ground water

Technical Note/

Analysis of Steady Ground Water Flow Toward Wells in a Confined–Unconfined Aquifer

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Abstract

A confined aquifer may become unconfined near the pumping wells when the water level falls below the confining unit in the case where the pumping rate is great and the excess hydraulic head over the top of the aquifer is small. Girinskii's potential function is applied to analyze the steady ground water flow induced by pumping wells with a constant-head boundary in a mixed confined-unconfined aquifer. The solution of the single-well problem is derived, and the critical radial distance at which the flow changes from confined to unconfined condition is obtained. Using image wells and the superposition method, an analytic solution is presented to study steady ground water flow induced by a group of pumping wells in an aquifer bounded by a river with constant head. A dimensionless function is introduced to determine whether a water table condition exists or not near the pumping wells. An example with three pumping wells is used to demonstrate the patterns of potentiometric surface and development of water table around the wells.

Introduction

When a confined aquifer is heavily developed with pumping wells, drawdown may be great enough to cause a transition from confined to unconfined flow near the pumping wells while the water level falls below the confining unit of the aquifer. The problem is usually neglected when Dupuit's equation is applied for radial steady flow or the Theis equation is applied for transient radial flow in a confined aquifer. However, this transition is common, especially for pumping in riverside aquifers underlying fine-grained deposits when the aquifers are exploited. In the natural state, the potentiometric surface in such aquifers is usually higher than the top of the aquifers. Water table near the wells can be easily developed by pumping wells with the decrease of hydraulic head.

Mixed confined and unconfined ground water flow in an initially confined aquifer cannot be sufficiently described through a single Dupuit's equation or Theis equation. Chen et al. (1961) gave an analytic solution using the so-called subsection method for an aquifer undergoing conversion from confined to unconfined conditions induced by a single pumping well. With this method, different equations were applied to the confined and unconfined zones. Moench and Prickett (1972) presented a model of the artesian water table conversion due to a single pumping well. The thickness of the aquifer in the model is assumed to be much larger than the difference between the water table and the top of the aquifer so that the transmissivity could be treated as a constant. This assumption may not be appropriate for relatively thin aquifers. Electrical analog (Rushton and Wedderburn 1971) and numerical method (Elango and Swaminathan 1980) have also been applied to analyze the behavior of aquifers that change from the confined to unconfined states. To our best knowledge, analytic solutions for a well-group problem with confined-unconfined zones have not been found in the literature. In this paper, an analytic solution of steady ground water flow

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toward pumping wells in an aquifer undergoing conversion from confined to unconfined conditions is presented. The solution is applied to study the patterns of potentiometric surface and development of water table around the wells near a constant-head boundary.

Application of Girinskii's Potential Function

Girinskii's potential function (Girinskii 1946; also can be seen in Chen 1966; Bear 1972) is applied here to analyze ground water flow in confined and unconfined aquifers. This function is robust for horizontally extended, multiple-layered aquifers. According to the definition of Girinskii's potential function, the potential of a uniform confined aquifer can be described as

$$\varphi = Kb(H - b/2) \tag{1}$$

For a uniform unconfined aquifer, the potential is written as

$$\varphi = Kh^2/2 \tag{2}$$

Where φ is the Girinskii's potential, K is the hydraulic conductivity of the aquifer, b and H are thickness and hydraulic head of the confined aquifer, respectively, and h is the thickness of the saturated area in the unconfined aquifer.

For a confined aquifer exploited by pumping wells, Equation 2 is valid when it is converted from fully saturated condition to water table condition. The critical Girinskii's potential, or the potential at the location where confined flow is converted to unconfined flow, is given as

$$\varphi_c = Kb^2/2 \tag{3}$$

If φ is less than φ_c, the confined-unconfined conversion occurs.

Steady radial flow to a single pumping well in an axial symmetrical confined or unconfined aquifer with constant thickness can then be described with Girinskii's potential as follows

$$Q = 2\pi r \cdot \mathrm{d}\varphi/\mathrm{d}r \tag{4}$$

The general solution of Equation 4 is

$$\varphi = Q/2\pi \cdot \ln r + C_1 \tag{5}$$

Where C_1 is a constant determined by boundary conditions.

