# Physics of Processes

## Electricity and magnetism

# Electric charge

- experiments with friction of bodies - force action
- carrier of one of the fundamental physical forces
- elementary charge $e \cong 1.6.10^{-19}$ C (Coulomb), fundamental role in technical applications
- atomic structure
- quarks with charge *e*/3
- origin of the word electron – in Greek "amber"

# Coulomb's law

## (For the force acting between two point charges $Q_1$ and $Q_2$)

$$F = \frac{1}{4\pi\varepsilon}\frac{Q_1 Q_2}{r^2}$$

$$\varepsilon = \varepsilon_0 \varepsilon_r$$

$$\varepsilon_0 = 8{,}85.10^{-12}\, F.m^{-1}$$

$$\varepsilon_r \geq 1$$

$\varepsilon$ – permittivity

$\varepsilon_0$ – permittivity of vacuum

$\varepsilon_r$ – relative permittivity

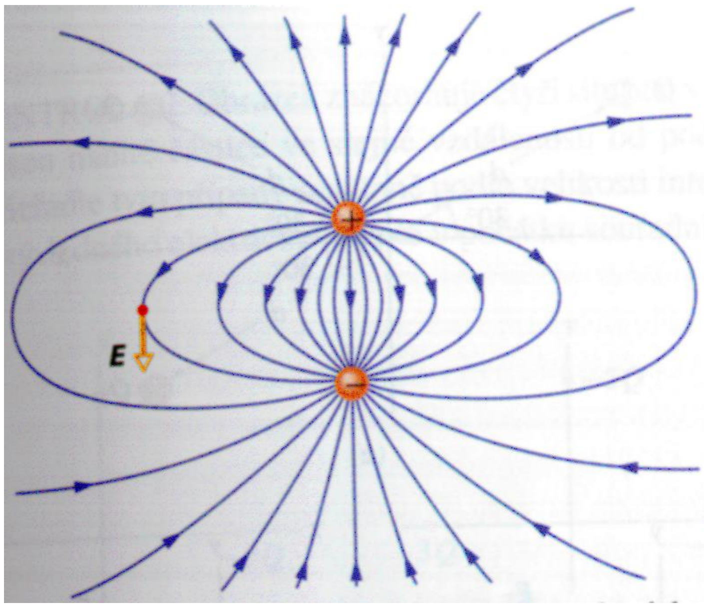
$r$ – distance of charges

**attractive force** between charges of different signs

**repulsive force** between charges of the same signs

# Electric Field - EF Intensity

**Intensity of electric field (*E*)** is equal to the force exerted by the field on the positive unit charge




$$\vec{E} = \frac{\vec{F}}{Q}$$

$$[E] = \mathrm{V.m}^{-1}$$

$$\vec{E} = \frac{1}{4\pi\varepsilon}\frac{Q_{sour.}}{r^2}\vec{r}$$

**Electric force lines** are the curves to which the electric field intensity vectors at all points are tangent.

# Area electric charge density

$$\sigma = \frac{\mathrm{d}Q}{\mathrm{d}S}$$

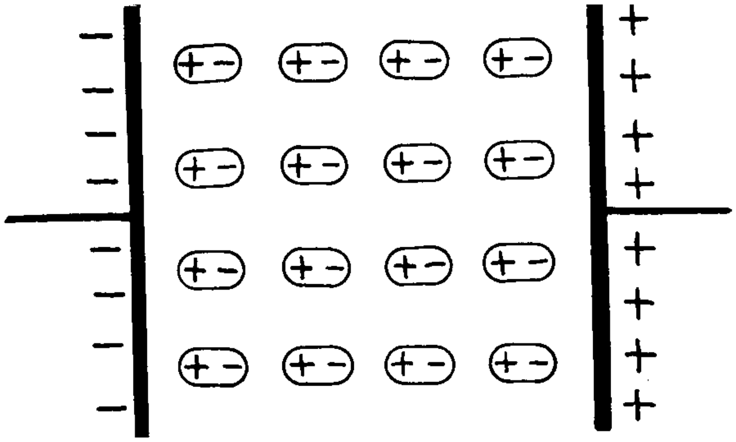
**[$\sigma$] = C.m$^{-2}$**

$$\sigma = \frac{Q}{S} = \frac{Q}{4\pi r^2}$$

**sphere**

**$\sigma$ grows with increasing surface curvature - <u>spikes</u>**

**conductors and insulators, polarization of dielectrics:**



**electric sliding current -**
**charging current of capacitors**

# Electric field - electric potential $\varphi$

***EP is the work done by the field forces in transferring a positive unit charge from point A to a point of zero potential***

*zero potential: at infinite distance from the charge (physics)*

*on Earth (electrical engineering)*

$$\varphi_{bn} = \frac{A_{AB}}{Q} = \frac{1}{Q}\int_A^\infty F\mathrm{d}r = \int_A^\infty E\mathrm{d}r = \int_A^\infty \frac{Q\mathrm{d}r}{4\pi\varepsilon r^2} = \frac{Q}{4\pi\varepsilon r_A}$$

$$[U] = [\varphi] = \mathrm{V}$$

$$A_{AB} = Q(\varphi_A - \varphi_B) = QU_{AB}$$

$$U_{AB} = \varphi_A - \varphi_B$$

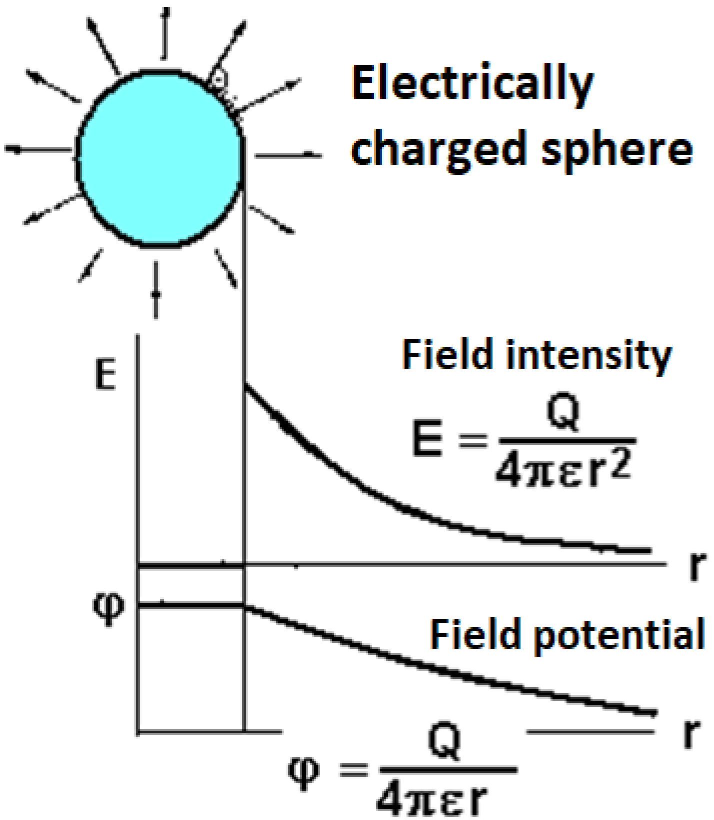
***U* - voltage**

unit 1 eV = $1{,}6.10^{-19}$ J

the work done by the field in moving the electron to a location with an EP 1 V higher

$\varphi_{\mathrm{bn}}$ - EP of the point charge

# Field of charged sphere




*r* – distance from the centre of the sphere

the sphere is charged with a positive charge

in a conductor without the influence of an external field, charge transport occurs so that all parts have the same potential

<u>the principle of superposition</u> applies to the potential of multiple charges

# Comparison of electric and gravitational fields

- **Potentials**
- **Intensities**

$$\varphi = \frac{Q}{4\pi\varepsilon r}$$

$$W_{po} = \kappa \frac{M}{r}$$

$$E = -\frac{\mathrm{d}\varphi}{\mathrm{d}r} = \frac{Q}{4\pi\varepsilon r^2}$$

$$g = -\frac{\mathrm{d}W_{po}}{\mathrm{d}r} = \kappa \frac{M}{r^2}$$

- Significant **similarity** between the two fields, more pronounced in the negative charge electric field with attractive forces
- Both fields are **conservative**
- In the case of a **gravitational field**, the gravitational acceleration vector plays the role of the field intensity

# Conductor capacity

*Ability to collect electric charge through a conductor*

$$C = \frac{\mathrm{d}Q}{\mathrm{d}\varphi}$$

$$[C] = F$$

$$C = \varepsilon \frac{S}{d}$$

$$C_{par} = \sum_i C_i$$

$$\frac{1}{C_{ser}} = \sum_i \frac{1}{C_i}$$

The unit is Farad (F)

**Capacitor:**

several conductors arranged to have an increased capacitance value

$S$ - plate overlap area, $d$ - distance of the plates,

$\varepsilon$ - permittivity of the dielectric between the plates

$C_{\mathrm{par}}$ - capacitance of capacitors connected in parallel

$C_{\mathrm{ser}}$ - capacitance of series connected capacitors

# Example: capacitance of a conducting sphere

$$C = \frac{Q}{\varphi} = \frac{Q}{\dfrac{Q}{4\pi\varepsilon r}} = 4\pi\varepsilon r$$

**<u>Capacity of a globe-sized sphere in vakuum:</u>**

$C = 4\pi\varepsilon r = 4\pi .\ 8{,}85.10^{-12}$ F.m$^{-1}$ . $6{,}4.10^{6} m \approx 7{,}1.10^{-4}$ F

= 0,71 mF

# Capacitor energy

$dA = U\,dQ = Q/C\,dQ$

$W = \int dA = {}_0\int^Q Q/C\,dQ = Q^2/(2C) = QU/2 = CU^2/2$

$$W = \frac{1}{2}CU^2$$

Example: Capacitor energy in a photographic flash

$U = 500$ V, $C = 400$ μF

$W = CU^2/2 = 4.10^2.10^{-6}.25.10^4/2 = 50$ J

# Electric current

# Definition

*The orderly movement of electric charge carriers, manifested by force action and different losses in different materials*

$$I = \frac{\mathrm{d}Q}{\mathrm{d}t}$$

$$[I] = \mathrm{A}$$

The unit is the **Ampere**, the basic unit of the SI system

It is the balancing of the potential gradient, which is renewed by the EP sources in the long term, the main sources of EP are chemical cells and especially electromagnetic induction in power plants.

