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Constant-head test in a leaky aquifer with a finite-thickness skin

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SUMMARY

Constant-head test is a commonly used aquifer testing method in groundwater hydrology. A mathematical model for constant-head test in a leaky aquifer with a finite-thickness skin was developed in this study. Three different aquifer–aquitard systems were considered and the Laplace-domain solutions were obtained and then inverted numerically with the Stehfest method to yield the time-domain solutions. The well discharges for different cases were computed and a sensitivity analysis of the well discharge on different parameters was performed. The results indicated that the dimensionless transmissivity of the aquitard had little effect on the well discharge at early times while a larger transmissivity of the aquitard leaded to a larger well discharge at late times. The well discharge for the positive skin was smaller than that without the skin while the well discharge for the negative skin was larger than that without the skin, where positive and negative skins refer to the cases in which the permeability values of the skin zones are less and greater than that of the formation zone, respectively. A thicker skin resulted in a smaller well discharge for the positive skin case but leaded to a larger well discharge for the negative skin case at late times. We also found that the drawdown for the positive skin case was less than that for the negative skin case at the same time, and a positive skin might result in delayed response of the aquifer to pumping. The sensitivity analysis indicated that the well discharge was sensitive to the properties of the skin zone, but not sensitive to the properties of the aquitards for the aquifer–aquitard system presented in this study.

1. Introduction

Constant-head test is a technique commonly used to estimate the aquifer parameters such as the storage coefficient and the hydraulic conductivity (Chen and Chang, 2002). For a constant-head test, the hydraulic head or the drawdown in the test well remains constant and the well discharge is measured as a function of time. The measured well discharge versus time data can be used to evaluate the aquifer parameters using an appropriate flow theory. For instance, Jacob and Lohman (1952) obtained an analytical solution of the well discharge for a constant-head test in a confined aquifer and developed a method to estimate the storage coefficient and the transmissivity. Hantush (1964) obtained a similar solution for a constant-head test in leaky aquifers. In addition, many researchers studied the constant-head test problem (e.g., Hantush, 1959; Mishra and Guyonnet, 1992; Hiller and Levy, 1994; Murdoch and Franco, 1994; Chen and Chang, 2002; Chang and Chen, 2002). Jones et al. (1992) and Jones (1993) pointed out that the constant-head test was particularly useful when the transmissivity of the aquifer was relatively small. Constant-head test was also performed in boreholes or piezometers to determine the hydraulic conductivity of the clays (Wilkinson, 1968; Tavenas et al., 1990).

A problem that needs to be considered for a constant-head test is the well skin. The well skin is a small region surrounding the well and its permeability is different from that of the formation zone. This skin zone can be caused by the drilling mud and/or the formation damage during the well drilling and installation procedure (Chen and Chang, 2006). The well skin is generally classified into two types: a positive skin and a negative skin. If the permeability of the skin zone is less than that of the formation zone, the skin is called positive; while if the permeability of the skin zone is greater than that of the formation zone, the skin is called negative. The well skin has been studied extensively in hydrological sciences and petroleum engineering (e.g., Hurst, 1953; Hurst et al., 1969; Motz, 2002; Park and Zhan, 2002; Yang and Yeh, 2002, 2005; Chen and Chang, 2006; Walton, 2007; Pasandi et al., 2008).

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For instance, Hurst et al. (1969) presented the concept of the effective well radius to deal with a negative skin. Pasandi et al. (2008) investigated the flow to a partially penetrating well in a phreatic aquifer considering a finite-thickness skin. Motz (2002) obtained an analytical solution for one-dimensional flow in a leaky aquifer considering the effect of a low-permeability skin. Chen and Chang (2006) investigated a more realistic skin problem by proposing a mathematical model for non-uniform skin effect on the aquifer response due to the constant-rate pumping. Yang and Yeh (2005) obtained the Laplace-domain solutions for a constant-head test conducted in a partially penetrating well with a finite-thickness skin. It is probably worthwhile to mention that most studies of well skin are referred to vertical wells. However, the well skin also exists around a horizontal well (Park and Zhan, 2002, 2003). Park and Zhan (2002) investigated the flow to a finite-diameter horizontal well considering a skin with an infinitesimal thickness.

