the wellbore, while p_D represents pressures on the formation side of the skin effect.

Solution of Eqs. 1 through 5 follows readily using the Laplace transformation as shown by Blackwell.[10] We will review only portions of the solution of interest to this study. The transform of the dimensionless flowing pressure may be written as

$$L \left\{ p_{wD} \right\} =$$

$$\frac{K_o(\sqrt{p}) + s\sqrt{p} \, K_1(\sqrt{p})}{p \left[\sqrt{p} \, K_1(\sqrt{p}) + \bar{C}p \left\{ K_o(\sqrt{p}) + s \sqrt{p} \, K_1(\sqrt{p}) \right\} \right]}$$

$$\cdots \cdots \cdots \cdots \cdots \quad (8)$$

where K_0 and K_1 are the modified Bessel functions of the second kind of zero and unit orders. An identical transform was published recently by Russell.[15]

Jaeger also considered determination of the pressure distribution within the radial system, the sand face flux, and the pressure drop across the skin (within the heat conduction analog, of course). However, these quantities are only of incidental interest to the present purpose of this study. Blackwell and Jaeger presented the heat conduction analog to the following real inversion integral solution to Eq. 8:

$$p_{wD}(s,\bar{C},t_D) =$$

$$\frac{4}{\pi^2} \int_0^\infty \left[(1 - e^{-u^2 t_D}) du \div \right.$$

$$u^3 \left\{ \left[u\bar{C}J_0(u) - (1-\bar{C}su^2)J_1(u) \right]^2 \right.$$

$$\left. + \left[u\bar{C}Y_0(u) - (1-\bar{C}su^2)Y_1(u) \right]^2 \right\} \right], \quad (9)$$

where $J_0(u)$ and $J_1(u)$ are the Bessel functions of the first kind of zero and unit orders, and $Y_0(u)$ and $Y_1(u)$ are the Bessel functions of the second kind of the respective orders. Both numerical evaluations and short- and long-time approximations for the integral in Eq. 9 have been presented by a number of authors in connection with other problems. However, the relationships between them seem to have been largely overlooked. Table 1 presents a comparison of the symbols used by various authors who have presented pertinent evaluations of the the integrals or related derivations. Table 2 presents the ranges of parameters in numerical evaluations considered by various investigators.

It is apparent from Tables 1 and 2 that most evaluations of the integral in Eq. 9 have concerned the special case of a zero skin effect — no-flow resistance at the sand face. It is also apparent that,

although there has been some overlap in evaluations, there has been little duplication because various authors have been interested in different ranges of storage constant, $\bar{C}$, values. Note also that the special case of zero storage leads to the well-known constant rate solution. (Jaeger a and Hurst σ values of infinity). Jaeger[9] has presented the most complete information for finite skin effect values, but the range of both dimensionless times and storage constants is far too limited to be of use in well-test analysis.

Another related solution has appeared in the petroleum literature.[1-3] By using the superposition principle and representing the well as a continuous line source, the Laplace transform of the dimensionless well pressure may be shown to be:

$$L\left\{p_{wD}\right\} = \frac{K_0(\sqrt{p}) + s}{p\left[1 + \bar{C}p K_0(\sqrt{p}) + s\bar{C}p\right]} . \quad (10)$$

This same result may be obtained from Eq. 8 by noting that the product $[\sqrt{p} K_1(\sqrt{p})]$ approaches unity as the argument p becomes smaller. Thus Eq. 10 is a long-time approximation for Eq. 8.

Eq. 10 produces the real inversion integral:

$$p_{wD}(s,\bar{C},t_D) =$$

$$\int_0^\infty \left[(1 - e^{-u^2 t_D}) J_0(u) du \div \right.$$

$$u\left\{ \left[1 - u^2\bar{C}s + \frac{1}{2}\pi u^2\bar{C}Y_0(u) \right]^2 \right.$$

$$\left. + \left[\frac{1}{2}\pi \bar{C}u^2 J_0(u) \right]^2 \right\} \right] \cdot \cdots \cdots (11)$$

TABLE 1 — CORRELATION OF VARIOUS DIMENSIONLESS GROUPS USED BY OTHER INVESTIGATORS

This Study	Blackwell[10]	Jaeger[6,9]	Lesem et al.[7]	Hurst[8]
$p_{wD}(s,\bar{C},t_D)$	$\theta_1(t)$	$2\pi G(h,a,\tau)$	$\frac{4k^2}{\pi^2}I_1$	$\sigma N(\sigma,t_D)$
$\bar{C}$	$\frac{1}{2}a$	$1/a$	$1/k$	$1/\sigma$
t_D	T, t	τ	θ	t_D
s	$1/H$	h	$s = 0$	$s = 0$

Note: For the fluid flow analog of Blackwell's study, h_2, b, K_2 and Q' were all taken as unity (in his symbolism). Both van Everdingen[2] and Hurst[3] used symbolism similar to this study in their line-source approximation, except $\bar{p}$ was used in place of p_{wD}.

TABLE 2 — RANGES OF PARAMETERS CONSIDERED BY VARIOUS INVESTIGATORS IN NUMERICAL EVALUATIONS

	s	$\bar{C}$	t_D	Comments
Blackwell[10]				Analytical study only*
Jaeger[6,9]	0	1/8, 1/6, 1/4, 1/2, 2/3, 1, 2	0.2–2.0	Tabular form*
	0, 1/2, 1, 2, 3, 4, 5, 7, 10, 20	1/2, 1	0.2–20	Graphical form*
Lesem et al.[7]	0	1/6, 1/9, 1/12, 1/15, 1/18, 1/21, 1/24, 1/27, 1/30	1 – 1,000	Graphical form.
Hurst[8]	0	2.5, 5, 10, 12.5, 16-2/3, 25, 50	10^{-2}–10^4	Graphical form
van Everdingen and Hurst[1]	0	1,000 – 50,000	10^3–10^7	Line-source well approximation, graphical**

*Both short- and long-time approximating forms given.
**Long-time approximating form given.