2 basic conditions for the passage of electric current

# Current density

$$J = \frac{\mathrm{d}I}{\mathrm{d}S}$$

$S$ is the area through which the electric current passes

steady current **$J = I/S$** $[J] = \mathrm{A.m^{-2}}$

technical practice $J = 1 - 20\ \mathrm{A.mm^{-2}} = 1 - 20\ \mathrm{MA.m^{-2}}$

# Volumetric density of the electric charge

$$\rho_V = \frac{\mathrm{d}Q}{\mathrm{d}V}$$

$$\mathrm{d}Q_+ = \rho_+ \mathrm{d}V = \rho_+ \mathrm{d}S\, v_+ \mathrm{d}t$$

$$\frac{\mathrm{d}Q_+}{\mathrm{d}t} = \mathrm{d}I = \rho_+ \mathrm{d}S\, v_+$$

$$\frac{\mathrm{d}I}{\mathrm{d}S} = J_+ = \rho_+ v_+$$

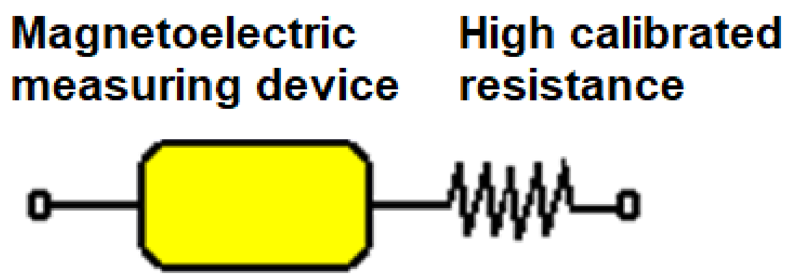
$\rho_V$ volumetric electric charge density (free charges)

$[\rho_V]$ = C.m$^{-3}$ = A.s.m$^{-3}$,

where d$Q$ is the charge in the elementary volume d$V$

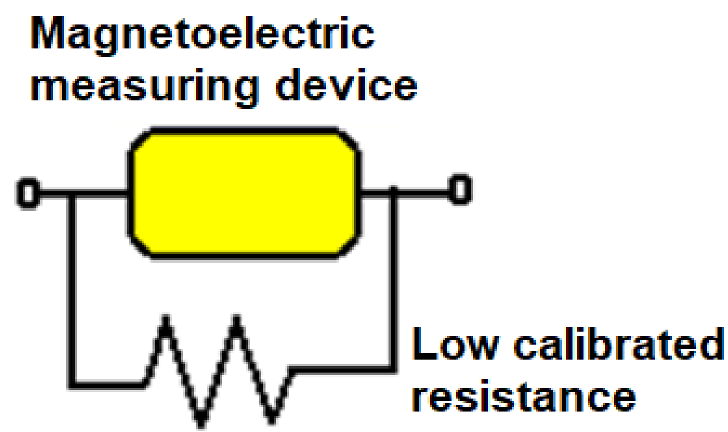
$$\vec{J} = \rho_V \vec{v} = \rho_{V(+)} \vec{v}_{(+)} - \rho_{V(-)} \vec{v}_{(-)}$$

# Voltage and current measurement



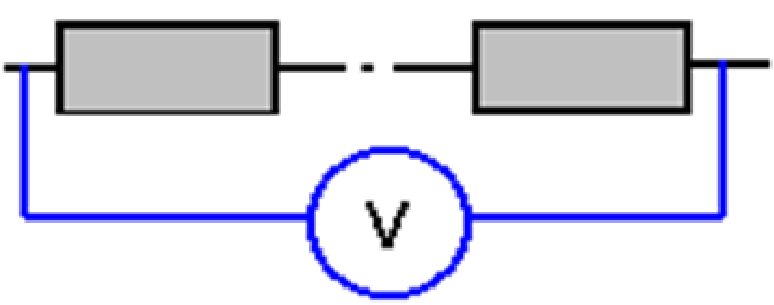

**Voltmeter (high internal resistance)**

series resistor




**Ammeter (low internal resistance)**

parallel resistor



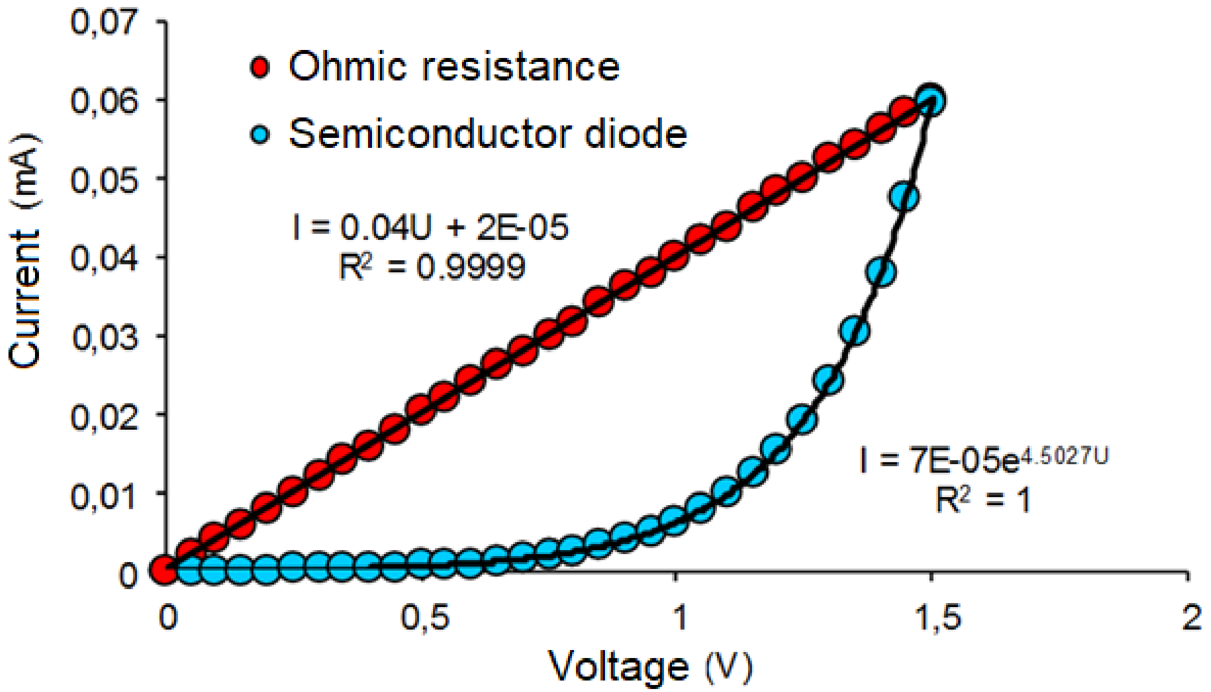

# Voltampere characteristic

**Apparent conductivity:**

$$G_z = \frac{\mathrm{d}I}{\mathrm{d}U}$$

**Conductivity:**

$$G = \frac{I}{U}$$



# Resistance of the conductor

Inverse conductivity value

$$R = \frac{1}{G}$$

$$[R] = \Omega$$

$$R = \rho \frac{l}{S}$$

$$[\rho] = \Omega.\mathrm{m}$$

for conductors with **linear** VA characteristic

$\rho$ - **resistivity (specific electrical resistance)** ($10^{-8}$ - $10^{-6}$) Ω.m - vodiče

($10^{-6}$ - $10^{8}$) Ω.m - semiconductors

($10^{8}$ - $10^{10}$) Ω.m - non-conductors

**Temperature dependence**

linear growth - metals: $\rho = \rho_0(1 + \alpha\Delta t)$

$\alpha$ - temperature coefficient of resistance ($K^{-1}$)

drop - for example semiconductors

# Ohm's law

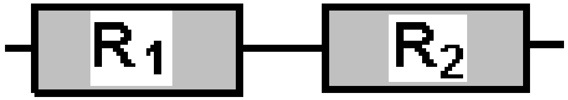
For conductors with linear VA characteristic:

$$U = RI$$

Resulting resistance when connected

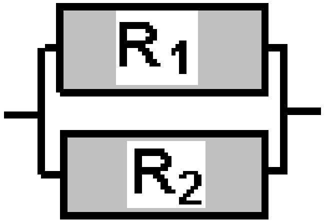
to series

$$R = R_1 + R_2$$




to parallel

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2}$$



# Ohm's law in differential form

$$\vec{F} = \vec{E}e$$

the force acting on a charge in an electric field

$$\vec{F}t = m_e\vec{v}$$

impulse equation (velocity changes in time *t* from zero to *v*)

$$\vec{v}_{med} = \frac{\vec{E}te}{2m_e}$$

medium speed of movement of the charge

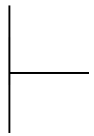
$$\vec{J} = \rho_V\vec{v}_{med} = ne\vec{v}_{med} = \frac{ne^2t\vec{E}}{2m_e} = \gamma\vec{E}$$

***n*** **- number of elementary charges (*e*) per unit volume**
***t*** **- time between collisions of moving elementary charges with obstacles**
$\gamma$ **- conductivity (specific electrical conductivity),** $[\gamma] = \Omega^{-1}.m^{-1}$

$$\vec{J} = \gamma\vec{E}$$

# Kirchhoff's Laws

1. **<u>Behaviour of electric current in nodes</u>**

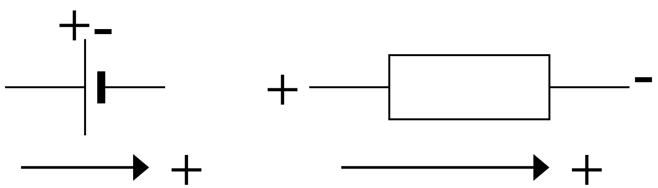


The sum of the currents entering the node is equal to the sum of the currents leaving the node.

2. **<u>Behaviour of electric current in circuits (loops)</u>**

The sum of the voltages of the sources of electrical energy in a closed electrical circuit (loop), together with the voltage drop across the conductors and resistors included in it, is equal to zero.