Although the well skin was commonly assumed to be infinitesimal (e.g., Hurst, 1953; Dougherty and Babu, 1984; Kabala and Cassiani, 1997; Park and Zhan, 2002, 2003), the thickness of the skin might vary from nearly zero to a few meters (Barker and Herbert, 1982). For a finite-thickness skin, the problem should be considered as a composite aquifer system. Up to now, many studies have been devoted to study the finite-thickness skin (e.g., Moench and Hsieh, 1985; Yang and Yeh, 2005, 2006, 2009; Chiu et al., 2007; Yeh and Yang, 2006; Yeh et al., 2008). For instance, Yang and Yeh (2006) obtained an analytical solution for a constant-head test considering a finite-thickness skin. Chiu et al. (2007) developed a mathematical model for a constant-rate test in a partially penetrating well with a finite-thickness skin in a confined aquifer. Yeh and Yang (2006) investigated the slug test conducted in a well with a finite-thickness skin in a confined aquifer. Yang and Yeh (2009) investigated the finite-thickness skin effect on the constant-rate test in leaky aquifers.

A careful review of the present literatures shows that there are limited researches on the constant-head test in a leaky aquifer with a finite-thickness skin, which will be the purpose of this study. Similar to the study of Hantush (1960), three different aquifer–aquitard systems will be discussed. The well discharge for different cases will be analyzed and a sensitivity analysis will be performed. The results of this investigation are also compared with previous studies to exhibit the new features of the constant-head test in a leaky aquifer with a finite-thickness skin.

2. Problem statement and solutions

2.1. Mathematical model

The schematic diagram of the investigated problem is shown in Fig. 1, the main aquifer is bounded by two aquitards. The coordinate system is set up as follows. The *x*-axis is horizontal, the *z*-axis is oriented upward along the axis of the well, and the origin of the