Analytic Solution for Ground Water Flow toward Pumping Wells in a Riverside Aquifer

For a pumping well near a constant-head linear boundary (for example, a river fully penetrates through the aquifer), the solution can be obtained by considering the superimposed effect of the pumping well and an image injection well with the same pumping rate (Figures 1a and 1b). Girinskii's potential satisfies the Laplace equation, and the superposition rule can be applied here. Applying Equation 5 to the pumping well and the image well and considering the constant-head boundary, the solution of potential is given by

$$\varphi = \varphi_0 - \frac{Q}{2\pi}\ln\left(\frac{r'}{r}\right) \tag{6}$$

Where $\varphi_0 = Kb(H_0 - 0.5b)$ is the Girinskii's potential at the constant-head boundary, r and r' are the distance from the observation well $p(x, y)$ to the pumping well and the image well, respectively. Equation 6 is the solution for a single river-neighboring well. The location where the conversion of confined to unconfined condition occurs can be found by $\varphi = \varphi_c$. Applying Equation 3, Equation 6 becomes

$$\frac{1}{2}Kb^2 = Kb\left(H_0 - \frac{b}{2}\right) - \frac{Q}{2\pi}\ln\sqrt{\frac{(x-L)^2 + y^2}{(x+L)^2 + y^2}} \tag{7}$$

The curve traced by point (x, y) that satisfies Equation 7 can then be described as

$$\left(x + L\frac{\lambda^2+1}{\lambda^2-1}\right)^2 + y^2 = \left(\frac{2L\lambda}{\lambda^2-1}\right)^2 \tag{8}$$

It is a circle around the pumping well (see the circle in Figure 1b). The geometry of the circle is controlled by the following geometric factors:

$$a_1 = 2L/(\lambda - 1), \quad a_2 = 2L/(\lambda + 1), \quad R = 2L\lambda/(\lambda^2 - 1) \tag{9}$$

Where

$$\lambda = \exp[2\pi Kb(H_0 - b)/Q] \tag{10}$$

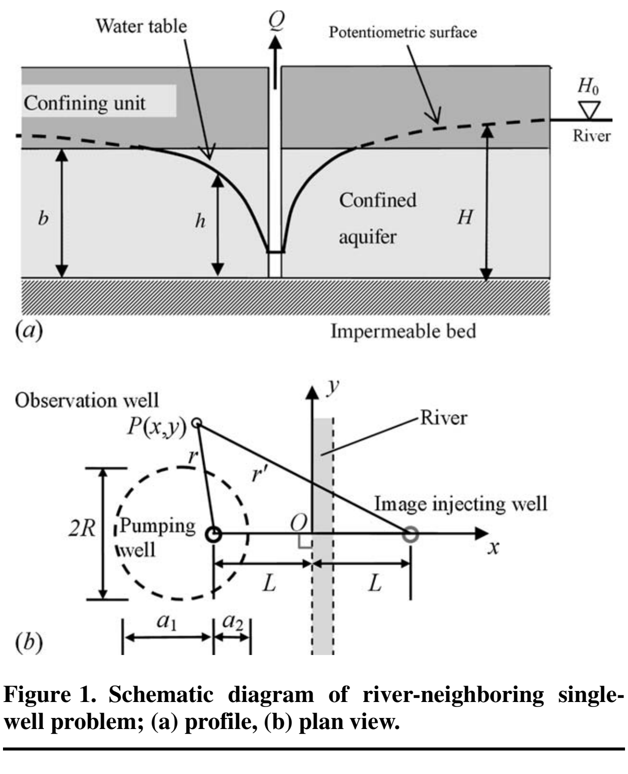

Figure 1. Schematic diagram of river-neighboring single-well problem; (a) profile, (b) plan view.

For the problem of a wellfield with n wells near a linear constant-head boundary, assuming the pumping rates at the wells are Q_1, Q_2, …, Q_n, the steady ground water flow can be obtained using Equation 6 and the superposition method as follows

$$\varphi = \varphi_0 - \sum_{i=1}^{n} \frac{Q_i}{2\pi} \ln\left(\frac{r_i'}{r_i}\right) \quad (11)$$

Where r_i is the distance from the pumping well i to the observation point p and r_i' is the distance from the image well i to the observation point.