**<u>Sign convention:</u>** - choice of orientation in the loop

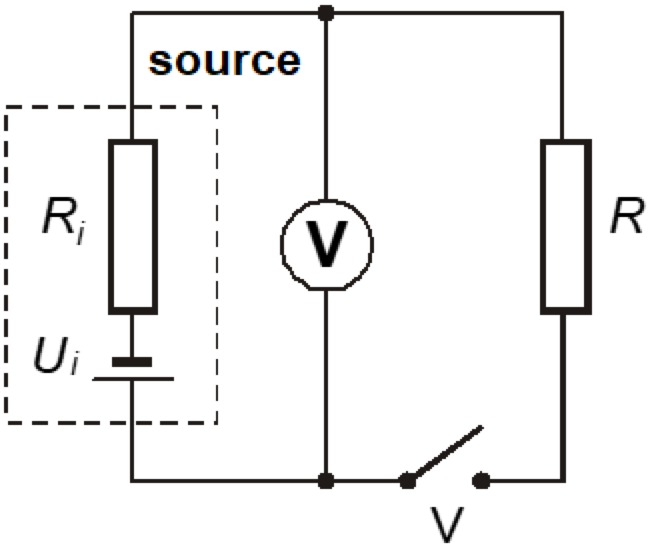



<u>Opposite orientation - opposite sign</u>

Loop orientation

# Sources of electric current

## Ohm's law for closed circuit




**U** clamping voltage

$R_i$ - internal resistance

R - load resistance

$U_i$ - source voltage (without load)

$$I = \frac{U}{R} = \frac{U_i}{R_i + R}$$

$$I_k = \frac{U_i}{R_i}$$

$I_k$ - short circuit current

$$E = -U_i$$

electromotive force

$$U = U_i - R_i I = \frac{R}{R_i + R}$$

hard x soft source: $I_k$ (high x low)

$R_i$ (low x high)

# Work and power of electric current

Definitional relations:

$$\mathrm{d}A = U\mathrm{d}Q = UI\mathrm{d}t = RI^2\mathrm{d}t = \frac{U^2}{R}\mathrm{d}t$$

$$P = \frac{\mathrm{d}A}{\mathrm{d}t} = UI = RI^2 = \frac{U^2}{R}$$

expressions with resistance (according to Ohm's law) are suitable for appliances with **linear VA characteristic**

**- electric energy in kWh (1 kWh = 3,6.10$^6$ J)**

# Electric current in liquids

Normal state: insulators (water - **$\rho$ = 2.10$^5$ Ω.m**)

charge carriers: ions in the form of impurities (salts, acids, bases)
in the dissociated state - **electrolytes**

conductivity: - by concentrations of dissociated ions and their mobility
- **$\rho$ = 1 Ω.m (Limited validity of Ohm's law)**

Electrolysis - passage of electric current - transport of substances

Faraday's Laws

$$I. \quad m = AQ = AIt$$

$$II. \quad A = \frac{M}{Fv} = \frac{M}{N_A e v}$$

[$A$] = kg.C$^{-1}$ - electrochemical equivalent

$F$ = 96,5.10$^3$ C.mol$^{-1}$ - Faraday constant

$M$ - molar mass, $v$ - valency

$N_A$ - Avogadro's constant

# Electric current in gases

Normal state: insulators

charge carriers: ions - formed by **ionization** of gas (electrons are released from the molecule by external action) - heating, UV radiation, radioactive radiation, cathode rays, impact

- electric current in gas: **discharge (limitation of Ohm's law):**

non-self-contained - the ioniser must operate continuously

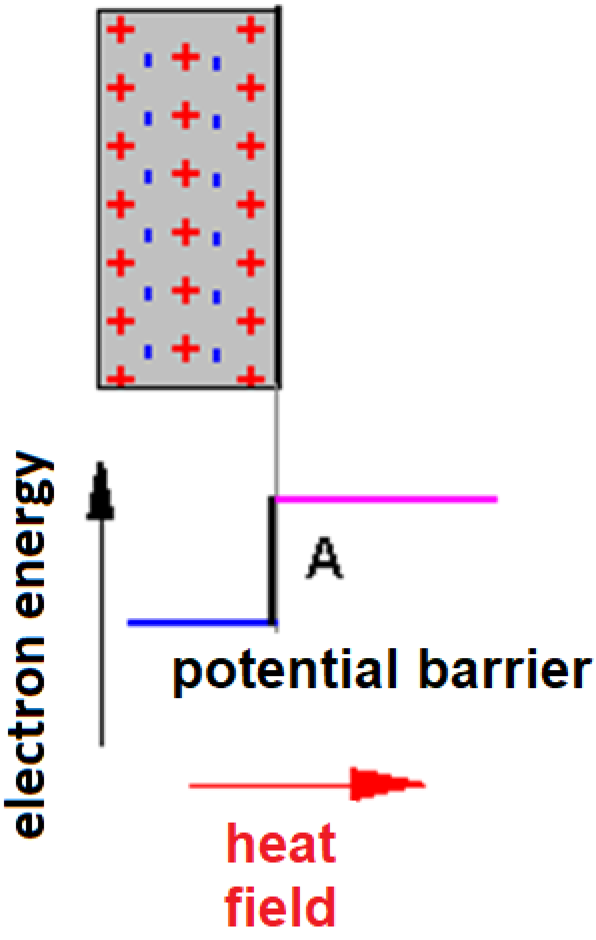
self-contained – stimulated mainly by impact - acceleration of ions and electrons by **electric field** and **reduced pressure** - smouldering (small currents), spark (small distance between electrodes), arc

effect of pressure: lengthening of the free path of ions and thus increasing their energy by diluting the gas

cathode ray: the flow of electrons emitted from the cathode in an depleted discharge tube

Thermoemission: the release of electrons from the surface of bodies at high temperature

# Electron emission from metal



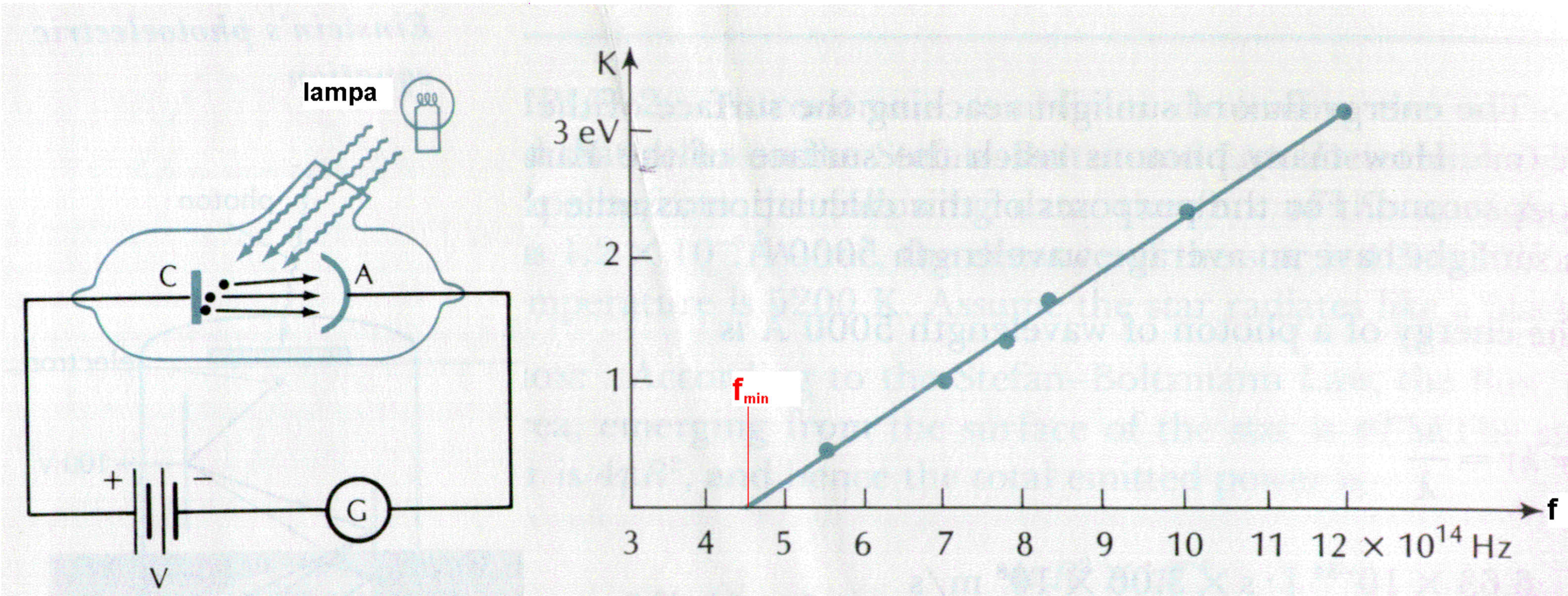

*A* - output work (height of the potential barrier) to overcome the binding force

*E* - the energy supplied to the electron for its exit from the metal:

$$E = A + \frac{1}{2} m_e v^2$$

# External photoelectric effect

## (Einstein's Nobel Prize in 1921)




Photoemission of electrons - the release of electrons from a metal due to radiation
- does not depend on radiation intensity
- for each metal there is a limiting frequency of radiation for electron release

