Engineering Physics
Physics for Forestry

Work and Energy
Rotation

Work and Energy

- **Work – the force acting on a body along a distance**
- **Work d*W* – the scalar product of the force $\vec{F}$ and the displacement $d\vec{r}$:**
- $dW = \vec{F} \cdot d\vec{r}$
- **If $\vec{F}$ = constant, then $W = \vec{F} \cdot \vec{r}$ = *F r* cosα (*α* - the angle between $\vec{F}$ and $\vec{r}$)**
- **Work is done only by the component of force in the direction of motion *F cos α***

- **Work depends on angle *α*:**
- **0° ≤ *α* < 90°: *W* > 0 - work is done by a force**
- ***α* = 90° ($\vec{F} \perp \vec{v}$): *W* = 0 - no work is done by a force (for example - centripetal force)**
- ***α* < 90° ≤ 180°: *W* < 0 - work is consumed by a force**

Work and Energy

- In a unit vector notation ($\vec{F} = F_x\vec{i} + F_y\vec{j} + F_z\vec{k}$, $d\vec{r} = dx\vec{i} + dy\vec{j} + dz\vec{k}$):
- $dW = \vec{F} \cdot d\vec{r} = F_x dx + F_y dy + F_z dz$
- If a body is moving from point “A” to point “B”, total work *W*:
- $W = \int_A^B \vec{F} \cdot d\vec{r} = \int_A^B \left(F_x dx + F_y dy + F_z dz\right)$
- SI unit of work [*W*] = [*F*] . [*s*] = N . m = J (joule)
- <u>Power</u> – defined as the work done per a unit of time: $P = \frac{dW}{dt}$
- <u>Average power</u>: $\bar{P} = \frac{W(t)}{t}$, *W(t)* is work done in time *t*
- <u>Instantaneous power</u>: $P = \frac{dW}{dt} = \frac{d\left(\vec{F} \cdot \vec{r}\right)}{dt} = \vec{F} \cdot \frac{d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$, ($\vec{F} = \text{constant}$)
- $\vec{v}$ – instantaneous velocity of body
- SI unit of power [*P*] = [*W*] / [*t*] = J/s = W (watt)

Work and Energy

- <u>**Efficiency:**</u>
- **The useful power done by machine *P* (output) is always less than the power absorbed by the machine P_0 (input)**

$$\text{POWER INPUT } P_0 \longrightarrow \underset{\text{(losses)}}{\boxed{\text{MACHINE}}} \longrightarrow \text{POWER OUTPUT } P$$

- **The efficiency *η* – the ratio of output power *P* to input power P_0:**
- $\eta = \frac{P}{P_0} < 1$ **– non-dimensional physical quantity**
- **Sometimes the efficiency is expressed in percents:**
- $\eta = \frac{P}{P_0} \cdot 100\ \%$

Work and Energy

- **Energy:**
- **Energy can be converted into work**
- **Work W is done, when the energy of a system ΔE is changed: $\Delta E = W$**
- ***Mechanical energy***
- **Kinetic energy E_k – the energy of a moving body – energy of motion**
- **We define kinetic energy of translational motion:**
- **The work done by force $\vec{F}$ along the curve of length ℓ:**
- $W = \int_\ell \vec{F} \cdot d\vec{r} = \int_\ell F cos\alpha \, dr$; $F cos\alpha = ma_t = m\frac{dv}{dt} = m\frac{dr}{dt}\frac{dv}{dr} = mv\frac{dv}{dr}$,

then: $W = \int_{v_1}^{v_2} mv \, dv = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2 = \Delta E_k$