Eq. 11 was evaluated numerically and presented in graphical form for the special case of zero skin effect by van Everdingen and Hurst[1] and later by Chatas.[12] Storage constant, $\bar{C}$, range was 1,000 to 50,000 in Ref. 1 and 1,000 to 75,000 in Ref. 12.

Originally, both Blackwell and Jaeger solved Eqs. 1 through 5 in the classical manner. That is, the Laplace transforms of the equations were taken and the resulting equations solved simultaneously to provide the transform space solution given by Eq. 8. Although Jaeger mentioned that classical super-position could have been used, he did not employ this essentially simpler technique; van Everdingen and Hurst did employ superposition in their development of Eq. 10. Thus, one solution leading to Eq. 8 can be developed employing superposition:

$$p_{wD}(t_D) =$$

$$\int_0^{t_D} \left[1 - \bar{C}\frac{dp_{wD}(t_D')}{dt_D'} \right]\left[\frac{dp_D}{dt_D}(t_D - t_D')\right]$$

$$dt_D' + s\left[1 - \bar{C}\frac{dp_{wD}(t_D)}{dt_D} \right] \quad . . \quad (12)$$

Throughout Eq. 12, the dimensionless time within brackets simply indicates that the preceding term is a function of time. The prime mark indicates a variable of integration. Eq. 12 is an integro-differential equation of the convolution type. The Laplace transform can be taken directly. If the constant-rate line source is used to obtain the Laplace transform needed for the second term in brackets in the integral, Eq. 10 results. If the constant-rate, finite radius cylinder transform (Ref. 1) is substituted, Eq. 8 results. This approach is enlightening because the true nature of the wellbore storage problem is indicated clearly by Eq. 12. Reference to Eq. 7 shows that the first term within brackets in the integral in Eq. 12 is the sand-face production rate at any given time. Thus, Eq. 12 expresses the wellbore pressure caused by a changing production rate which results from the wellbore storage condition. More will be said about this point later.

Both Jaeger[9] and Blackwell[10] presented short- and long-time approximate inversions for Eq. 8. Changing their solutions into the fluid-flow nomenclature, the long-time approximation is:

$$p_{wD}(s,\bar{C},t_D) = \frac{1}{2}\left\{ \ln 4t_D \right.$$

$$- \gamma + 2s + \frac{1}{2t_D}\left[\ln 4t_D - \gamma + 1 \right.$$

$$\left. -2\bar{C}(\ln 4t_D - \gamma + 2s) \right] + O(t_D^{-2})\left. \right\}$$

$$. . . . . (13)$$

Short-time approximations are

$$p_{wD}(s,\bar{C},t_D) = \frac{1}{\bar{C}}\left\{ t_D - \frac{t_D^2}{2\bar{C}s} \right.$$

$$\left. + \frac{8\, t_D^{5/2}}{15\sqrt{\pi}\ \bar{C}s^2} + O(t_D^3) \right\} \quad ; \quad s,\bar{C} \neq 0$$

$$. . . . . (14)$$

$$p_{wD}(0,\bar{C},t_D) = \frac{1}{\bar{C}}\left\{ t_D - \frac{4\, t_D^{3/2}}{3\,\bar{C}\sqrt{\pi}} \right.$$

$$\left. + O(t_D^2) \right\} \quad ; \quad s = 0,\ \bar{C} \neq 0 \quad . . . \quad (15)$$

where γ is Euler's constant 0.57722. The well known constant rate solution (no storage or skin effect) was shown by van Everdingen and Hurst[1] to approach the following form at long times:

$$p_D(t_D) \approx \frac{1}{2}\left[\ln(4t_D) \right.$$

$$\left. - \gamma \right], \quad t_D > 100 \quad . . \quad (16)$$

Comparison of Eqs. 13 and 16 indicates that at very long times

$$p_{wD}(s,\bar{C},t_D) \approx p_D(t_D) + s \cdot \quad (17)$$

as was concluded by van Everdingen[2] and Hurst.[3] This can be seen a little easier if Eq. 13 is rearranged and the substitution for $p_D(t_D)$ made from Eq. 16:

$$p_{wD}(s,\bar{C},t_D) =$$

$$\left[p_D(t_D) + s \right]\left[1 + \frac{1}{2t_D} - \frac{\bar{C}}{t_D} \right]$$

$$- \frac{(2s-1)}{4t_D} + \frac{1}{2}O(t_D^{-2}), \quad t_D > 100$$

$$. . . . . (18)$$

In this form, several interesting features of the long-time solution can be recognized. First, the term $[1/(2t_D)]$ within brackets will always be negligible for t_D greater than 100. Terms of the order of t_D^{-2} will be negligible. Ramey[11] pointed out that the effect of wellbore storage essentially dies for zero skin effect cases by a dimensionless time of

$$t_D \geq 60\,\bar{C} \quad . . . . \quad (19)$$

It is clear that this approximation will also hold for finite skins (either positive or negative in sign) for all practical purposes. That is, when Eq. 19 is valid, the term $(\overline{C}/t_D)$ is only 0.017 as compared to unity, and the next to last term involving "s" will always be negligible compared to s. Of course, it is *not* apparent that Eq. 18 is a valid approximation for times specified by Eq. 19. This can only be established by comparison of results from Eq. 18 with the rigorous solution provided by Eq. 9. This comparison will be discussed later. Jaeger[9] offered two interesting limiting forms correct for any value of time:

$$p_{wD}\ (s,\overline{C},t_D)\ =\ \frac{t_D}{\overline{C}},\ \text{for}\ \ s\ =\ \infty$$
$$\dots \dots (20)$$

and

$$p_{wD}\ (s,\overline{C},t_D)\ =\ p_D(t_D)\ +\ s,$$

$$\text{for}\ \ \overline{C}\ =\ 0 \ \dots \ (21)$$

Eq. 20 would represent the condition of the sand face completely blocked to flow, or depletion of the wellbore volume only. Note that both short-time approximating forms (Eqs. 14 and 15), contain $t_D/\overline{C}$ as the first term of the series. Eq. 21 presents constant-rate production with a skin effect but no storage. This solution is in fact the basis for most of the pressure buildup and drawdown testing commonly used today. There is one aspect of this solution which has not been discussed. Since $p_D(t_D)$ approaches zero as time approaches zero, Eq. 21 indicates that there would be a finite pressure difference between the formation and the wellbore as time approaches zero. Although the solution is mathematically correct, this condition does not represent the physical fluid-flow problem. That is,

$$\begin{array}{c} \text{lim} \\ t_D \ \rightarrow \ +\ 0 \end{array} \left\{ p_{wD}\ (s,\overline{C},t_D) \right\} = \ 0 \ . \ (22)$$