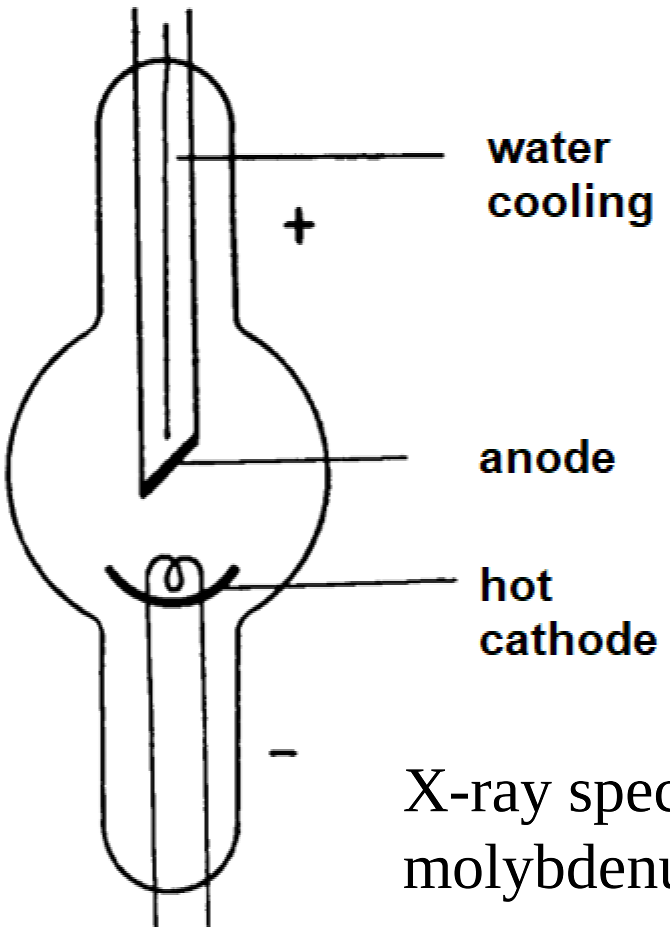
$$hf = A + \frac{1}{2} m_e v^2$$

$$hf_{\min} = A$$

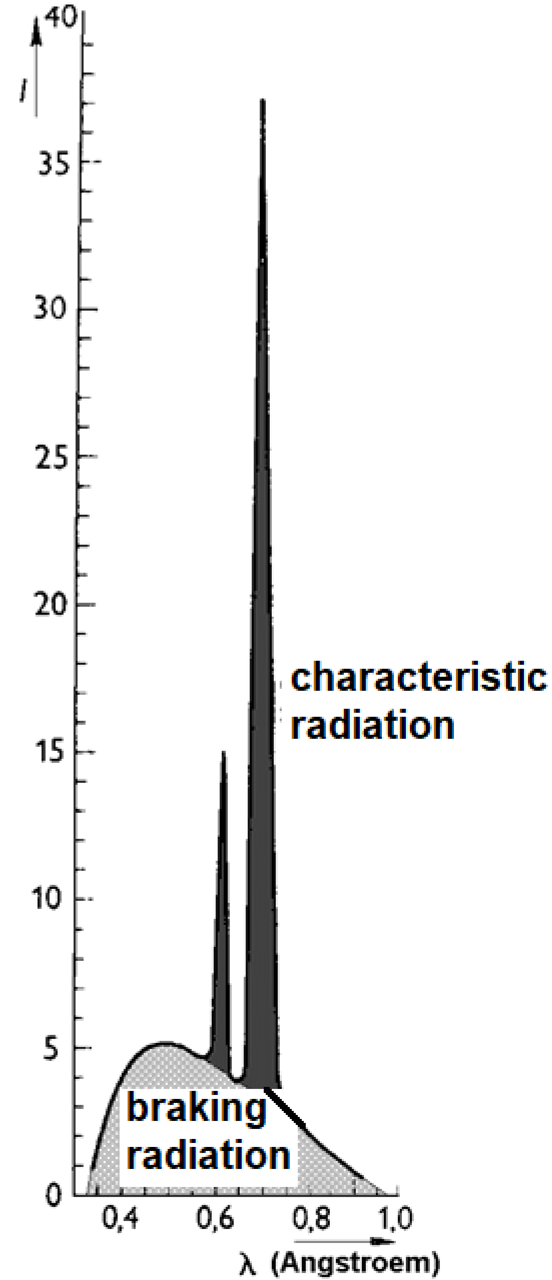
# X-ray radiation

**- when cathode radiation (accelerated electrons) hits the anode**

**- very penetrating ($10^{-8}$ - $10^{-12}$ m)**




X-ray spectrum from molybdenum anticathode:



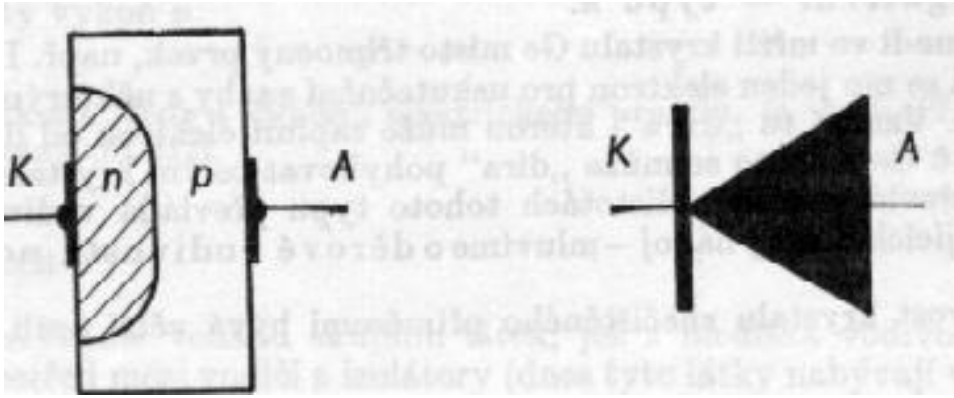
# Semiconductors

- resistivity from$10^{-6}$ Ω.m to $10^{8}$ Ω.m
- rapid decrease of resistance with temperature
- intrinsic and extrinsic semiconductors (admixtures - N and P)

- **Semiconductor technology**

• use of PN transition properties
• use of temperature changes of PN transition properties
• semiconductor diode - rectifier
• transistor - amplifier

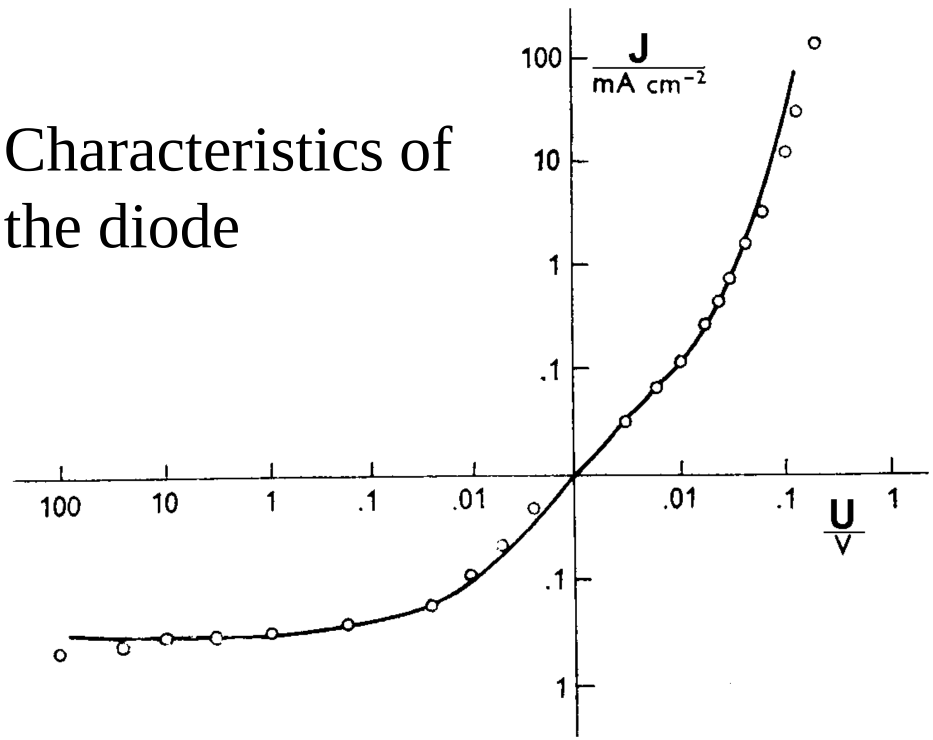
# Semiconductor diode



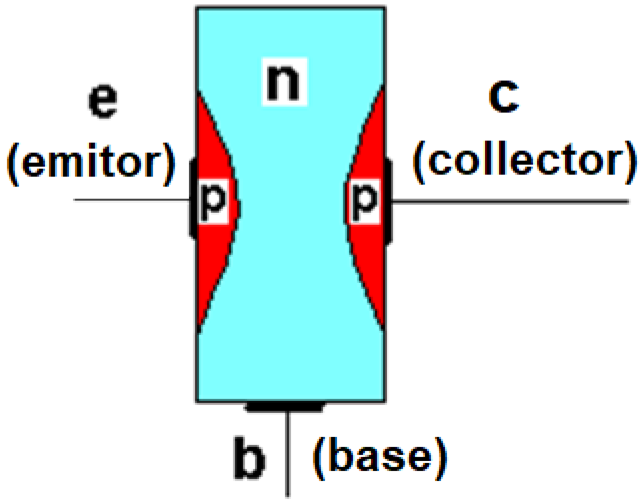

$$I_p = I_0\left[e^{\frac{eV}{kT}} - 1\right]$$

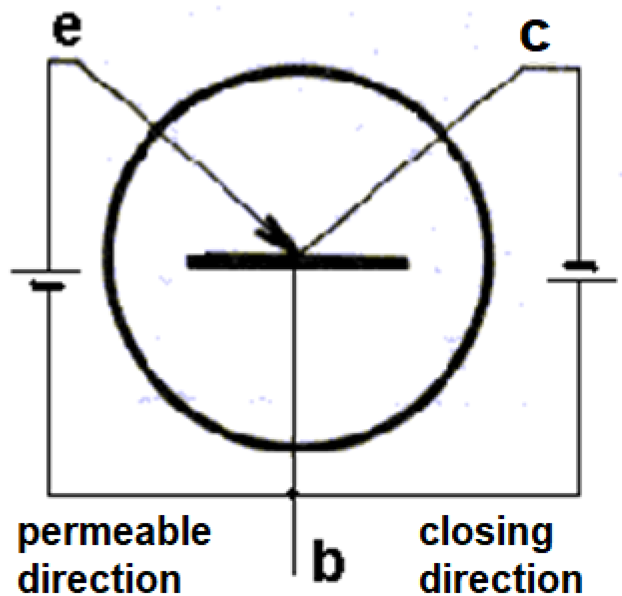
$$I_z = I_0\left[1 - e^{-\frac{eV}{kT}}\right]$$

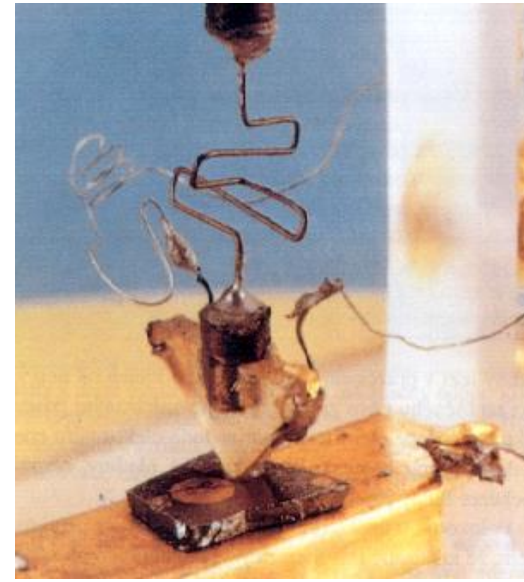
Characteristics of the diode



# Transistor
## (example of PNP transistor)



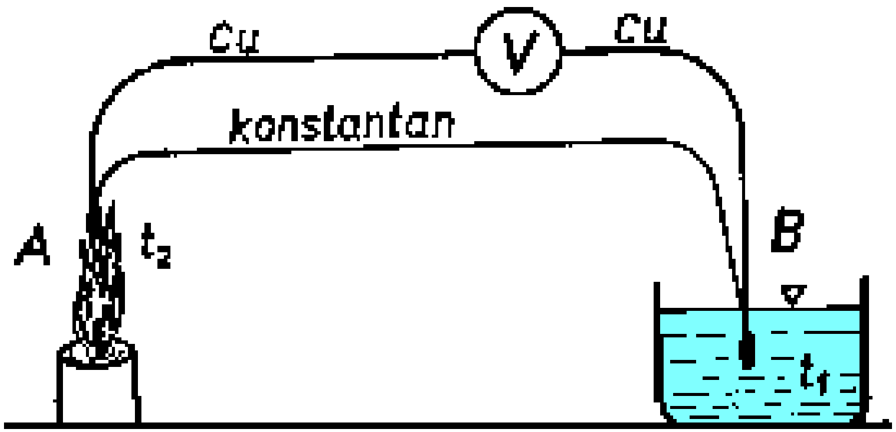



- low voltage at the e-b transition in the pass (permeable) direction,
- the resistance in the c-b transition is reduced
- higher voltage level at the e-b transition causes power amplification of the c-b signal: $U_{eb}I_{eb} \ll U_{cb}I_{cb}$

