Engineering Physics
Physics for Forestry

Kinematics

Kinematics

- **MECHANICS – investigates the interactions between matter and the acting forces**

- **It is divided into two parts:**

- **Kinematics – describes HOW bodies move (position, velocity, acceleration)**

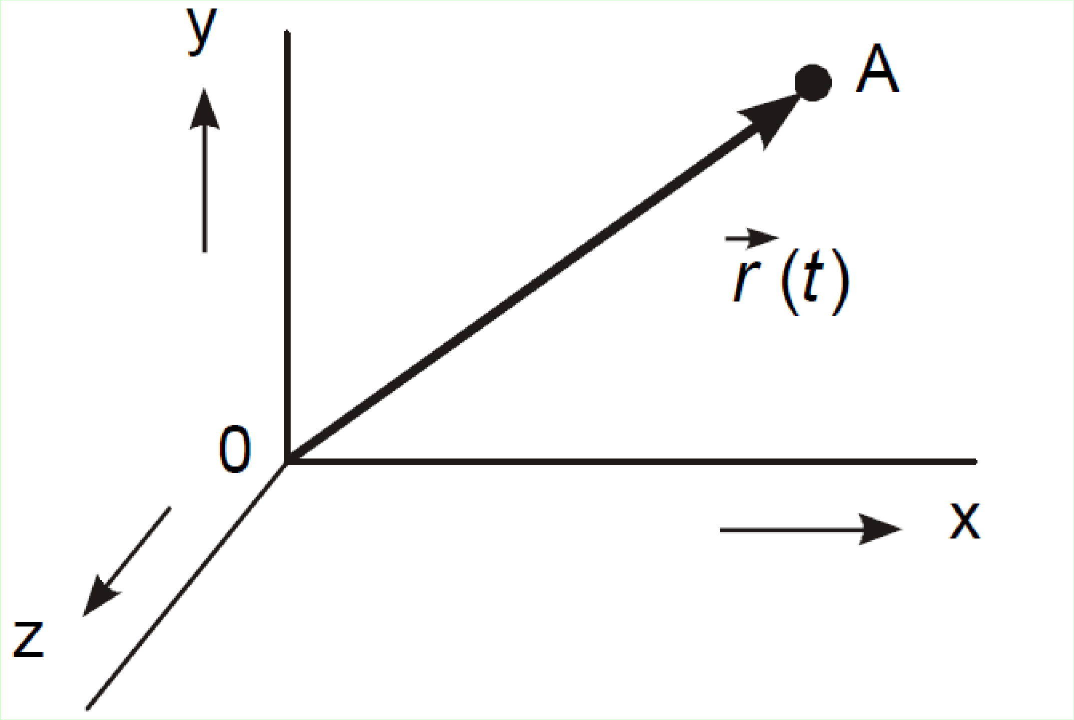
- **Dynamics – explains HOW and WHY bodies move (by acting force)**
- **Statics – a part of dynamics related to the conditions under which the body is at rest (state of equilibrium)**

Kinematics

- **Kinematics**
- **Rest and motion – relative terms**
- **All motions are relative to the observer**
- **The description of the motion of any object (body) is always related to some reference system**
- **For simplicity, we will start with a small body (mass point - particle) whose position in a space can be fully expressed by only one position**
- **Its mass will be described by *m***
- **Later we will describe the real body as a system of particles**

- **The location of a particle relative to the origin of a rectangular coordinate system is given by the position vector $\vec{r}$ – see Fig.**

Kinematics



- If the particle moves, its position vector changes in time,

  then: $\vec{r}(t) = x(t)\vec{i} + y(t)\vec{j} + z(t)\vec{k}$ (function of time),

- where *x (t), y (t), z (t)* are functions, how the motion of the particle along axes *x, y, z* depends on the time

Kinematics

- **The end point of the position vector $\vec{r}(t)$ moves along a curve called the <u>trajectory (path)</u> of moving particle**