Eq. 21 indicates that the previous solutions do not satisfy Eq. 22 for the special case of zero wellbore storage. Eq. 22 *is* satisfied if the storage constant is finite. This results because the sand-face flow rate increases gradually from zero to a constant value for finite storage, but increases instantaneously from zero to a constant rate if wellbore storage (unloading) is non-existent.

The fact that solutions with finite storage constants appear to make more physical sense at short times does not indicate validity of the solutions. The problem lies in the basic definition of the skin effect as a pressure drop across an infinitesimal skin (zero storage capacity, or steady-state flow in the skin region). Obviously, real skins which affect well performance must have

some fluid storage capacity (pore space) or they could not permit fluid flow. Thus there must *always* be a period of unsteady-state flow through the skin at short times for the physical problem posed by well behavior. But nothing very general can be said about the potential duration of this transient period without specifying the nature of the skin effect. Hawkins[13] pointed out that one physical interpretation of the skin effect is an annular region of permeability k_1 and radius r_1 immediately adjacent to the well. Hawkins showed that

$$s \ = \ \left(\frac{k}{k_1}\ -\ 1 \right)\ \ln\ \left(\frac{r_1}{r_w} \right), \ \dots \ (23)$$

where k is the permeability of the formation proper. If k_1 is assumed to be infinitely greater than k (perhaps 500 times as great), Eq. 23 becomes:

$$s \ = \ -\ln\left(\frac{r_1}{r_w} \right), \ \dots \dots \dots (24)$$

or

$$r_1 \ = \ r_w\ e^{-s}\ , \ \dots \dots \dots \dots (25)$$

a relationship which is often cited to indicate that the skin effect can be interpreted simply in terms of an "effective" well radius, r_1.

The point we wish to make is that, even if an annular region of altered permeability *is* visualized as physically responsible for the appearance of a skin effect, the finite dimensions of the annular region would lead to unsteady flow within the skin at short times. This is the "composite reservoir" problem described by Jaeger[9] in connection with heat conduction and by Louckes and Guerrero[14] in connection with fluid flow.

The steady-state skin effect concept is a useful one, and it has been established in previous studies that flow in the region near the well does become steady a short time after production begins.

However, there still is a basic problem. As Eq. 23 indicates, there is an infinite number of pairs of values (k_1, r_1) that will produce the same constant skin effect, s. Furthermore, there is undoubtedly an infinite variety of conditions near a wellbore which can also lead to the appearance of a skin effect — whether positive or negative in sign. The problem regarding the variety of annular configurations which can produce a specific skin effect value is not quite as bad as it first appears. This problem is discussed by Wattenbarger and Ramey.[22] It is beyond the purpose of this paper to pursue further the problems involved in the skin-effect concept. It is important that the inherent nature of the skin effect be realized so that appropriate methods will be used in well-test analysis.

Finally, Jaeger[9] also presented the solution to a heat-conduction problem analogous to that of the fluid-flow problem. The solution involved the

sudden opening of a bottom-hole choke in a well where the tubing had been pressured initially and the wellhead valve closed. A skin effect was included; and Jaeger's solution would permit estimation of the dissipation of the wellbore pressure to the formation. Although this problem has no common useful significance in well-test analysis, the solution is related to the time derivative of $p_{wD}(t_D)$; thus can be used to compute the wellbore unloading flow rate, and consequently the sand-face flow rate for the subject problem of this paper. Moreover, there is a useful physical interpretation. *We can picture the flow rate variation caused by the wellbore storage problem to be the superposition of the depletion of well casing fluid to a formation upon the behavior of a well produced at constant rate.*

Converting Jaeger's solution to the fluid-flow nomenclature of the subject problem and multiplying by appropriate constants yields:

$$\overline{C}\ \frac{dp_{wD}}{dt_D} = \frac{4\overline{C}}{\pi^2} \int_0^\infty \left[e^{-u^2 t_D}\, du \quad \div \right.$$

$$\left. u\left\{ \left[u\overline{C}J_0(u)-(1-\overline{C}su^2)J_1(u)\right]^2 + \left[u\overline{C}Y_0(u)-(1-\overline{C}su^2)Y_1(u)\right]^{\,2}\right\}\right] . \quad (26)$$

Reference to Eq. 7 indicates that Eq. 26 provides the *fraction of the total surface flow rate produced from the casing at any time* (and thus the sand-face flow rate by difference). It is clear that Eq. 26 is simply the product of $\overline{C}$ and the *time derivative of Eq. 9.* Of greater interest, Jaeger provided short- and long-time approximating forms, as well as limited tabulations and graphical solutions, for Eq. 26:

For short time and skin effect, s, not zero:

$$\overline{C}\ \frac{dp_{wD}}{dt_D} = 1 - \frac{t_D}{\overline{C}s} + 0(t_D^{\frac{3}{2}}),\ \overline{C} \neq 0$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (27)$$

For short time and skin effect, s, of zero:

$$\overline{C}\ \frac{dp_{wD}}{dt_D} = 1 - \frac{2}{\overline{C}}\sqrt{\frac{t_D}{\pi}} + \frac{t_D}{\overline{C}}\left(\frac{1}{\overline{C}} - \frac{1}{2}\right)$$

$$+ 0(t_D^{\frac{3}{2}}),\ \overline{C} \neq 0 \quad . \quad . \quad (28)$$

For long times,

$$\overline{C}\ \frac{dp_{wD}}{dt_D} = \frac{\overline{C}}{2t_D} + \frac{\overline{C}^2}{2t_D^2} (2s-1) - \frac{\overline{C}(1-2\overline{C})}{4t_D^2}$$

$$\cdot \left[\ln(4t_D) - \gamma\right] + 0(\frac{\ln t_D}{t_D^3}) \cdot \cdot \ (29)$$