# Thermoelectricity - thermocouple

- conversion of internal energy into electrical energy
- the voltage that arises at the junction of two different metallic conductors depends on the temperature of the junction




$$U = \varphi_A - \varphi_B = \alpha\,(t_1 - t_2)$$

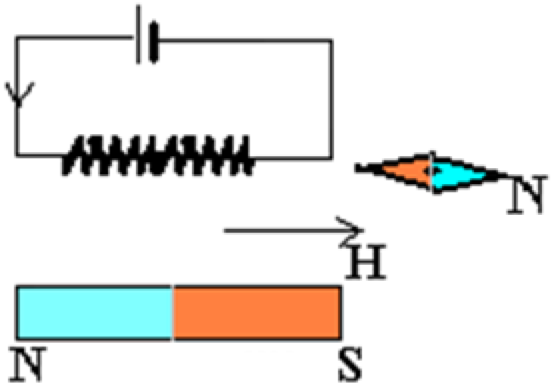
$U$ - thermoelectric voltage

$\alpha$ - thermocouple coefficient (tens to hundreds μV.K$^{-1}$)

- the voltage is dependent on the material pair and the temperature

# Magnetism

## Magnetic field - examples




**Magnetic field in the surroundings of a permanent magnet and conductors with electric current**

**Animation – bar magnet, straight conductor, Lorentz force**

**electrical charges at rest – electrostatics**
**electric charges in motion – electrodynamics**

**electric and magnetic field = electromagnetic field**

# Magnetic field

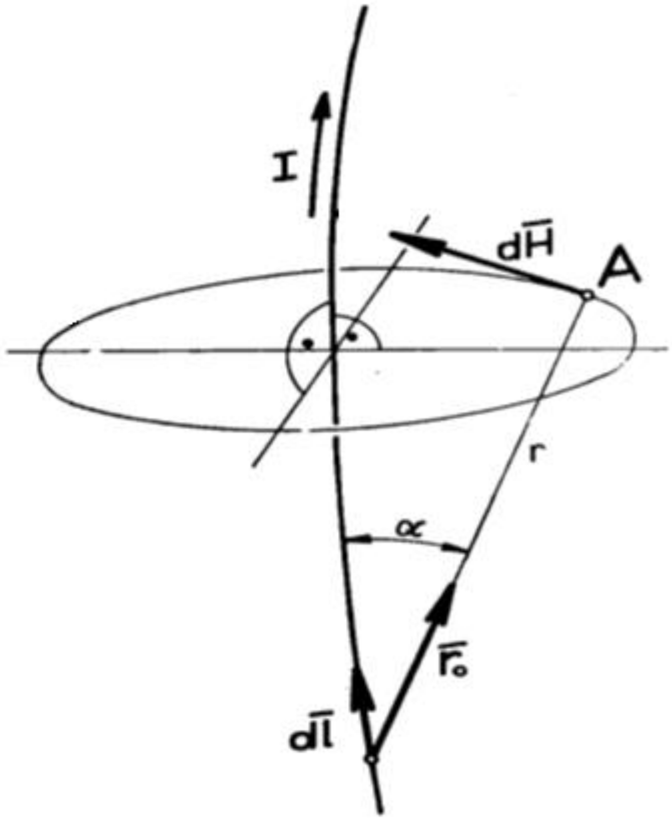
**description: magnetic field intensity**

**[*H*] = A.m$^{-1}$**

**<u>Biot-Savart law</u>**

$$\mathrm{d}\vec{H} = \frac{I}{4\pi r^2}\left(\mathrm{d}\vec{l} \times \vec{r}_0\right)$$

**<u>Right hand rule for vector product</u>**



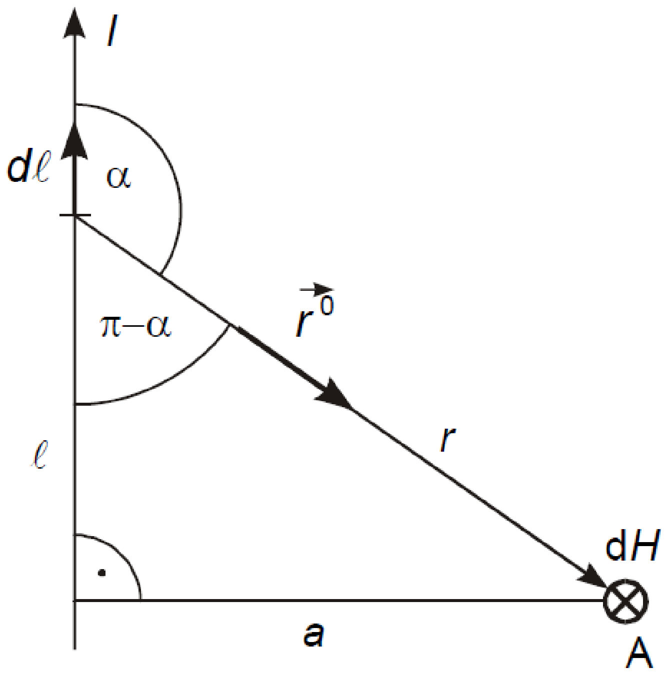

**Magnetic field superposition**

**$H = \int \mathrm{d}H$**

**<u>magnetic field curve</u>** –

closed line always tangent to *H*

# Magnetic field of infinite straight conductor




$$\left|\vec{H}\right| = \frac{I}{2\pi a}$$

**<u>Right hand rule for vector product:</u>**

**all dH have the same direction, so we can add them algebraically.**

**- derive:**

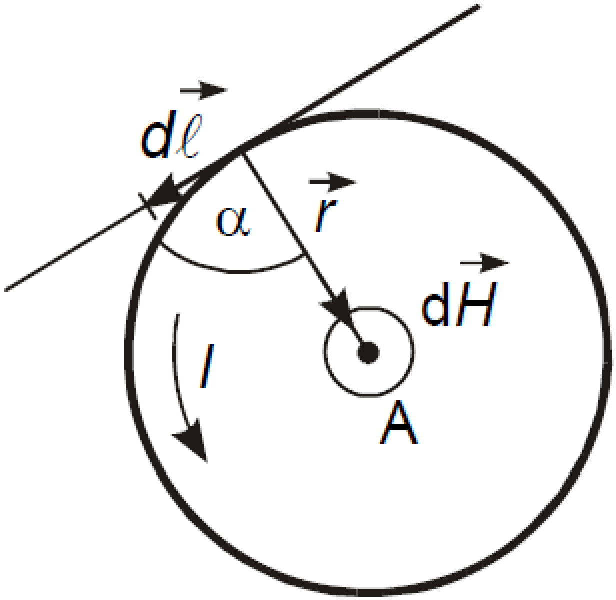
$$H = \int_0^H \mathrm{d}H = \int_{-\infty}^{\infty} \frac{I \mathrm{d}l \sin\alpha}{4\pi r^2} = \frac{I}{4\pi a}\int_0^{\pi} \sin\alpha \,\mathrm{d}\alpha = \frac{I}{2\pi a}$$

$$\frac{l}{a} = \mathrm{cotg}(\pi - \alpha) = \mathrm{cotg}(-\alpha) = -\mathrm{cotg}\,\alpha$$

$$l = -a\,\mathrm{cotg}\,\alpha,\ \mathrm{d}l = \frac{a}{\sin^2\alpha}\mathrm{d}\alpha$$

$$\frac{a}{r} = \sin(\pi - \alpha) = \sin\alpha,\ r = \frac{a}{\sin\alpha}$$

# Magnetic field at the centre of the circular loop




$$H = \frac{I}{2R}$$

**Right hand rule for vector product:**

**all dH have the same direction, so we can add them algebraically.**

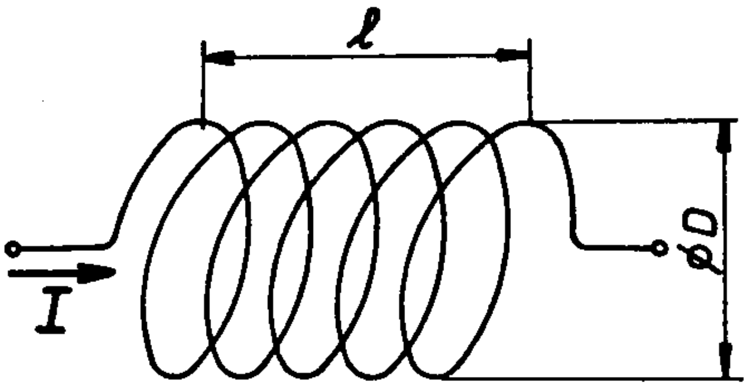
**- derive:**

$$\mathrm{d}H = \frac{I\mathrm{d}l\sin\alpha}{4\pi r^2}$$

$$\alpha = \frac{\pi}{2} \Rightarrow \sin\alpha = 1$$

$$H = \int_0^H \mathrm{d}H = \int_0^{2\pi R} \frac{I\mathrm{d}l}{4\pi r^2} = \frac{I}{2R}$$