- **<u>Distance</u> (scalar quantity) – the length of the path of the moving particle in elapsed time**

- **<u>Displacement</u> (vector quantity) is the distance travelled in a given direction**
- **If a particle moves, its position vector $\vec{r}$ changes from $\vec{r}_1$ to $\vec{r}_2$**
- **Its displacement: $\Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = (x_2 - x_1)\vec{i} + (y_2 - y_1)\vec{j} + (z_2 - z_1)\vec{k}$**
- **($\vec{r}_1$, $\vec{r}_2$ – position vectors of particle at time t_1 and t_2)**

Kinematics

- **Velocity and speed**

- **Velocity** $\vec{v}$ **(vector quantity) – the rate of change of position**

- **Speed** $v = \vec{v} = |\vec{v}|$ **(scalar quantity) – the magnitude of velocity**
- **Let us suppose that a particle moves along *x* axis – one dimension motion, then:**
- **Average speed** $\overline{v}_x$ **– defined as the displacement Δ*x* divided by the time interval Δ*t*:** $\overline{v}_x = \frac{\Delta x}{\Delta t}$

Kinematics

- <u>Instant speed</u> v_x – defined as the limit of the average speed at a particular time as $\Delta t \to 0$ (tends to zero)

- Limiting value $v_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{\mathrm{d}x}{\mathrm{d}t}$ is first derivative of *x* with respect to *t*

- Similar for other coordinates: $v_y = \frac{\mathrm{d}y}{\mathrm{d}t}$, $v_z = \frac{\mathrm{d}z}{\mathrm{d}t}$

- Instant velocity in a unit vector notation:

- $\vec{v} = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} = \frac{\mathrm{d}\left(x\vec{i}+y\vec{j}+z\vec{k}\right)}{\mathrm{d}t} = \frac{\mathrm{d}x}{\mathrm{d}t}\vec{i} + \frac{\mathrm{d}y}{\mathrm{d}t}\vec{j} + \frac{\mathrm{d}z}{\mathrm{d}t}\vec{k} = v_x\vec{i} + v_y\vec{j} + v_z\vec{k}$

- Magnitude of velocity (speed):

- $|\vec{v}| = v = \sqrt{v_x^2 + v_y^2 + v_z^2} = \sqrt{\left(\frac{\mathrm{d}x}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}y}{\mathrm{d}t}\right)^2 + \left(\frac{\mathrm{d}z}{\mathrm{d}t}\right)^2}$

Kinematics

- **SI unit of velocity (speed): [*v*] = m.s^{-1} (meter per second)**

- **<u>Acceleration</u> – the rate of change of velocity**
- **If $\vec{v}$ is not constant, a particle whose velocity changes over time is accelerating (decelerating)**
- **A particle's velocity changes from $\vec{v}_1$ to $\vec{v}_2$ in time Δ*t***
- **Its *average acceleration* during time Δ*t*:** $\vec{a} = \frac{\vec{v}_2 - \vec{v}_1}{\Delta t} = \frac{\Delta\vec{v}}{\Delta t}$
- ***instant acceleration* – limiting value of average acceleration:**
- $\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta\vec{v}}{\Delta t} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t}$
- **instant acceleration in a unit vector notation:**
- $\vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}\left(v_x\vec{i} + v_y\vec{j} + v_z\vec{k}\right)}{\mathrm{d}t} = \frac{\mathrm{d}v_x}{\mathrm{d}t}\vec{i} + \frac{\mathrm{d}v_y}{\mathrm{d}t}\vec{j} + \frac{\mathrm{d}v_z}{\mathrm{d}t}\vec{k} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$

Kinematics

- **The acceleration coordinates are calculated as the first derivative of the velocity components with respect to time or as the second derivative of the position vector components with respect to time**

- **Magnitude of acceleration:** $|\vec{a}| = a = \sqrt{a_x^2 + a_y^2 + a_z^2}$

- **SI unit of acceleration: [*a*] = m.s^{-2} (metre per second squared)**

Kinematics

- **<u>Tangential and centripetal acceleration</u>**
- **Velocity $\vec{v}$ is always tangent to the particles trajectory – see next Fig.**