Eq. 29 may also be written in terms of the $p_D(t_D)$ function as per Eq. 16:

$$\overline{C}\ \frac{dp_{wD}}{dt_D} = - \left[p_D(t_D)+s\right] \left[\frac{\overline{C}(1-2\overline{C})}{2t_D^2}\right]$$

$$+ \frac{\overline{C}}{2t_D^2} \left[t_D-\overline{C}+s\right] + 0(\frac{\ln t_D}{t_D^3}),\ t_D \geq 100$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (30)$$

Comparison of Eqs. 26 through 29 with Jaeger's Eqs. 6 through 10 indicates that:[9]

$$\overline{C}\ \frac{dp_{wD}}{dt_D} = F(h, \alpha, \tau) \cdot \cdot \ (31)$$

where $F(h, \alpha, \tau)$ is a function evaluated by Jaeger, and h, α, and τ have the same significance as indicated in Table 1.

DISCUSSION

EVALUATION OF INTEGRALS

In order to determine useful engineering approaches to interpretation of well-flow tests, the real inversion integral presented as Eq. 9 was evaluated numerically for a range of values of time, storage constant, and skin effect. The line-source approximation given by Eq. 11 was also evaluated for a range of conditions to determine the validity of the approximation for ranges of parameters met in well testing. This information is presented in Tables 3 through 8.

Evaluations were made with a high-speed digital computer using "Richardson's Deferred Approach to the Limit",[20] which is a *modified form of Simpson's rule.* Integrations were performed repeatedly, reducing the Simpson's rule interval until values of the integral agreed to within 10^{-8}. Results were also obtained by a more sophisticated method of numerical integration, the Romberg[21] method, to provide a check of the computed results. It is not possible to specify rigorously the accuracy of integration, but it is believed that results are good to at least three significant figures.

RESULTS

Tables 3 and 4 present results obtained from Eq. 9 for the cylindrical source. Table 3 presents results for a skin effect of zero, while Table 4 presents

results for a skin effect of +20. Tables 5 through 8 present results obtained from Eq. 11 for the line-source well, for skin effects of zero, +5, +10 and +20. Comparison of Tables 3 and 5 and 4 and 8 indicates that the results for the cylinder source and line source agree within 0.5 percent for all times, wellbore storage constants and skin effects considered. It is common procedure to produce short- and long-time approximations such as those given by Eqs. 13 through 15. In general, it is *not* possible to establish the applicable ranges of the approximations unless a comparison is made with the general solution. Eq. 11 is, in a sense, a long-time approximation for Eq. 9. We have just shown that it is an exceedingly good approximation over the ranges of parameters of interest in well testing. Short-time approximations such as Eqs. 14 and 15 are usually given in ascending powers of the dimensionless time and, strictly speaking, converge only for values of the dimensionless time less than unity. But comparison with the general solution often will indicate that the first few terms may provide an excellent approximation for a much greater time

TABLE 3 — $p_{wD}(s, \bar{C}, t_D)$ VS t_D FOR $s = 0$, CYLINDRICAL SOURCE WELL, $p_{wD}(0, \bar{C}, t_D)$ FOR $\bar{C}$ OF

t_D	10^2	10^3	10^4	10^5
100	0.7975	0.09763	0.00998	0.00100
200	1.3724	0.1919	0.01992	0.00200
500	2.4357	0.4585	0.04956	0.00500
1,000	3.2681	0.8585	0.0984	0.00999
2,000	3.9274	1.5298	0.1944	0.01995
5,000	4.5585	2.8832	0.4697	0.0497
10,000	4.9567	4.0328	0.8925	0.0989
20,000	5.3288	4.9350	1.6275	0.1958
50,000	5.8027	5.6762	3.2109	0.4765
100,000	6.1548	6.0940	4.6773	0.9141
200,000	6.5043	6.4736	5.8871	1.6931
500,000	6.9643	6.9515	6.7895	3.4571
1,000,000	7.3116	7.3049	7.2309	5.2164
2,000,000	7.6585	7.6550	7.6185	6.7731
5,000,000	8.1168	8.1154	8.1004	7.8983
10,000,000	8.4635	8.4627	8.4550	8.3701
20,000,000	8.8101	8.8097	8.8057	8.7663
50,000,000	9.2683	9.2681	9.2664	9.2523
100,000,000	9.6149	9.6148	9.6139	9.6082

TABLE 4 — $p_{wD}(s, \bar{C}, t_D)$ VS t_D FOR $s = +20$, CYLINDRICAL SOURCE WELL, $p_{wD}(20, \bar{C}, t_D)$ FOR $\bar{C}$ OF

t_D	10^2	10^3	10^4	10^5
100	0.9777	0.09978	0.01000	0.00100
200	1.9132	0.1991	0.01999	0.00200
500	4.4900	0.4946	0.0499	0.00500
1,000	8.1220	0.9787	0.0998	0.0100
2,000	13.479	1.9172	0.1992	0.0200
5,000	21.102	4.5125	0.4948	0.0500
10,000	24.241	8.1987	0.9797	0.0998
20,000	25.186	13.710	1.9209	0.1993
50,000	25.758	21.786	4.5333	0.4953
100,000	26.134	25.271	8.2698	0.9810
200,000	26.494	26.324	13.925	1.9252
500,000	26.960	26.907	22.443	4.5545
1,000,000	27.310	27.284	26.286	8.3394
2,000,000	27.657	27.645	27.460	14.133
5,000,000	28.116	28.112	28.055	23.085
10,000,000	28.463	28.461	28.434	27.297
20,000,000	28.810	28.809	28.795	28.606
50,000,000	29.268	29.268	29.262	29.216
100,000,000	29.615	29.615	29.612	29.596