# Magnetic field in the centre of the coil




1. **D >> *l*; short coil**

$H \approx n$ - circular loop field

$$H = \frac{nI}{D}$$

2. ***l* >>D; long thin coil**

$$H = \frac{nI}{l} = zI$$

***n*** **- number of threads**

***z*** **- density of threads; [*z*] = m$^{-1}$**

# Magnetic induction

**Magnetic induction:**

$$\vec{B} = \mu \vec{H}$$

$[B]$ = T = Wb.m$^{-2}$ $[\mu]$ = H.m$^{-1}$

$$\mu = \mu_0 \mu_r$$

***μ*** - **environmental permeability**

$\mu_o = 4\pi.10^{-7}$ H.m$^{-1}$; $\mu_o$ - vacuum permeability

$\boldsymbol{\mu_r}$ - **relative permeability**

- basic characteristics of magnetics as an environment

- is a manifestation of the internal magnetic moments of the electron shell and the nucleus of the atom

**magnetic field lines** - closed curves to which the magnetic induction vectors at all points are tangent

# Basic forms of magnetism

**Diamagnetism:** without the presence of an external mg. field the resulting magnetic moment of the diamagnetic atoms is zero, diamagnetism occurs only in the presence of an external mg. field - after inserting the substance into the mg. field, dipoles are formed in the substance, which are oriented against the direction of the external mg. field and thus the mg. field is weakened in the substance

$\mu_r$ **is slightly less than 1** (carbon, copper, water)

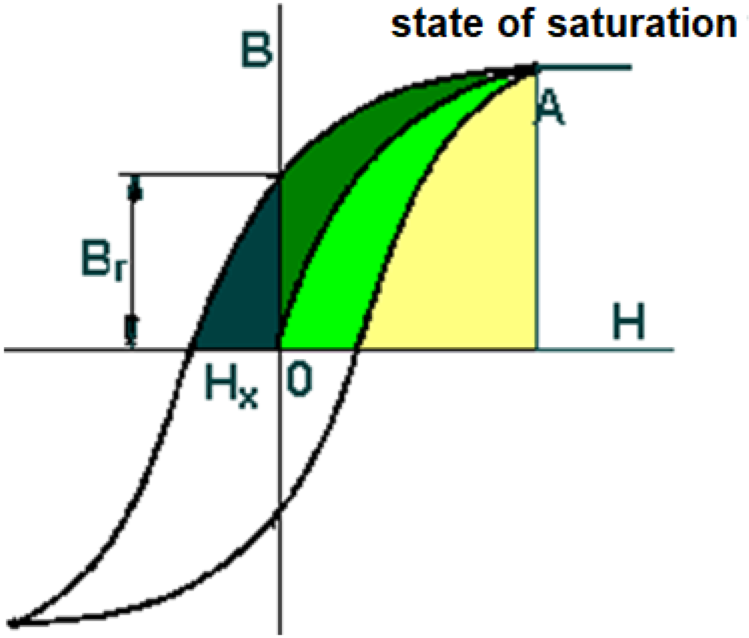
**Paramagnetism:** without the presence of an external mg. field the resulting magnetic moment of the paramagnetic atoms is zero, paramagnetism occurs only in the presence of an external mg. field - after inserting the substance into the mg. field, dipoles are formed in the substance, which are oriented in the direction of the external mg. field and thus the mg. field is amplified in the substance

$\mu_r$ **is slightly more than 1** (aluminium, calcium, oxygen)

**Ferromagnetism:** the action of an external magnetic field results in a large amplification of the mg. field in the substance due to the existence of domains (whole regions with the same orientation of the mg. dipoles) - they are oriented in the direction of the field

$\mu_r$ **is much larger than 1**, depends on the previous magnetic history of the substance (iron, nickel, cobalt)

# Ferromagnetics




**<u>Magnetic hysteresis</u>**

***B*$_r$ - residual magnetism**
**(magnetic remanence)**

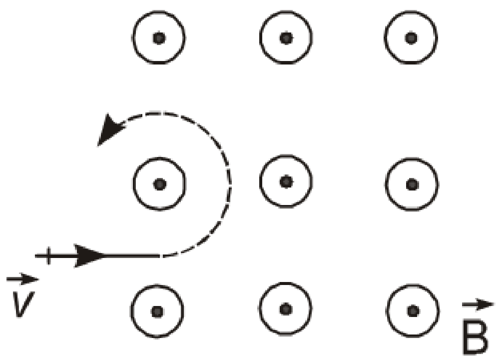
***H*$_x$ - coercive force**

**the area of the curve is proportional to the magnetization energy**

**large hysteresis loop area - large *B*$_r$ -**
**<u>magnetically hard materials</u> - permanent magnets,**
**their opposite - <u>magnetically soft materials</u> - transformers, motors**

# Lorentz force

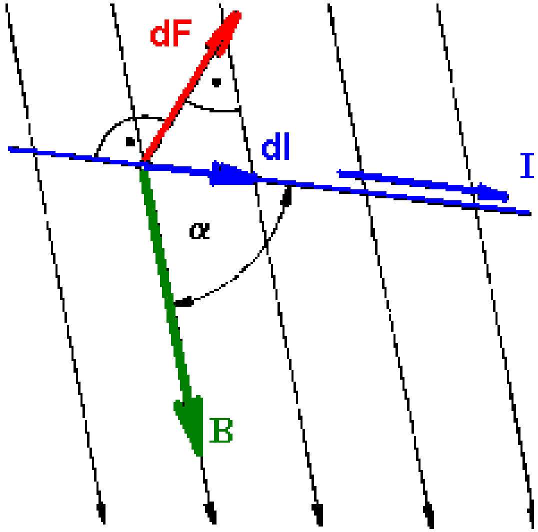
- magnetic force acting on a moving electrically charged particle in a magnetic field
- the right hand rule for the vector product applies – then: the magnetic force is <u>perpendicular</u> to the plane formed by the velocity and magnetic induction vectors
- see the picture - electron in mg. field




$$\vec{F} = Q\vec{v} \times \vec{B}$$

$$\left|\vec{F}\right| = QvB\sin\alpha$$

# Ampere force

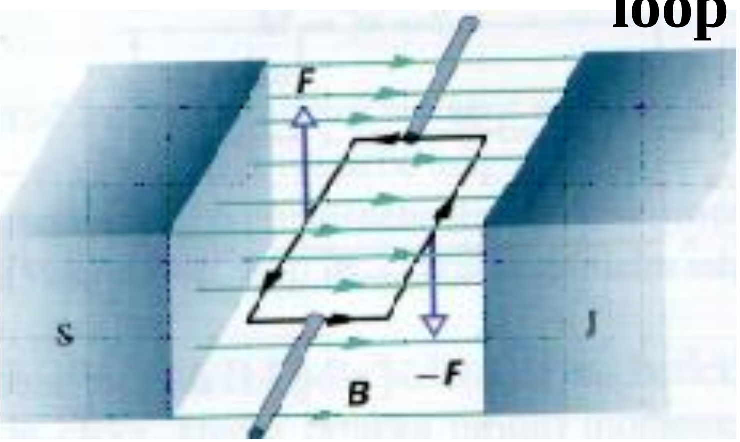



$$\vec{F} = I\int \mathrm{d}\vec{l} \times \vec{B}$$

$$\left|\vec{F}\right| = BIl\sin\alpha$$

**- the right hand rule for the vector product applies – see the picture**

**Moment of force acting on a rectangular loop in a magnetic field**




$$M = a\,F = aBIl$$

$$M = BIS$$

# Forces between parallel conductors

The force acting on a conductor located in the magnetic field created by the other conductor:

$$F_{21} = I_1 l B_2 = I_1 l \frac{\mu I_2}{2\pi r} =$$

$$= \frac{\mu I_1 I_2 l}{2\pi r}$$

**Definition of the SI unit of electric current**

***1 ampere* is the current which, when flowing steadily through two parallel infinite straight conductors of negligible cross-section, 1 m apart, produces a force of $2.10^{-7}$ N per metre of conductor length.**

# Magnetic induction flux Φ

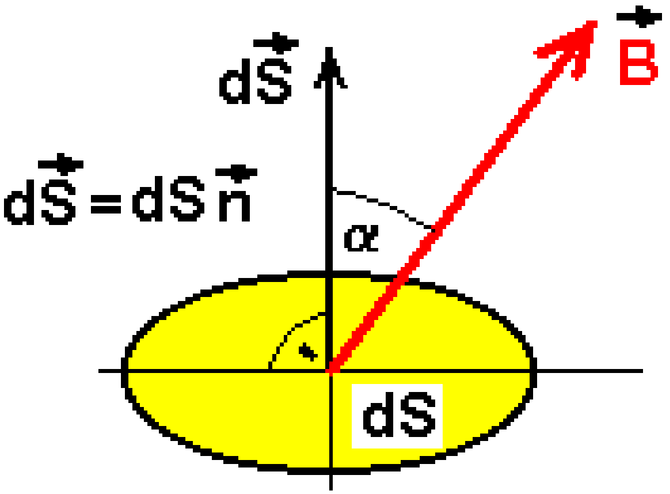
$\alpha$ - angle between $B$ a $S$

$S$ - closed area

$$\mathrm{d}\Phi = \vec{B}\cdot \mathrm{d}\vec{S} = B\,\mathrm{d}S\cos\alpha$$

$$\Phi = \int_S \vec{B}\,\mathrm{d}\vec{S}$$

$$[\Phi] = \mathrm{Wb} = \mathrm{T.m}^2$$



# Electromagnetic induction

*The generation of an electric voltage in a conductor placed in a changing magnetic field.*

## Faraday's law of induction

**for electromotive voltage induced at the ends of the conductor $E_{mn}$**

$$E_{mn} = -\frac{\mathrm{d}\Phi}{\mathrm{d}t}$$

## Lenz rule (law)

***the induced electric voltage has such polarity that the electric current induced by it prevents the change that caused it***

# Electromagnetic induction

$$\Phi = \vec{B} \cdot \vec{S} = BS\cos\alpha$$

$$E_{mn} = -\frac{d\Phi}{dt}$$

**- mg. induction or area or angle changes**

# EMI only by changing the magnetic induction

## I. Angle $\alpha$ and area S are constant

$$E_{mn} = -S\cos\alpha \frac{dB}{dt} = $$

$$= -K\frac{di}{dt}$$

$[K]$ = H (Henry)