- **We can express the vector of velocity as $\vec{v} = \vec{\tau}_0 v$, where**

$\vec{\tau}_0 = \frac{\vec{v}}{v}$ – the unit vector of vector $\vec{v}$; *v* is the magnitude of velocity (speed)

- **Acceleration: $\vec{a} = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}(v\,\vec{\tau}_0)}{\mathrm{d}t} = \frac{\mathrm{d}v}{\mathrm{d}t}\vec{\tau}_0 + v\frac{\mathrm{d}\vec{\tau}_0}{\mathrm{d}t} = \vec{a}_t + \vec{a}_c$**

Total acceleration $\vec{a}$ – the vector sum of both – tangential acceleration $\vec{a}_t$ and centripetal acceleration $\vec{a}_c$

Kinematics

- **<u>Tangential acceleration</u> $\vec{a}_t$ is given by the change in magnitude of the velocity and is always in the direction tangent to the trajectory:**

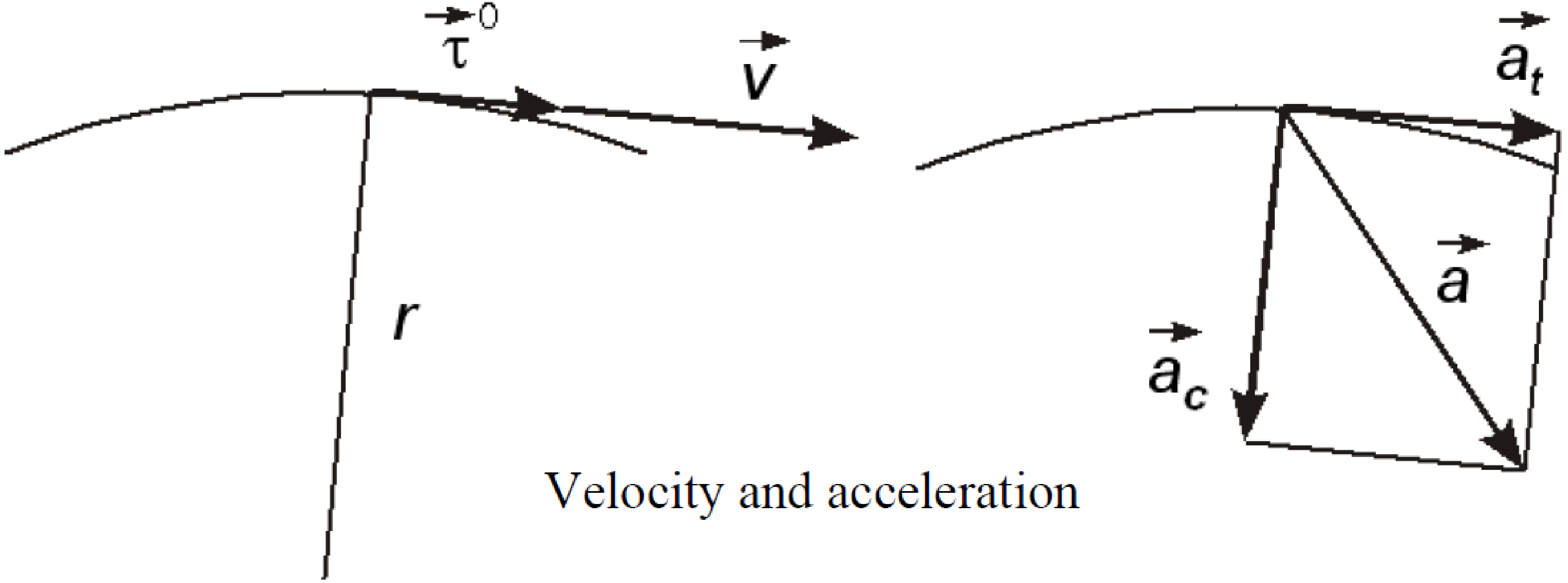
$$\vec{a}_t = \frac{\mathrm{d}v}{\mathrm{d}t}\vec{\tau}_0 \quad (\vec{\tau}_0 \parallel \vec{v})$$