TABLE 5 — $p_{wD}(s, \bar{C}, t_D)$ VS t_D FOR $s = 0$, LINE SOURCE WELL, $p_{wD}(0, \bar{C}, t_D)$ FOR $\bar{C}$ OF

t_D	10^2	10^3	10^4	10^5
100	0.7938	0.09758	0.00998	0.00100
200	1.3671	0.1918	0.01992	0.00200
500	2.4299	0.4582	0.04956	0.00500
1,000	3.2640	0.8580	0.0984	0.00999
2,000	3.9254	1.5292	0.1944	0.01995
5,000	4.5579	2.8824	0.4697	0.0497
10,000	4.9564	4.0321	0.8925	0.0989
20,000	5.3286	4.9347	1.6274	0.1958
50,000	5.8026	5.6761	3.2108	0.4765
100,000	6.1548	6.0940	4.6772	0.9141
200,000	6.5043	6.4736	5.8870	1.6931
500,000	6.9643	6.9515	6.7894	3.4571
1,000,000	7.3116	7.3049	7.2309	5.2163
2,000,000	7.6585	7.6550	7.6185	6.7731
5,000,000	8.1169	8.1154	8.1104	7.8983
10,000,000	8.4635	8.4627	8.4550	8.3701
20,000,000	8.8101	8.8097	8.8057	8.7663
50,000,000	9.2683	9.2681	9.2664	9.2523
100,000,000	9.6149	9.6148	9.6139	9.6082

TABLE 6 — $p_{wD}(s, \bar{C}, t_D)$ VS t_D FOR $s = +5$, LINE SOURCE WELL, $p_{wD}(5, \bar{C}, t_D)$ FOR $\bar{C}$ OF

t_D	10^2	10^3	10^4	10^5
100	0.9319	0.09929	0.009993	0.00100
200	1.7512	0.1973	0.01997	0.00200
500	3.6982	0.4843	0.04984	0.00500
1,000	5.7984	0.9410	0.0994	0.00999
2,000	7.8403	1.7820	0.1977	0.01998
5,000	9.3823	3.8349	0.4863	0.0499
10,000	9.8913	6.1533	0.9480	0.0995
20,000	10.300	8.5524	1.8062	0.1979
50,000	10.792	10.436	3.9463	0.4878
100,000	11.150	11.025	6.4558	0.9536
200,000	11.502	11.445	9.1982	1.8256
500,000	11.963	11.941	11.488	4.0388
1,000,000	12.311	12.300	12.156	6.7163
2,000,000	12.658	12.652	12.589	9.7845
5,000,000	13.117	13.114	13.090	12.517
10,000,000	13.463	13.462	13.450	13.286
20,000,000	13.810	13.809	13.803	13.734
50,000,000	14.268	14.268	14.265	14.239
100,000,000	14.615	14.615	14.613	14.601

TABLE 7 — $p_{wD}(s, \bar{C}, t_D)$ VS t_D FOR $s = +10$, LINE SOURCE WELL, $p_{wD}(10, \bar{C}, t_D)$ FOR $\bar{C}$ OF

t_D	10^2	10^3	10^4	10^5
100	0.9594	0.09958	0.01000	0.00100
200	1.8463	0.1984	0.01998	0.00200
500	4.1401	0.4904	0.04990	0.00500
1,000	7.0124	0.9629	0.0996	0.0100
2,000	10.487	1.8587	0.1985	0.0200
5,000	13.852	4.2027	0.4911	0.0499
10,000	14.797	7.2010	0.9658	0.0997
20,000	15.269	10.955	1.8693	0.1986
50,000	15.781	14.811	4.2568	0.4918
100,000	16.144	15.917	7.3677	0.9683
200,000	16.499	16.413	11.382	1.8785
500,000	16.962	16.930	15.737	4.3043
1,000,000	17.311	17.295	17.031	7.5162
2,000,000	17.658	17.650	17.556	11.773
5,000,000	18.117	18.113	18.079	16.631
10,000,000	18.463	18.462	18.445	18.138
20,000,000	18.810	18.809	18.801	18.699
50,000,000	19.268	19.268	19.264	19.227
100,000,000	19.615	19.615	19.613	19.595

TABLE 8 — $p_{wD}(s, \bar{C}, t_D)$ VS t_D FOR $s=+20$, LINE SOURCE WELL, $p_{wD}(20, \bar{C}, t_D)$ FOR $\bar{C}$ OF

t_D	10^2	10^3	10^4	10^5
100	0.9776	0.09977	0.01000	0.00100
200	1.9130	0.1991	0.02000	0.00200
500	4.4896	0.4946	0.0499	0.00500
1,000	8.1212	0.9787	0.0998	0.0100
2,000	13.478	1.9172	0.1992	0.0200
5,000	21.101	4.5125	0.4948	0.0500
10,000	24.241	8.1986	0.9797	0.0998
20,000	25.186	13.709	1.9209	0.1993
50,000	25.758	21.786	4.5333	0.4953
100,000	26.134	25.271	8.2698	0.9810
200,000	26.494	26.324	13.925	1.9252
500,000	26.960	26.907	22.443	4.5545
1,000,000	27.310	27.284	26.286	8.3394
2,000,000	27.657	27.645	27.460	14.133
5,000,000	28.116	28.112	28.055	23.085
10,000,000	28.463	28.461	28.434	27.297
20,000,000	28.810	28.809	28.795	28.606
50,000,000	29.268	29.268	29.262	29.216
100,000,000	29.615	29.615	29.612	29.596

range. The range of validity of long-time approximations such as Eq. 13 can only be established by comparison with the general solution.