In the confined zone, applying Equation 1 to Equation 11, the hydraulic head is

$$H = H_0 - \frac{1}{2\pi Kb} \sum_{i=1}^{n} Q_i \ln\left(\frac{r_i'}{r_i}\right) \quad (12)$$

In the unconfined zone, applying Equation 2 to Equation 11, the hydraulic head is calculated as

$$h^2 = 2bH_0 - b^2 - \frac{1}{\pi K} \sum_{i=1}^{n} Q_i \ln\left(\frac{r_i'}{r_i}\right) \quad (13)$$

A dimensionless function is introduced herein to determine whether water table exists or not at a location $p(x, y)$ as follows

$$f(x,y) = \left(1 - \frac{b}{H_0}\right) - \frac{1}{2\pi KbH_0} \sum_{i=1}^{n} Q_i \ln\left(\frac{r_i'}{r_i}\right) \quad (14)$$

Introducing Equation 14 to Equations 12 and 13, the dimensionless hydraulic head at location $p(x, y)$ is given as follows

$$H_D = \frac{H}{H_0} = \frac{b}{H_0} + f(x,y), \quad f(x,y) \geq 0 \quad (15\text{-}1)$$

$$H_D = \frac{h}{H_0} = \frac{b}{H_0}\sqrt{1 + \frac{2H_0 f(x,y)}{b}}, \quad f(x,y) < 0 \quad (15\text{-}2)$$

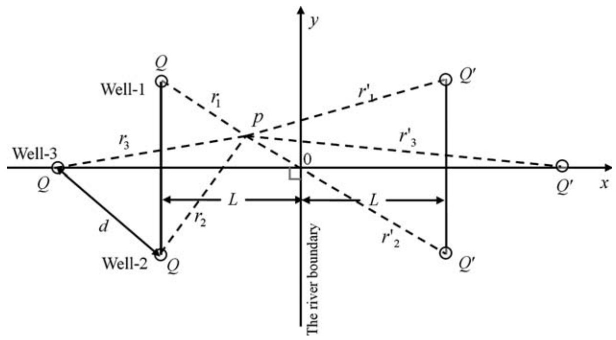

Figure 2. Example of the river-neighboring well group with three pumping wells.

The interfaces where the aquifer is converted from a confined to unconfined state can be derived from $f(x, y) = 0$. While $f(x, y) > 0$, the hydraulic head is higher than the top of the aquifer and satisfies Equation 15-1; while $f(x, y) < 0$, water table condition occurs and the thickness of the saturated area is given by Equation 15-2.

Example

To investigate the patterns of hydraulic head distribution while the conversion occurs, a typical well-group example is considered herein. In this example, there are three pumping wells arranged at vertexes of an equilateral triangle as shown in Figure 2. Pumping rates at the wells are the same and equal to Q. The side length of the equilateral triangle is d. Then, the critical function $f(x, y)$ defined in Equation 14 can be rewritten as

$$f(x,y) = \left(1 - \frac{b}{H_0}\right) - \frac{Q}{2\pi KbH_0} \ln\left(\frac{r_1' r_2' r_3'}{r_1 r_2 r_3}\right) \quad (16)$$

Locations where conversion occurs are given by $f(x, y) = 0$. However, while $b \geq H_0$, the aquifer becomes a fully

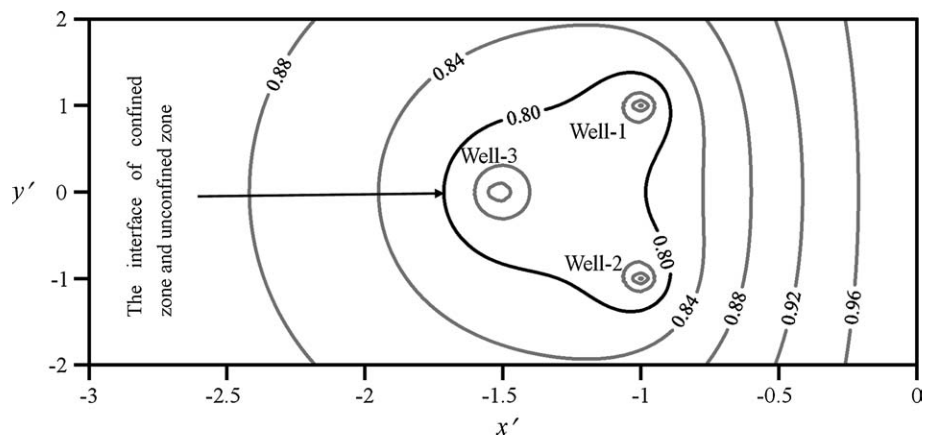

Figure 3. Contours of dimensionless hydraulic head H_D around the well group. The dimensionless factors are $\beta = 0.03$, $\gamma = 0.3$, and $\alpha = 0.8$.