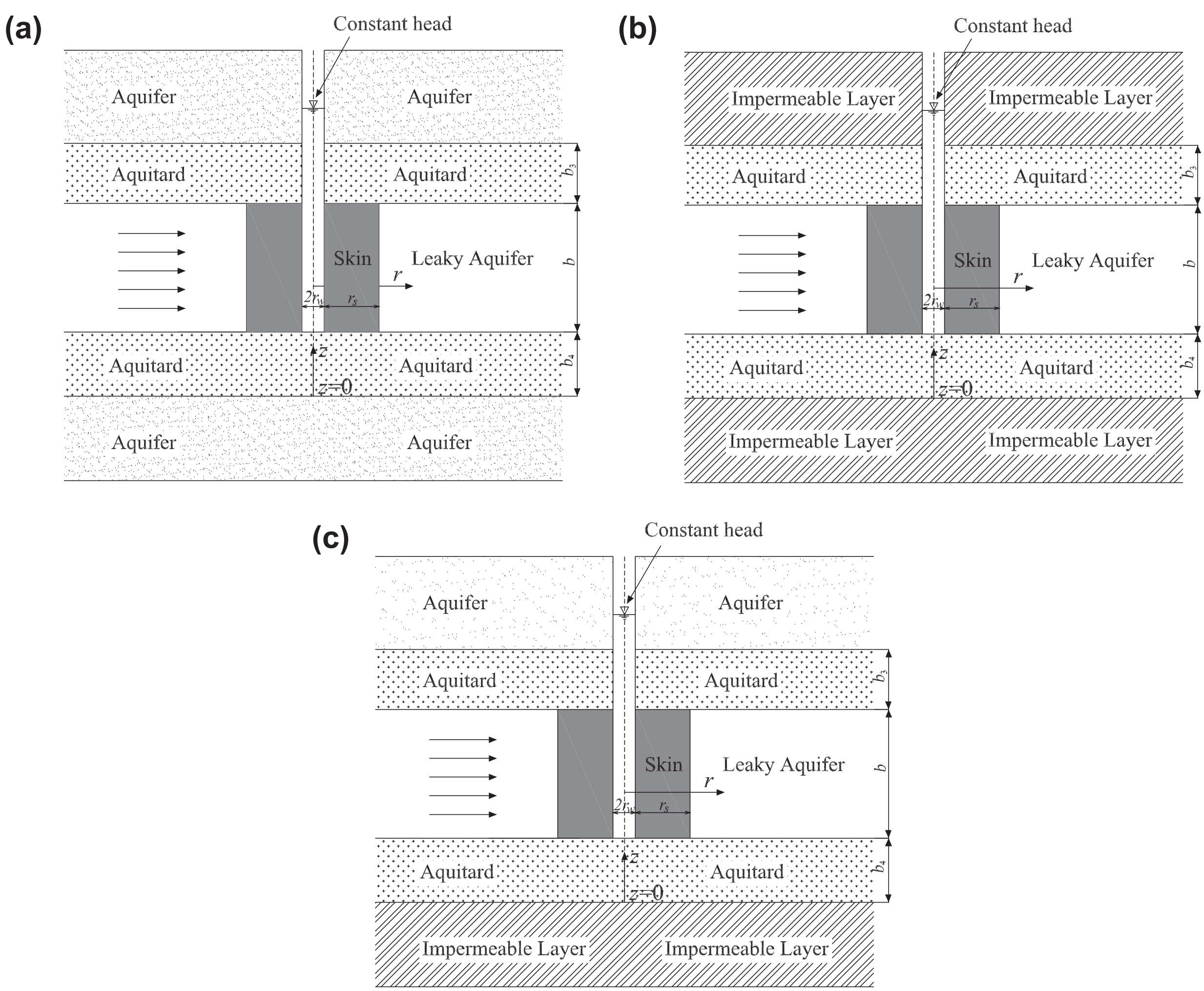

Fig. 1. The schematic diagram of the system: (a) case A: the two aquitards are over- and underlying two aquifers in which the hydraulic heads are constants; (b) case B: the two aquitards are over- and underlying two impermeable layers; (c) case C: one aquitard is bounded by an impermeable layer and the other is bounded by an aquifer in which the hydraulic head is constant.

system is at the bottom of the lower aquitard. The pumping well fully penetrates the main aquifer and has a constant head. There is a finite-thickness skin around the wellbore. As presented in Hantush (1960), there are three possible cases for such an aquifer–aquitard system. They are: (A) the two aquitards are over- and underlying two aquifers in which the hydraulic heads are constants; (B) the two aquitards are over- and underlying two impermeable layers; (C) one aquitard is bounded by an impermeable layer and the other is bounded by an aquifer in which the hydraulic head is constant. For the sake of mathematical tractability, several assumptions similar to those used by Yang and Yeh (2009) were used in this study as well. First, the aquifer, the aquitard and the skin zone each is homogeneous, isotropic and with a constant thickness although the heterogeneity and anisotropy might be very important issue for such an aquifer–aquitard system. Second, the flow in the aquifer is horizontal and the flow in the aquitards are vertical. Third, the radius of the well is sufficiently small thus the wellbore storage can be neglected. Fourth, the storage of the aquitards can not be ignored. The second assumption is valid when the hydraulic conductivity of the aquitard is at least two orders of magnitude smaller than that of the aquifer, which is often true in real applications (Hantush, 1960, 1964; Zhan et al., 2009a,b). Based on these assumptions, flow in the aquifer–aquitard system can be described by the following equations.

Case A: For the aquifer,

$$\frac{\partial^2 s_1(r,t)}{\partial r^2}+\frac{1}{r}\frac{\partial s_1(r,t)}{\partial r}+\frac{q_3}{T_1}-\frac{q_4}{T_1}=\frac{S_1}{T_1}\frac{\partial s_1(r,t)}{\partial t},\quad r_w\leqslant r\leqslant r_s, \tag{1}$$

$$\frac{\partial^2 s_2(r,t)}{\partial r^2}+\frac{1}{r}\frac{\partial s_2(r,t)}{\partial r}+\frac{q_3}{T_2}-\frac{q_4}{T_2}=\frac{S_2}{T_2}\frac{\partial s_2(r,t)}{\partial t},\quad r_s\leqslant r<\infty, \tag{2}$$

$$s_1(r,0)=s_2(r,0)=0, \tag{3}$$

$$s_2(\infty,t)=0, \tag{4}$$

$$s_1(r_s,t)=s_2(r_s,t), \tag{5}$$

$$T_1\frac{\partial s_1(r_s,t)}{\partial r}=T_2\frac{\partial s_2(r_s,t)}{\partial r}, \tag{6}$$

$$s_1(r_w,t)=s_w. \tag{7}$$

For the upper aquitard,

$$\frac{\partial^2 s_3(z,t)}{\partial z^2}=\frac{S_3}{T_3}\frac{\partial s_3(z,t)}{\partial t}, \tag{8}$$

$$s_3(z,0)=0, \tag{9}$$

$$s_3(z,t)|_{z=b+b_4}=\begin{cases}s_1(r,t), & r_w<r<r_s\\ s_2(r,t), & r_s<r<\infty\end{cases}, \tag{10}$$

$$s_3(z,t)|_{z=b+b_3+b_4}=0. \tag{11}$$

For the lower aquitard,

$$\frac{\partial^2 s_4(z,t)}{\partial z^2}=\frac{S_4}{T_4}\frac{\partial s_4(z,t)}{\partial t}, \tag{12}$$