Portions of the solutions are shown on Fig. 1.* Fig. 1 is a log-log plot such as would be required if "type-curve" matching of field performance were to be used. Beck et al.[16] illustrated type-curve matching of borehole unsteady-temperature data to determine thermal constants of well systems. The similar appearance of the computed curves for various storage constants and skin effects indicates that this sort of performance data interpretation might be rather poor for fluid flow systems. There is one useful technique illustrated by Fig. 1, however. All curves approach a unit slope at small times for finite, positive skin effects, and a skin effect of zero. This condition results because only the first term of the series in Eqs. 14 and 15 is important at very small times. Thus at very early times,

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*A full-size plot of Fig. 1 (with a grid) will be available through the American Documentation Institute, Library of Congress, Washington, D.C. At present, it may be obtained by writing to H. J. Ramey, Jr., Dept. of Petroleum Engineering, Stanford U., Stanford, Calif. 94305.
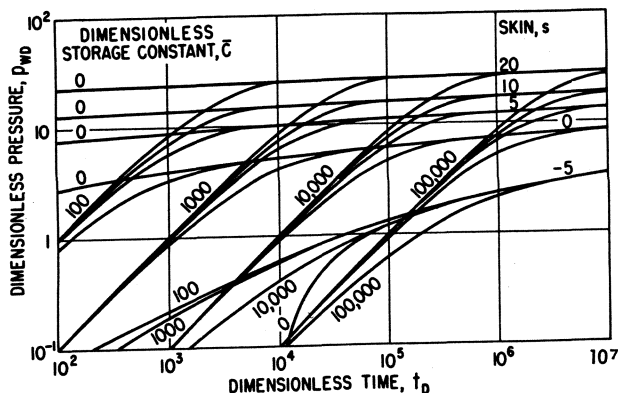

FIG. 1 — p_{wD} VS t_D FOR INFINITE RADIAL SYSTEM WITH STORAGE AND SKIN EFFECT.

$$p_{wD}(s, \bar{C}, t_D) = \frac{t_D}{\bar{C}} \qquad (32)$$

Fig. 1 indicates that the approximation given by Eq. 32 holds to dimensionless times on the order of 0.5 $\bar{C}$ to $\bar{C}$. This indicates that early performance is controlled entirely by wellbore unloading. It further indicates that the storage constant (but *only* the storage constant) might be determined from early field performance data. That is, a plot of the logarithm of $(p_i - p_{wf})$ vs the logarithm of flowing time should have a unit slope and a value of (q/C) at $t = 1$. Alternatively, the flowing pressure could be plotted vs flowing time on Cartesian coordinates and the slope would be $-(q/C)$. These interpretations result because we may substitute Eq. 6 for $\bar{C}$ in Eq. 32, and the usual identities for dimensionless pressure and time:

$$p_{wD} = \frac{2\pi k h (p_i - p_{wf})}{q \mu} \qquad (33)$$

$$t_D = \frac{kt}{\phi \mu c r_w^2} \qquad (34)$$

to achieve

$$p_i - p_{wf} = \left(\frac{q}{C}\right) t \qquad (35)$$

This approach may be useful to determine the storage constant where specific information is not available. Perhaps more important, this analysis indicates that well pressure data *cannot* be used to determine either skin effect or flow capacity before times when the dimensionless pressure on Fig. 1 begins to bend below the early straight line dictated by Eq. 32.

A pertinent question is what *is* the duration of this initial period? If we assume that a time sufficient for the sand-face flow rate to become 20 percent of the surface flow rate is adequate, Eq. 27 can be solved approximately to provide

$$t_D = 0.2 \bar{C} s, \quad s \neq 0 \qquad (36)$$

Using the same criterion, Eq. 28 may be solved approximately to yield

$$t_D = 0.4 \bar{C}, \quad s = 0 \qquad (37)$$

It is recommended that Eq. 37 be used for skin effects ranging from zero to +2 and Eq. 36 be used to estimate the time for skin effects greater than +2. The reason for this recommendation may be seen by inspection of Fig. 1. The dimensionless times that pressures depart from Eq. 32 increase as the skin effect increases above zero. Because Eq. 36 is only an approximate simplification of a very complex form, it would forecast smaller times than Eq. 37 if the skin effect is between zero and +2.

In summary, data obtained from the start of a test until dimensionless times specified by the appropriate Eq. 36 or 37 may be used to estimate the wellbore storage coefficient, C. Well-test data obtained for times between the appropriate Eqs. 36 or 37 and Eq. 19 may hopefully be interpreted for skin effect and formation flow capacity. Well-test data obtained for times after that specified by Eq. 19 can be analyzed in the normal fashion. The method of interpreting the intermediate time data remains to be shown, although we would expect that the modified Gladfelter et al.[17] method discussed by Ramey[11] would be appropriate.

One problem remains. Nothing yet has been said concerning results for negative skin effects. In the heat-conduction analog to the subject fluid-flow problem, the skin effect is analogous to a convection heat transfer film coefficient. Heat transfer coefficients always have values equal to or greater than zero. There is nothing in the statement of the fluid flow problem (Eqs. 1 through 5) that precludes the possibility of a negative skin effect. The transform solution, Eq. 8, and the line-source transform, Eq. 10, are both valid for a negative skin effect. Furthermore, the skin effect term in all of the long- and short-time approximations may be positive or negative. But the real inversion integrals given by Eqs. 9 and 11 apply only for skin effects equal to, or greater than, zero.

Eqs. 8 and 10 can be inverted for negative skin effects, but this does not appear worthwhile. An alternate procedure that makes more physical sense is to postulate that a negative skin effect can be represented physically by an increased wellbore radius. We recognize that many factors could be the actual cause of the appearance of a negative skin effect. For example, one would expect that the result of acidizing a well would be to increase the specific permeability of the rock matrix adjacent to the well. This situation could be approximated by Eq. 23. If the increase in permeability were large, then Eqs. 24 or 25 could be used to represent this physical situation. The proper procedure would be to use either of the real inversion integrals given by Eqs. 9 or 11 for a zero skin effect, but evaluated for a dimensionless time, t_D'', and a wellbore storage constant, $\bar{C}''$, defined as follows.

$$ t_D'' = t_D (r_w/r_1)^2 = t_D e^{2s}. \quad (38) $$

$$ \bar{C}'' = \bar{C} (r_w/r_1)^2 = \bar{C} e^{2s}. \quad (39) $$

Evaluation of Eq. 9 for a skin effect, s, of -5 and a range of storage constants, $\bar{C}$, is presented in Table 9 and also shown on Fig. 1. These dimensionless pressures are given as functions of t_D and $\bar{C}$, since a skin effect of -5 listed as a parameter would be observed in field operations. It should be emphasized that dimensionless pressures were found using Eq. 9, $s = 0$, and t_D'' and $\bar{C}''$ values which are e^{-10} times the t_D and $\bar{C}$ tabulated. At first glance, it would appear that a similar

procedure could be used for positive skin effects and that only a tabulation for a skin effect of zero is necessary. Even if such is true, and it is a good approximation, this would not be practical because tabulations for a tremendous range in dimensionless time would be required.