- **<u>Centripetal acceleration</u> $\vec{a}_c$ is given by the change in direction of the velocity and is always perpendicular to the trajectory:**

$\vec{a}_c = v\frac{\mathrm{d}\vec{\tau}_0}{\mathrm{d}t} \quad (\mathrm{d}\vec{\tau}_0 \perp \vec{v})$, **Magnitude of centripetal acceleration:** $a_c = \frac{v^2}{r}$

- **Since tangential and centripetal acceleration are always perpendicular to each other $\vec{a}_t \perp \vec{a}_c$; for the magnitude of total acceleration *a* we use the Pythagorean Theorem:** $a^2 = a_t^2 + a_c^2 \Rightarrow a = \sqrt{a_t^2 + a_c^2}$

Kinematics



Velocity and acceleration

Kinematics

- ***Some special types of motion***

- **Uniform straight-line motion – $\vec{v}$ = constant:**
- ***magnitude* (speed) is constant (*v* = constant – uniform motion)**
- ***direction* is not changed (one dimensional motion in straight line)**

- **Uniform curvilinear motion – $\vec{v} \neq$ constant:**
- ***magnitude* (speed) is constant (v = constant)**
- ***direction* is changed (curvilinear motion)**

Kinematics

- **Uniformly accelerated straight-line motion – $\vec{a}$ = constant:**
- ***magnitude* of acceleration is constant (*a* = constant)**
- ***direction* is not changed (motion in straight line)**

- **Unevenly accelerated motion – $\vec{a} \neq$ constant**

- **If the magnitude of the velocity (speed) increases, the acceleration is positive: $a > 0$**
- **If the magnitude of the velocity (speed) decreases, the acceleration is negative: $a < 0$ (*decelerated* motion)**

Kinematics

- **If we know position vector $\vec{r}$ as a function of time *t*, we can determine velocity $\vec{v}$ and acceleration $\vec{a}$ as:**

- $\vec{r}(t) \rightarrow \vec{v}(t) = \frac{\mathrm{d}\vec{r}}{\mathrm{d}t} \rightarrow \vec{a}(t) = \frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}^2\vec{r}}{\mathrm{d}t^2}$

- **From acceleration $\vec{a}$ we determinate velocity $\vec{v}$ and position vector $\vec{r}$ (functions of motion *x (t), y (t), z (t)*) using integration:**

- $\vec{a}(t) \rightarrow \vec{v}(t) = \int \vec{a}\,\mathrm{d}t \rightarrow \vec{r}(t) = \int \vec{v}\,\mathrm{d}t$ **(see example: *a* = constant ⇒ …)**

$$\begin{pmatrix} v_x = \int a_x \,\mathrm{d}t & , & x = \int v_x \,\mathrm{d}t \\ v_y = \int a_y \,\mathrm{d}t & , & y = \int v_y \,\mathrm{d}t \\ v_z = \int a_z \,\mathrm{d}t & , & z = \int v_z \,\mathrm{d}t \end{pmatrix}$$

https://www.walter-fendt.de/html5/phen/acceleration_en.htm

Dynamics

Example 2.9:

The tram started from the stop with a constant acceleration a_1 = 0.8 m/s^{-2} for time t_1 = 15 s; it continued to move for 50 s with a uniform motion and stopped at the second stop after another 10 s.

Calculate:

a) the speed of the tram after starting,

b) the distance between the two stops,

c) the average speed of the tram,

d) acceleration at stops.

Show the dependence of the speed and acceleration of the tram on time.

[a) v_1 = 12 m·s^{-1} ; b) s = 750 m ; c) $\overline{v}$ = 10 m·s^{-1} ; d) a_3 = -1.2 m·s^{-2}]