$$s_4(z,0)=0, \tag{13}$$

$$s_4(z,t)|_{z=b_4}=\begin{cases}s_1(r,t), r_w<r<r_s\\ s_2(r,t), r_s<r<\infty\end{cases}, \tag{14}$$

$$s_4(z,t)|_{z=0}=0, \tag{15}$$

where $s(r, t)$ and $s(z, t)$ are the drawdowns at time t at the radial distance r and vertical distance z respectively; T is the transmissivity; S is the storage coefficient; b is the thickness of the aquifer, b_3 and b_4 are the thickness of the upper and lower aquitards, respectively; r_s is the thickness of the skin zone in the horizontal direction; the subscripts 1, 2, 3 and 4 denote skin zone, formation zone, upper aquitard and lower aquitard, respectively; q_3 and q_4 are the leakages of the upper and lower aquitards, respectively, and are expressed as:

Table 1
Dimensionless variables used in this study.

$r_D=\frac{r}{r_w}$	$z_D=\frac{z}{r_w}$	$r_{sD}=\frac{r_s}{r_w}$
$b_D=\frac{b}{r_w}$	$b_{3D}=\frac{b_3}{r_w}$	$b_{4D}=\frac{b_4}{r_w}$
$T_D=\frac{T_1}{T_2}$	$T_{3D}=\frac{T_3}{T_2}$	$T_{4D}=\frac{T_4}{T_2}$
$S_D=\frac{S_1}{S_2}$	$S_{3D}=\frac{S_3}{S_2}$	$S_{4D}=\frac{S_4}{S_2}$
$s_{iD}=\frac{s_i}{s_w}$	$q_{3D}=\frac{q_3 r_w}{k_3 s_w}$	$q_{4D}=\frac{q_4 r_w}{k_4 s_w}$
$t_D=\frac{T_2 t}{r_w^2 S_2}$	$Q_D(t_D)=\frac{Q(t)}{2\pi b k_{2w}}$	i = 1, 2, 3, 4

$$q_3=k_3\frac{ds_3(z,t)}{dz}\bigg|_{z=b+b_4}, \tag{16}$$

$$q_4=k_4\frac{ds_4(z,t)}{dz}\bigg|_{z=b_4}, \tag{17}$$

in which, k_3 and k_4 are the hydraulic conductivities of the upper and lower aquitards, respectively.

Case B: Same as case A except that the boundary conditions Eqs. (11) and (15) will be replaced by:

$$\frac{\partial s_3(z,t)}{\partial z}\bigg|_{z=b+b_3+b_4}=0, \tag{18}$$

$$\frac{\partial s_4(z,t)}{\partial z}\bigg|_{z=0}=0. \tag{19}$$

Case C: Same as case A except that the boundary condition Eq. (15) will be replaced by Eq. (19).

2.2. Dimensionless transform

Defining the dimensionless variables as shown in Table 1, the problem can be transformed to the following equations.

For the aquifer,

$$\frac{\partial^2 s_{1D}(r_D,t_D)}{\partial r_D^2}+\frac{1}{r_D}\frac{\partial s_{1D}(r_D,t_D)}{\partial r_D}+\frac{q_{3D}T_{3D}}{T_D b_{3D}}-\frac{q_{4D}T_{4D}}{T_D b_{4D}}=\frac{S_D}{T_D}\frac{\partial s_{1D}(r_D,t_D)}{\partial t_D},\quad 1\leqslant r_D\leqslant r_{sD}, \tag{20}$$

$$\frac{\partial^2 s_{2D}(r_D,t_D)}{\partial r_D^2}+\frac{1}{r_D}\frac{\partial s_{2D}(r_D,t_D)}{\partial r_D}+\frac{q_{3D}T_{3D}}{b_{3D}}-\frac{q_{4D}T_{4D}}{b_{4D}}=\frac{\partial s_{2D}(r_D,t_D)}{\partial t_D},\quad r_{sD}\leqslant r_D<\infty, \tag{21}$$

$$s_{1D}(r_D,0)=s_{2D}(r_D,0)=0, \tag{22}$$

$$s_{2D}(\infty,t_D)=0, \tag{23}$$

$$s_{1D}(r_{sD},t_D)=s_{2D}(r_{sD},t_D), \tag{24}$$

$$T_D\frac{\partial s_{1D}(r_{sD},t_D)}{\partial r_D}=\frac{\partial s_{2D}(r_{sD},t_D)}{\partial r_D}, \tag{25}$$