Finally, the fact that the infinitesimal skin effect is not a valid concept for short times is even more evident for the negative skin effect case shown on Fig. 1 than for the positive skin effect cases. Note the peculiar behavior for the zero storage constant case on Fig. 1 for $s = -5$.

INTERPRETATION OF SHORT-TIME WELL-TEST DATA

It has already been shown that the initial pressure data obtained from the start of a test until dimensionless times of the order of those from Eqs. 36 or 37 may be interpreted to obtain the wellbore storage constant only. The problem we consider here is whether the intermediate short-time pressure data can be interpreted for the formation flow capacity and the skin effect.

One technique that has been used extensively in the field of groundwater hydrology is "type-curve matching".[18] Drawdown data would be plotted as $p_i - p_{wf}$ in any convenient units vs time in any convenient units on log-log paper of the same size cycle as Fig. 1. The curve would be moved over Fig. 1 until the best match with one of the precomputed curves was obtained. The storage constant and skin effect could be read directly from the curve matched, and the flow capacity could be obtained from any pair of matching dimensionless time and real time from the two graphs, or from any pair by matching dimensionless pressure, p_{wD}, and pressure difference, $p_i - p_{wf}$. This procedure has the theoretical capability of also yielding the hydraulic diffusivity, $k/\phi \mu c$, from matching times. As in pressure buildup or drawdown, an accurate value of r_w is required.

In order to apply the preceding to pressure buildup, superposition may be used in the normal

TABLE 9 — $p_{wD}(s, \bar{C}, t_D)$ VS t_D FOR $s = -5$, CYLINDRICAL SOURCE WELL, $p_{wD}(-5, \bar{C}, t_D)$ FOR $\bar{C}$ OF

t_D	10^2	10^3	10^4	10^5
100	0.0697	0.0447	0.00896	0.00099
200	0.0992	0.0715	0.0172	0.00197
500	0.1557	0.1263	0.0394	0.00487
1,000	0.2164	0.1872	0.0718	0.00963
2,000	0.2977	0.2697	0.1267	0.01896
5,000	0.4446	0.4199	0.2518	0.0458
10,000	0.5913	0.5701	0.3990	0.0879
20,000	0.7722	0.7548	0.5972	0.1655
50,000	1.0646	1.0523	0.9313	0.3622
100,000	1.3232	1.3145	1.2254	0.6219
200,000	1.6086	1.6028	1.5422	0.9926
500,000	2.0170	2.0139	1.9806	1.6088
1,000,000	2.3420	2.3401	2.3201	2.0895
2,000,000	2.6757	2.6747	2.6630	2.5324
5,000,000	3.1248	3.1243	3.1187	3.0598
10,000,000	3.4677	3.4675	3.4644	3.4323
20,000,000	3.8124	3.8123	3.8107	3.7932
50,000,000	4.2693	4.2693	4.2685	4.2608
100,000,000	4.6154	4.6154	4.6150	4.6108

manner. That is,

$$\frac{2\pi kh}{q\,\mu}\left[p_i - p_w(t+\Delta t)\right] = p_{wD}(s,\bar{C},t+\Delta t)$$

$$- p_{wD}(s,\bar{C},\Delta t) \quad . \quad . \quad . \quad (40)$$

$$\frac{2\pi kh}{q\,\mu}(p_i - p_{wf}) = p_{wD}(s,\bar{C},t) \quad . \quad . \quad . \quad (41)$$

where t is the flowing time before shut-in, and Δt is the time since shut-in. Combination of Eqs. 40 and 41 yields:

$$\frac{2\pi kh}{q\,\mu}\left[p_w(t+\Delta t) - p_{wf}\right] = p_{wD}(s,\bar{C},t)$$

$$- p_{wD}(s,\bar{C},t+\Delta t) + p_{wD}(s,\bar{C},\Delta t) \quad . \quad . \quad (42)$$

If Δt is small compared to t, $t+\Delta t$ is approximately equal to t and the first two terms on the right in Eq. 42 are nearly equal. Thus,

$$\frac{2\pi kh}{q\,\mu}\left[p_w(t+\Delta t) - p_{wf}\right] \approx p_{wD}(s,\bar{C},\Delta t)$$

$$. \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (43)$$

The log of $(p_w(t+\Delta t) - p_{wf})$ should be plotted vs the log of Δt on a transparent log-log coordinate of the same size as Fig. 1. If the initial buildup data display a line of unit slope, q/C may be read from the pressure scale at a Δt of unity as discussed below Eq. 32. Eq. 6 may be used to compute the appropriate $\bar{C}$. The plot may then be placed over Fig. 1 using the family of curves for the proper $\bar{C}$. The curve should be positioned such that it matches the family of precomputed curves on Fig. 1. Then the permeability, k, may be determined from Eq. 33 using the corresponding values of p_{wD} and $[p_w(t+\Delta t) - p_{wf}]$; and corresponding values of t_D and Δt and Eq. 34 evaluated for $t = \Delta t$. An example calculation is shown in Ref. 19.