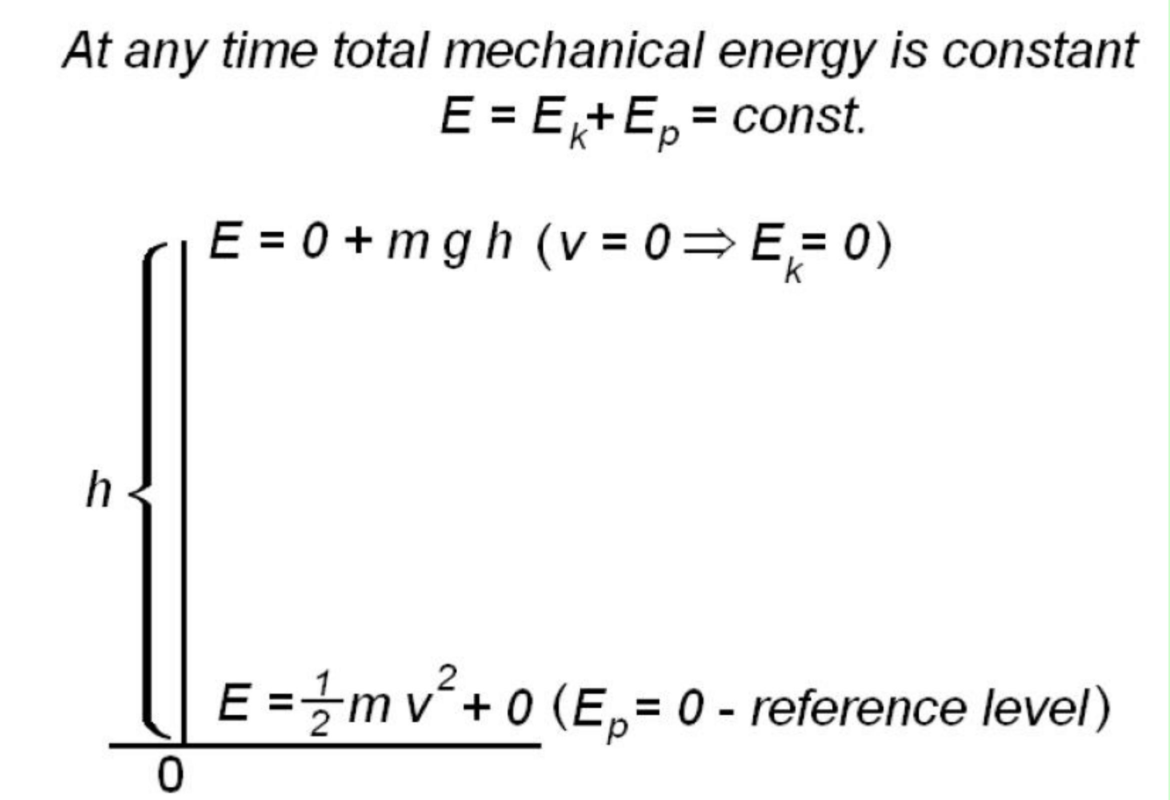
- **v_1 – the initial and v_2 – the final speed of a body**
- **ΔE_k – the change of kinetic energy of a body**

Work and Energy

- **The work done on a body is equal to its change in kinetic energy**
- **($\Delta E_k = W$): Work – energy theorem**
- **<u>Potential energy</u> E_p – associated with the mutual position of bodies in the system:**
- ***Example***
- **<u>Gravitational potential energy</u> for a system of two bodies, the Earth and a body of mass *m*:**
- **The change in gravitational potential energy ΔE_p – the work *W* consumed by the force of gravity $\vec{G}$ when a body of mass *m* moves from height y_2 to height y_1: $\Delta E_p = E_p(2) - E_p(1) = W = G\ \Delta h = mg\ (y_2 - y_1)$**
- **the gravitational potential energy $E_p(h)$ of body of mass *m* in height *h*: $E_p(h) = m.g.h$ (from a horizontal surface where the potential energy is zero; $E_p(h = 0) = 0$ - reference level)**

Work and Energy

- **<u>Conservation of mechanical energy</u>**
- **For any isolated system (the resultant of external forces is zero):**
- **The total mechanical energy ($E_k + E_p$) of an isolated system is constant**
- ***Example* – Free fall – see Fig.**



Work and Energy

- **The total mechanical energy at the top (potential energy) is equal to the total mechanical energy at a point at which the body hits the ground (kinetic energy):**
- $\boldsymbol{mgh = \frac{1}{2}mv^2}$
- $\Downarrow$
- **Then the speed v of a body falling from the height *h*:**
- $\boldsymbol{v = \sqrt{2gh}}$
- ***v* – independent of the mass of body *m* , air resistance is neglected**

Work and Energy

Example 3.25:

The 10 g projectile hits a fixed wooden block, which it penetrates to a depth of 12 cm. The mean drag force of the wood is 3000 N. At what speed did the projectile hit the block?