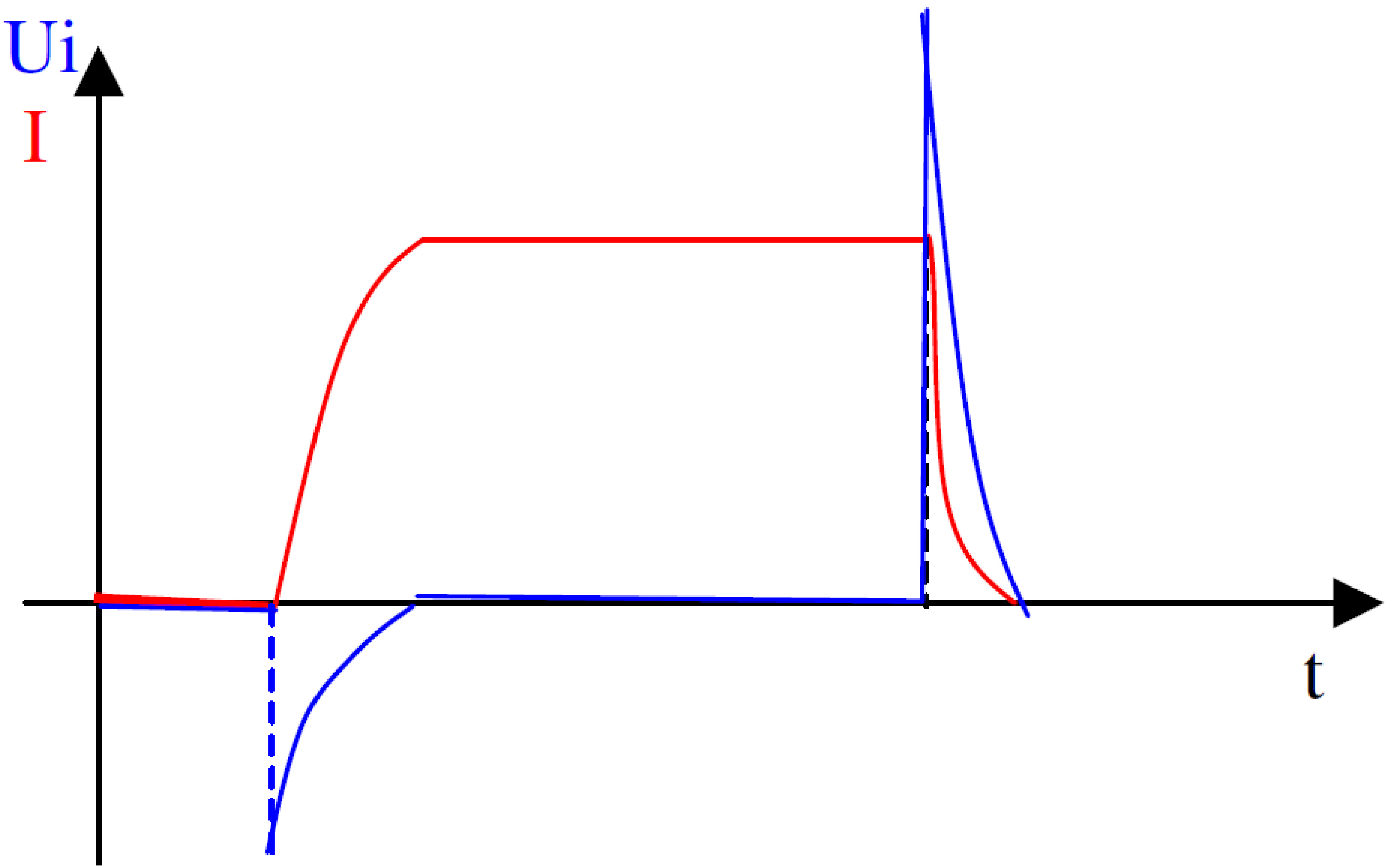
$i$ is the instantaneous value of the (excitation) current

$K$ is a constant ($S.\cos\alpha$ - conductor geometric factor)

***K* = *L*** (**self-inductance**) for inducting a voltage in the conductor that the magnetic field creates

***K* = *M*** (**mutual inductance**) for inducting of voltage in adjacent conductors

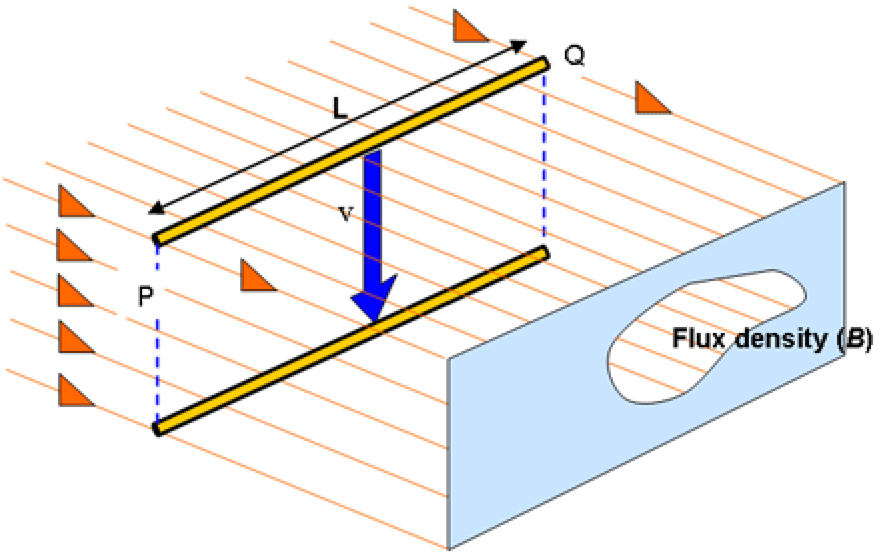
**Time dependence of current and induced voltage in the coil during switching on and off of DC current**




Source: http://www.gymkren.cz/download/ucebni-texty/fyzika/f19.pdf

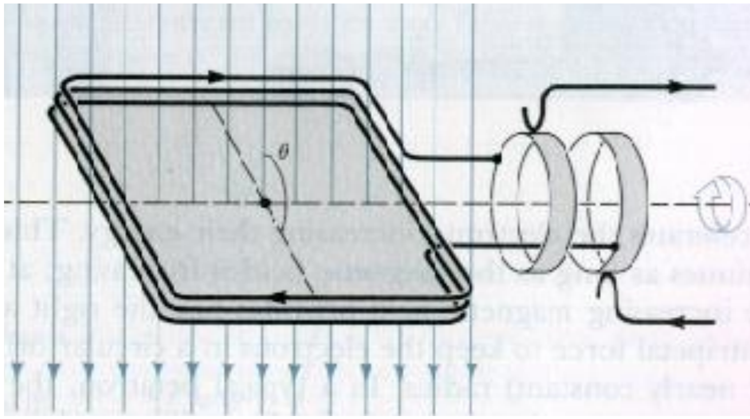
# EMI only by changing the area size

## II. Angle $\alpha$ and mag. induction B are constant




$$E_{mn} = -B\frac{\mathrm{d}S}{\mathrm{d}t} = -Blv$$

# EMI only by changing the angle $\alpha$



$$\Phi = \vec{B} \cdot \vec{S} = BS \cos\alpha$$

$$E_{mn} = -BS \frac{\mathrm{d}(\cos\alpha)}{\mathrm{d}t}$$

$$E_{mn} = BS\omega \sin(\omega t)$$

**III. Area *S* and mag. induction *B* are constant - alternating current production**

$$i = I_0 \sin(\omega t + \varphi)$$

$$u = U_0 \sin(\omega t + \psi)$$

**Amplitude, angular frequency, phase, phase shift**

**+ example with concrete values**

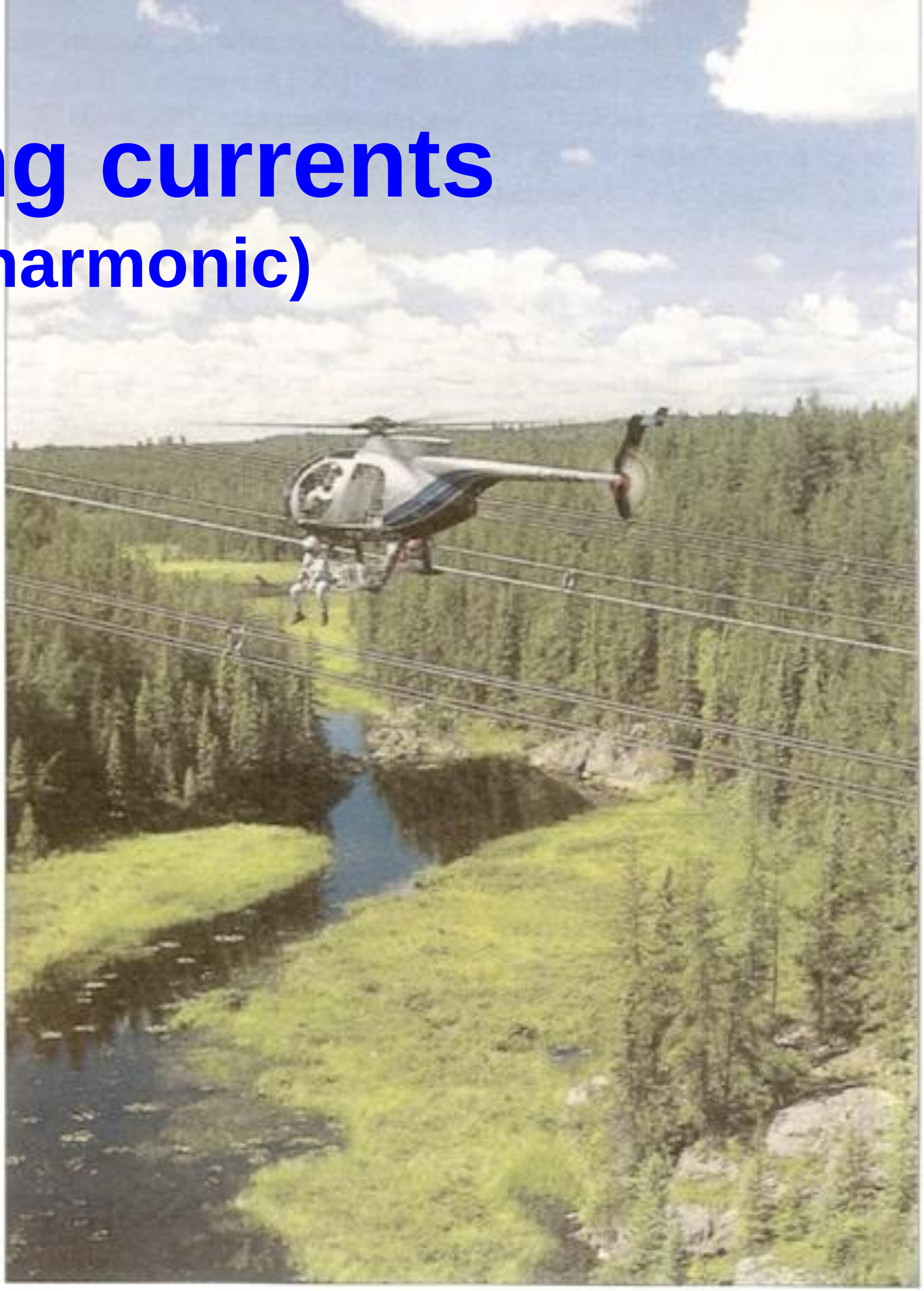
# Energy of the magnetic field

## (conductor of self-induction *L* at current flow *I*)

$$\mathrm{d}A = -E_{mn}\mathrm{d}Q = -E_{mn}i\mathrm{d}t = L\frac{\mathrm{d}i}{\mathrm{d}t}i\mathrm{d}t = Li\mathrm{d}i$$

$$A = \int_0^I Li\,di = \frac{LI^2}{2}$$

# Alternating currents
## (sinusoidal - harmonic)



# Basic characteristics

$$i = I_0 \sin(\omega t + \varphi)$$

$$u = U_0 \sin(\omega t)$$

*i, u* - instant values *I* a *U*

$I_0$, $U_0$ - amplitudes

$\omega = 2\pi f$ - angular frequency

$\varphi$ - phase shift of current with respect to voltage;