Another possible interpretative scheme is to match field data against some approximating equation such as the long-time expression given by Eq. 18. This is essentially the procedure recommended by Russell[15] for short-time pressure buildup analysis. Eq. 5 in Ref. 15 may be rearranged to the form:

$$\frac{2\pi kh}{q\,\mu}\left[p_w(t+\Delta t) - p_{wf}\right] =$$

$$\left[p_D + s\right]\left[1 - \frac{\bar{C}}{t_D}\right] \quad . \quad . \quad . \quad (44)$$

which was pointed out as an acceptable approximation for the equivalent of Eq. 43, employing the long-time approximation given by Eq. 18. This procedure is valid as long as Eq. 44 represents the rigorous

solution with reasonable accuracy. The storage constant $\bar{C}$ is equivalent to the reciprocal of the storage constant, C_1, used in Ref. 15. Of interest at this point is the range of validity of the approximation given by Eq. 44. Fig. 2 presents $p_{wD}(s,\bar{C},t_D)$ as determined from the rigorous Eq. 9, and the approximation given by Eq. 44. Results are shown for $s = 0$ and 20 and for $\bar{C} = 1,000$. As can be seen, the range of validity of the approximation is limited. But the long-time approximation has the same general shape as the rigorous solution, providing an empirical basis for the method.

Eq. 44 is interesting for another reason. It is clear that the time criterion given by Eq. 19 may be substituted in the last bracket on the right in Eq. 44 to yield:

$$\frac{2\pi kh}{q\,\mu}\left[p_w(t+\Delta t) - p_{wf}\right] = \left[p_D + s\right]$$

$$\cdot\left[0.9833\right] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (45)$$

This shows that the time criterion of Eq. 19 provides the time at which the approximated p_{wD} is within 1.7 percent of (p_D+s). A dimensionless time of $60\bar{C}$ is also shown on Fig. 2.

CONCLUSIONS

A thorough investigation of both the wellbore storage and skin effect concepts was performed. It was found that many studies have been made of these effects, although the connection between various studies has not been obvious. It is hoped that the review presented in this paper will help clarify this important problem.

In regard to wellbore storage, it has been shown that it is possible to forecast the duration of the initial flow period controlled by storage. During this time, it is possible to find the storage constant from well-test data, but the formation flow capacity and skin effect cannot be found. After the initial
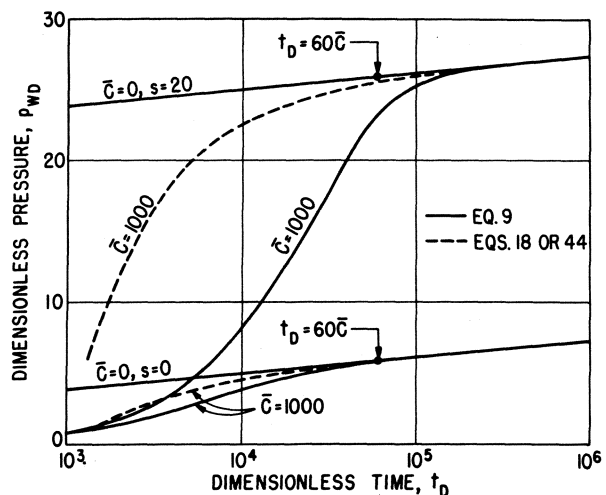

FIG. 2 — p_{wD} VS t_D FOR LONG-TIME APPROXIMATION AND RIGOROUS SOLUTION.

period controlled by storage, it is possible to perform analysis of well-test data for flow capacity and skin effect by type-curve matching techniques. This procedure extends well-test analysis into the short-time region and provides an additional useful interpretative tool.

It has been recognized, in regard to skin effect, that the steady-state skin effect concept becomes physically invalid at very short test times. If test data are to be interpreted properly, it will be necessary to generalize the skin effect concept to include a damaged region of a finite storage capacity.

Regarding negative skin effect, one possible interpretation is that the wellbore radius has been enlarged. Transient solutions have been presented for this case. It is also likely that negative skin effects arise more commonly as a result of fracture communication with the wellbore. This latter situation is not considered in this study.

In regard to the combined effect of wellbore storage and skin, it appears that the time required to reach the usual straight line is not usually affected significantly by a finite skin effect. The duration is mainly controlled by the wellbore storage.

A systematic inspection of a number of significant factors affecting short-time transients involved in well testing has been made. It is not implied, however, that *all* significant factors have been studied. Nevertheless, a number of pertinent findings have resulted. Wellbore storage should control the initial transients if the well is damaged or has a zero skin. If a well exhibits a negative skin, the effect of wellbore storage will not be prominent because the rate of pressure change in the wellbore may be quite low. It appears that proper interpretation of short-time well-test data can be made under favorable circumstances.

NOMENCLATURE

c = total system isothermal compressibility, vol/vol-atm

C = wellbore storage constant, cc/atm (see Eq. 6)

C_1 = equivalent to the reciprocal of $\bar{C}$ (used by Russell[15])

$\bar{C}$ = dimensionless wellbore storage constant (see Eq. 6)

d = differential operator

h = formation thickness

J_0 = Bessel function of first kind, order zero

J_1 = Bessel function of first kind, order one

k = permeability of the formation

k_1 = permeability of the region immediately adjacent to the well

K_0 = modified Bessel function of second kind, order zero

K_1 = modified Bessel function of first kind, order one

L = Laplace transform of the quantity

p = variable of Laplace transform

p_{wD} = dimensionless pressure drop within the wellbore

p_D = dimensionless pressure drop on the formation side of skin effect

p_i = initial formation pressure

p_w = well pressure

p_{wf} = flowing well pressure

q = surface flow rate

q_{sf} = sand face flow rate

r = radial distance

r_1 = radius of the region of permeability k_1

r_D = dimensionless radius, r/r_w

r_w = wellbore radius, cm

s = skin factor, dimensionless

t = flowing time, seconds

Δt = shut-in time, seconds

t_D = dimensionless time (see Eq. 34)

u = variable of integration

Y_0 = Bessel function of second kind, order zero

Y_1 = Bessel function of second kind, order one

α = reciprocal of $\bar{C}$ (see Table 1)

γ = Euler's constant, 0.57722

μ = viscosity, cp

σ = infinity value defined by Hurst (see Table 1)

τ = dimensionless time (see Table 1)

ϕ = porosity, fraction of bulk volume

SUBSCRIPTS

D = dimensionless quantity

i = refers to initial reservoir condition

sf = refers to conditions at sand face

w = refers to conditions at wellbore radius

wf = refers to flowing conditions at wellbore radius

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