[v = 270 m/s]

Rotation

- **<u>Rotational motion of a rigid body</u> (no deformation of body)**

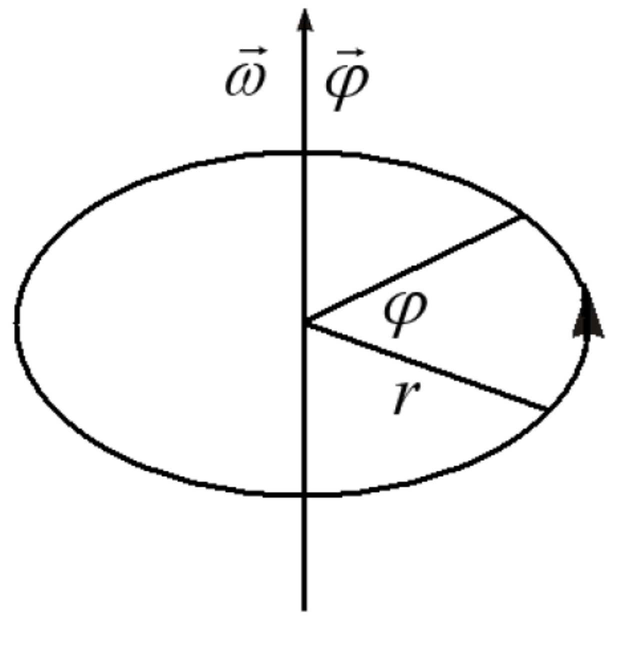
- **<u>General motion</u> of rigid body can be resolved into two parts:**

- **1 – <u>Translation motion</u> of body – determined by the motion of its centre of mass**

- **2 – <u>Rotational motion</u> - all mass points in the body move in circles, the centres of these circles lie on the line called the axis of rotation (rotational axis)**

Rotation

- **Similar to the displacement $d\vec{r}$ for translation motion; rotational motion of a rigid body is described by the angular displacement $d\vec{\varphi}$ - see Fig.**



Rotational motion

- **<u>Angle of rotation</u> $\vec{\varphi}(t)$ – a function of time**
- **[φ] = rad (radian) (circle 360° = 2π radians)**

Rotation

- **Angular velocity** $\vec{\omega} = \frac{d\vec{\varphi}}{dt}$ **, [ω] = rad/s**
- **Angular speed** $\omega = |\vec{\omega}|$ **(magnitude of the angular velocity)**
- **Angular acceleration** $\vec{\varepsilon} = \frac{d\vec{\omega}}{dt} = \frac{d^2\vec{\varphi}}{dt^2}$ **, [ε] = rad · s^{-2}**

- **Rotational kinematics**
- ***Constant angular acceleration***
- **Initial conditions: at time t = 0, angular speed $\omega(0)$ and angle $\varphi(0)$**
- **1) ε = constant ≠ 0**
- **Uniformly accelerated rotational motion (ω is increasing with time t, angular acceleration $\varepsilon > 0$)**
- **(decelerated - ω is decreasing with time t , $\varepsilon < 0$)**

Rotation

- **From** $\varepsilon = \frac{d\omega}{dt} \Rightarrow d\omega = \varepsilon\, dt \Rightarrow \omega(t) = \int \varepsilon\, dt = \varepsilon\, t + \omega(0)$ **– angular speed**
- **From** $\omega = \frac{d\varphi}{dt} \Rightarrow \varphi(t) = \frac{1}{2}\varepsilon\, t^2 + \omega(0)\, t + \varphi(0)$ **– angle of rotation**

- **If the angle of rotation after time *t* is φ(t), then the number of rotations *n(t)* during this time:** $n(t) = \frac{\varphi(t)}{2\pi}$ **, (one rotation - circle - is 2*π* radians)**

- **<u>2) If *ε* = 0</u> (angular acceleration is zero) ⇒ *ω* = *ω*(0) = constant ≠ 0**
- **Uniform rotational motion (angular speed is constant)**
- **Angle of rotation φ(t) = *ω(0) . t* + φ(0)**
- **If angular speed *ω* is constant, the speed of point *v*:**
- $v = \frac{2\pi r}{T} = 2\pi f r = \omega r$ **(2*πr* – path during one period *T*)**