- the current overtakes the voltage by $\varphi$

# Effective values of current and voltage

$$I_{ef} = \frac{I_0}{\sqrt{2}}$$

$$U_{ef} = \frac{U_0}{\sqrt{2}}$$

Effective value of AC current *is the value of DC current which, on an ohmic resistor, develops the same amount of heat as a given AC current in one period:*

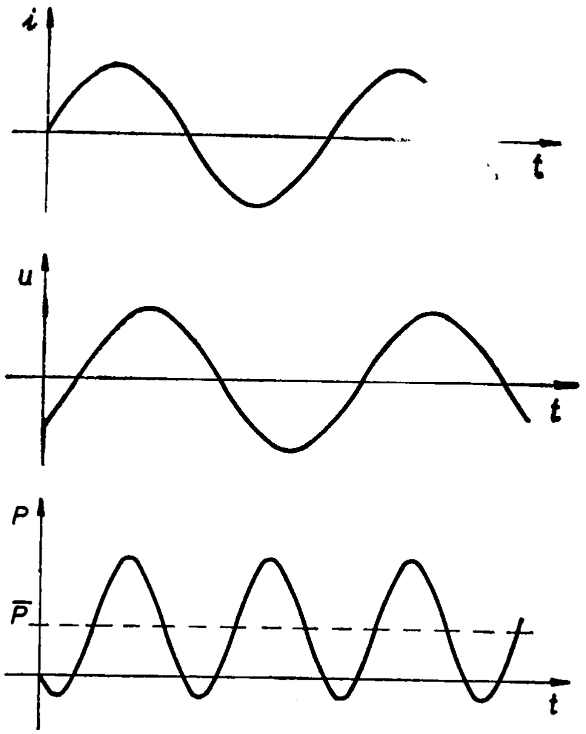
$$Q = RI_{ef}^2 T = R_0\int^T i^2\,\mathrm{d}t$$

**For sinusoidal AC current:**

$$RI_{ef}^2 T = RI_0^2 \int_0^T \sin^2(\omega t + \varphi)\,\mathrm{d}t = RI_0^2 \int_0^T \frac{1 + \cos 2(\omega t + \varphi)}{2}\,\mathrm{d}t =$$

$$= RI_0^2 \left[\frac{t}{2} + \frac{\sin 2(\omega t + \varphi)}{4\omega}\right]_0^T = \frac{RI_0^2 T}{2}$$

# Alternating current power




Instant:

$$P = ui = \frac{U_0 I_0}{2}\left[\cos\varphi - \cos\left(2\omega t + \varphi\right)\right]$$

Average (effective):

$$\overline{P} = U_{ef} I_{ef} \cos\varphi$$

$\cos\varphi$ – power factor

Electric network: $U_0 = U_{ef} \cdot 2^{1/2} = 230.1{,}414$ V ≈ 325 V

# Basic types of electric circuit elements

***R*** **- ohmic (active) resistance**

***L*** **- inductance**

***C*** **– capacity**

**Animation – AC circuit**

$$i = I_0 \sin(\omega t)$$

$$u_R = RI_0 \sin(\omega t)$$

Loop rule (2. KL):

$$u_L = L\frac{\mathrm{d}i}{\mathrm{d}t} = \omega L I_0 \cos(\omega t) = \omega L I_0 \sin\left(\omega t + \frac{\pi}{2}\right)$$

$$u_C = \int \frac{i\mathrm{d}t}{C} = -\frac{I_0}{\omega C} I_0 \cos(\omega t) = \frac{I_0}{\omega C}\sin\left(\omega t - \frac{\pi}{2}\right)$$

# Impedance

## $Z$, $[Z] = \Omega$

$$Z_R = R \qquad Z_L = \omega L \qquad Z_C = 1/\omega C$$

simple relationships for amplitudes and effective voltage values:

$$U_R = RI \qquad U_L = \omega LI \qquad U_C = I/(\omega C)$$

**significant differences are in the phase:**
**capacity: voltage delayed with respect to current by $\pi/2$**
**indukčnost: voltage overtakes current by $\pi/2$**

# Impedance as a complex number

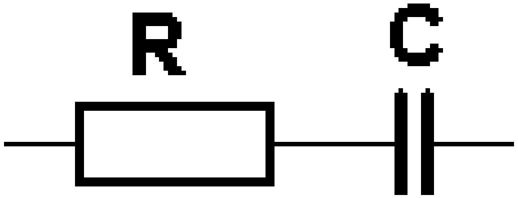
selection of axis directions for different types of impedance (they correspond to the corresponding voltages for time 0):

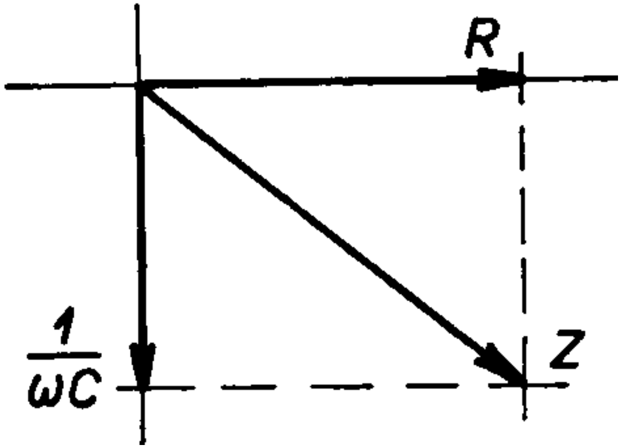
***R*** (reactance) has the usual horizontal axis direction, the real component of the impedance
$Z_L$ inductance) has the direction of the vertical axis, corresponding to the imaginary component

$Z_C$ (capacitance) has the direction opposite to the vertical axis, it corresponds to the negative of the imaginary component

**When working with impedances, we proceed as with ohmic resistances, except that we use operations for working with complex numbers.**

# Example:
# serial RC circuit



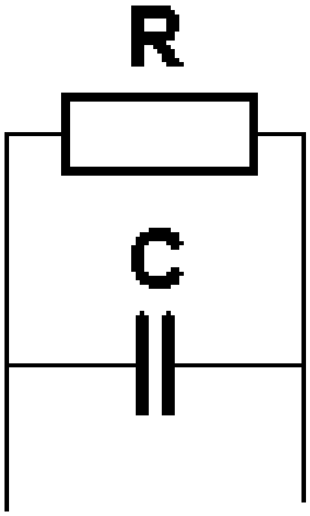

$$\hat{Z} = \hat{Z}_R + \hat{Z}_C = R - i\,\frac{1}{\omega C}$$

$$\left|\hat{Z}\right| = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$$

$$\tan\varphi = -\frac{1}{\omega RC}$$

**- serial RLC circuit + see picture**

# Example:
# parallel RC circuit




**Derive:**

$$\frac{1}{\hat{Z}} = \frac{1}{\hat{Z}_R} + \frac{1}{\hat{Z}_C} = \frac{1}{R} + \frac{1}{-\frac{i}{\omega C}} = \frac{1}{R} + i\omega C$$

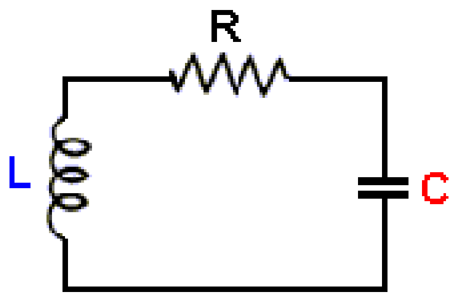
$$\hat{Z} = \frac{1}{\left(\frac{1}{R} + i\omega C\right)} = \frac{\left(\frac{1}{R} - i\omega C\right)}{\frac{1}{R^2} + \omega^2 C^2} =$$

$$= \frac{R}{1+R^2\omega^2C^2} - \frac{R^2\omega C}{1+R^2\omega^2C^2} i = Z_1 + iZ_2$$

$$Z_1 = \frac{R}{1+R^2\omega^2C^2}$$

$$Z_2 = -\frac{R^2\omega C}{1+R^2\omega^2C^2}$$

# Oscillating circuit




**<u>Oscillating RLC circuit - animation</u>:**

Alternating capacitor charging and discharging and inductive activity of coil:

$$iR + u_C + L\frac{di}{dt} = 0$$

(*<u>2. Kirchhoff's law</u>*: the sum of the magnitudes of the instantaneous voltages on the circuit elements must be equal to zero, because there is no voltage source)

$$R\frac{dq}{dt} + \frac{q}{C} + L\frac{d^2q}{dt^2} = 0$$

Neglect the dissipative term with ohmic resistance

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0$$

# Oscillating circuit

$$\frac{d^2q}{dt^2} + \frac{q}{CL} = 0$$

**Obtained equation for oscillating circuit**

$$\frac{d^2x}{dt^2} + \omega_0^{\ 2} x = 0$$

**Linear harmonic oscillator equation**

$$\omega_0^{\ 2} = \frac{1}{CL}$$

$$T = 2\pi\sqrt{CL}$$

Comparison

**Thomson formula** for oscillating circuit natural oscillations

**- theoretical basis for antenna circuits**

# Electromagnetic wave spectrum - animation

**Electromagnetic radiation** Electromagnetic radiation is a form of transverse waves in which changes in *electric and magnetic fields (electric field strength E and magnetic induction B) occur*