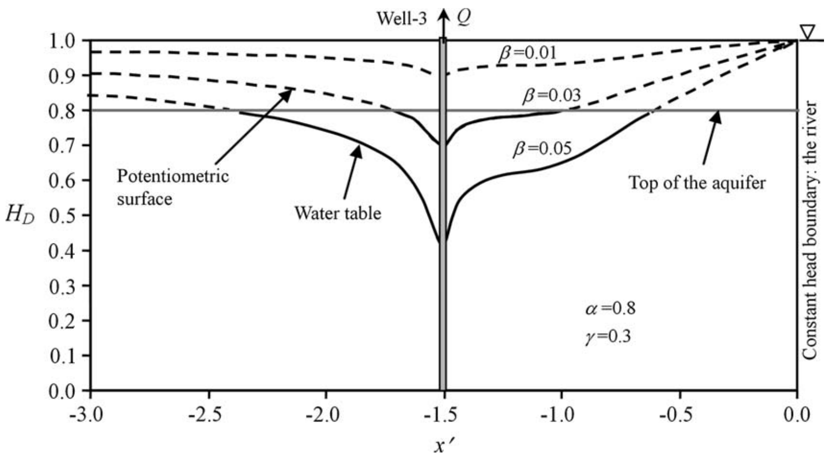

Figure 4. Potentiometric surface and water table indicated by H_D along the cross section of $y' = 0$ with different values of β.

unconfined aquifer and the hydraulic head can be obtained from Equation 15-2. From the distribution of pumping wells as shown in Figure 2, the distances from location $p(x, y)$ in the flow field to wells are $r_i = \sqrt{(x - x_i)^2 + (y - y_i)^2}$, $r_i' = \sqrt{(x + x_i)^2 + (y - y_i)^2}$, $i = 1,2,3$, where (x_i, y_i) is the location of well i determined by geometry factors d and L in Figure 2. Applying dimensionless coordinates

$$x' = x/L, \quad y' = 2y/d \tag{17}$$

the critical function can be rewritten as

$$f(x,y) = (1 - \alpha) - \frac{\beta}{2\alpha} \ln \left(\frac{(x' - 1)^2 + \gamma^2 (y' - 1)^2}{(x' + 1)^2 + \gamma^2 (y' - 1)^2} \times \frac{(x' - 1)^2 + \gamma^2 (y' + 1)^2}{(x' + 1)^2 + \gamma^2 (y' + 1)^2} \times \frac{(x' - 1 - \sqrt{3}\gamma)^2 + (\gamma y')^2}{(x' + 1 + \sqrt{3}\gamma)^2 + (\gamma y')^2} \right) \tag{18}$$

Where α, β, and γ are dimensionless factors defined as:

$$\alpha = b/H_0, \quad \beta = Q/2\pi K H_0^2, \quad \gamma = d/2L \tag{19}$$

α is the indicator for the status of the aquifer. If $\alpha \geq 1$, the aquifer is fully unconfined; while $\alpha < 1$, confined-unconfined conditions can be developed in the aquifer.

The distribution of dimensionless hydraulic head, H_D, along x' and y' for $\beta = 0.03$, $\gamma = 0.3$, and $\alpha = 0.8$ is shown in Figure 3. It is obvious that when $H_D < \alpha$, the water table occurs. Hence, the interface of the confined and unconfined zones is indicated by the contours of $H_D = \alpha = 0.8$ as shown in Figure 3, in which the three wells are encircled. In Figure 4, different patterns of potentiometric surface and water table vs. β (increases with increase of pumping rate) for $\alpha = 0.8$, $\gamma = 0.3$, and $y' = 0$ are shown. Water table is extensively developed around well 3 while $\beta \geq 0.03$.

Summary

An analytic approach based on the Girinskii's potential function is applied to analyze the steady ground water flow toward pumping wells in a confined-unconfined aquifer. The analytic solution can be used to predict the confined-unconfined conversion caused by a single pumping well or a group of wells when the water level near the pumping wells falls below the top of the originally confined aquifer. The shape of the conversion zone and the distribution of the hydraulic head are illustrated through a well-group example near a river that acts as a fixed-head boundary.

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