Rotation

- **<u>Moment of inertia</u> of a rigid body (relative to the axis of rotation):**
- $J = \int_m r^2 dm = \int_V r^2 \rho \, dV$ **(*m* – mass, *V* – volume, *ρ* - density of body)**
- ***r* – perpendicular distance from the axis of rotation to each mass element d*m* in the body**
- **<u>Moment of inertia describes how the mass of rotating body is distributed about its axis of rotation</u>**
- **SI unit of moment of inertia $[J] = [m].[r^2] = \mathrm{kg\ .\ m^2}$**

- **<u>Rotational kinetic energy of a rigid body</u>:**
- $E_k = \frac{1}{2} J \omega^2$ **(*J* - moment of inertia of rigid body)**
- **Experiment with two flywheels**

Rotation

Example 3.48:

The hoop of mass 1 kg is rolling on horizontal plane with speed 0.2 m/s. Find the kinetic energy of rolling hoop. Moment of inertia of the hoop

$J = m\,r^2$.

[E_k = 0.04 J]

Rotation

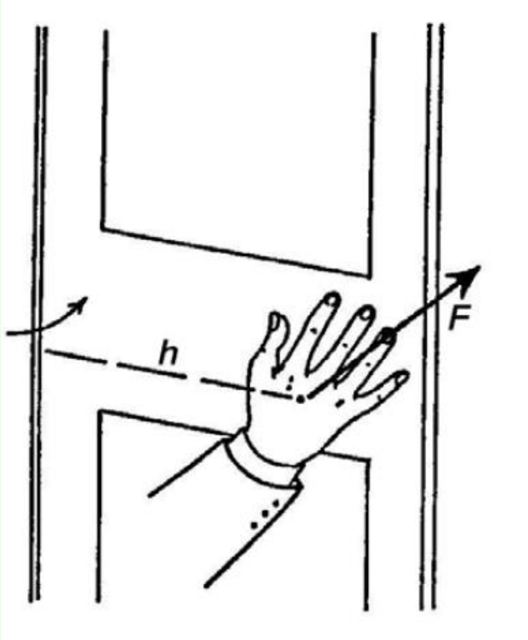
- **<u>Torque (moment of force)</u>:**
- **If we want to rotate a rigid body around a fixed axis, we have to apply a rotational force - a <u>torque</u> - to this body**
- **Torque $\vec{M}$ acting on body is defined as the vector product of a position vector $\vec{r}$ of force $\vec{F}$ to the axis of rotation *o* and the force $\vec{F}$: $\vec{M} = \vec{r} \times \vec{F}$**

Magnitude of the torque: $M = F \cdot r \sin\varphi = F \cdot h$ - see Fig.

SI unit of torque [*M*] = [*F*].[*r*] = N.m (newton-metre)

Application

To turn the door (see picture), press your hand on the edge of the door, not on the hinge. The rotational effect of the force (torque) when turning is proportional to our force and the distance of the force from the axis of rotation.



Rotation

- **<u>Angular momentum</u>**
- **Law of force is the basic law of dynamics for translation motion:**
- $\vec{F} = \frac{d\vec{p}}{dt}$ **(**$\vec{p} = m\,\vec{v}$ **– momentum of body)**
- **Using vector product of** $\vec{r}$ **both sides of this equation:**
- $\vec{r} \times \vec{F} = \vec{r} \times \frac{d\vec{p}}{dt}$
- **The left side of this equation: the torque** $\vec{M}$
- **the right side of this equation: so-called angular momentum** $\vec{b}$**:**
- $\vec{M} = \vec{r} \times \frac{d\vec{p}}{dt} = \vec{r} \times \frac{m\,d\vec{v}}{dt} = \frac{d(\vec{r} \times m\,\vec{v})}{dt} = \frac{d\vec{b}}{dt} \Rightarrow \vec{b} = \vec{r} \times m\,\vec{v}$

Rotation

- Analogical to the linear momentum $\vec{p} = m\,\vec{v}$, the angular momentum can be also expressed as $\vec{b} = J\,\vec{\omega}$

- We can write Newton's Second law of motion for rotational motion:

- $\overrightarrow{M} = \frac{d\vec{b}}{dt}$ or $\overrightarrow{M} = J\,\vec{\varepsilon}$,
- J – moment of inertia
- $\vec{\omega}$ – angular velocity
- $\vec{\varepsilon}$ – angular acceleration