$$s_{1D}(1,t_D)=1. \tag{26}$$

For the upper aquitard,

$$\frac{\partial^2 s_{3D}(z_D,t_D)}{\partial z_D^2}=\frac{S_{3D}}{T_{3D}}\frac{\partial s_{3D}(z_D,t_D)}{\partial t_D}, \tag{27}$$

$$s_{3D}(z_D, 0) = 0, \quad (28)$$

$$s_{3D}(z_D, t_D)|_{z_D=b_D+b_{4D}} = \begin{cases} s_{1D}(r_D, t_D), r_{wD} < r_D < r_{sD} \\ s_{2D}(r_D, t_D), r_{sD} < r_D < \infty \end{cases}, \quad (29)$$

$$s_{3D}(z_D, t_D)|_{z_D=b_D+b_{3D}+b_{4D}} = 0. \quad (30)$$

For the lower aquitard,

$$\frac{\partial^2 s_{4D}(z_D, t_D)}{\partial z_D^2} = \frac{S_{4D}}{T_{4D}} \frac{\partial s_{4D}(z_D, t_D)}{\partial t_D}, \quad (31)$$

$$s_{4D}(z_D, 0) = 0, \quad (32)$$

$$s_{4D}(z_D, t_D)|_{z_D=b_{4D}} = \begin{cases} s_{1D}(r_D, t_D), r_{wD} < r_D < r_{sD} \\ s_{2D}(r_D, t_D), r_{sD} < r_D < \infty \end{cases}, \quad (33)$$

$$s_{4D}(z_D, t_D)|_{z_D=0} = 0. \quad (34)$$

Accordingly, Eqs. (16)–(19) can also be rewritten in dimensionless format as:

$$q_{3D} = \left.\frac{ds_{3D}(z_D, t_D)}{dz_D}\right|_{z_D=b_D+b_{4D}}, \quad (35)$$

$$q_{4D} = \left.\frac{ds_{4D}(z_D, t_D)}{dz_D}\right|_{z_D=b_{4D}}, \quad (36)$$

$$\left.\frac{\partial s_{3D}(z_D, t_D)}{\partial z_D}\right|_{z_D=b_D+b_{3D}+b_{4D}} = 0, \quad (37)$$

$$\left.\frac{\partial s_{4D}(z_D, t_D)}{\partial z_D}\right|_{z_D=0} = 0. \quad (38)$$

3. Solutions in the Laplace domain

The solutions for case A in the Laplace domain can be expressed as follows. The details can be found in the Appendix.

$$\overline{s_{1D}}(r_D, p) = \frac{1}{p}\left[\frac{-\omega_1 I_0(\lambda_1 r_D) + \omega_2 K_0(\lambda_1 r_D)}{-\omega_1 I_0(\lambda_1) + \omega_2 K_0(\lambda_1)}\right], \quad (39)$$

and

$$\overline{s_{2D}}(r_D, p) = \frac{1}{p}\left[\frac{\omega_3 K_0(\lambda_2 r_D)}{-\omega_1 I_0(\lambda_1) + \omega_2 K_0(\lambda_1)}\right], \quad (40)$$

in which,

$$\omega_1 = \lambda_2 K_0(\lambda_1 r_{sD}) K_1(\lambda_2 r_{sD}) - T_D \lambda_1 K_0(\lambda_2 r_{sD}) K_1(\lambda_1 r_{sD}), \quad (41)$$

$$\omega_2 = \lambda_2 I_0(\lambda_1 r_{sD}) K_1(\lambda_2 r_{sD}) + T_D \lambda_1 I_1(\lambda_1 r_{sD}) K_0(\lambda_2 r_{sD}), \quad (42)$$

$$\omega_3 = T_D \lambda_1 I_0(\lambda_1 r_{sD}) K_1(\lambda_1 r_{sD}) + T_D \lambda_1 I_1(\lambda_1 r_{sD}) K_0(\lambda_1 r_{sD}), \quad (43)$$

where $\lambda_1^2 = (S_D/T_D)p + (AT_{3D}/T_D b_{3D})\coth(Ab_{3D}) + (BT_{4D}/T_D b_{4D})\coth(Bb_{4D})$, and $\lambda_2^2 = p + (AT_{3D}/b_{3D})\coth(Ab_{3D}) + (BT_{4D}/b_{4D})\coth(Bb_{4D})$, with $A^2 = pS_{3D}/T_{3D}$ and $B^2 = pS_{4D}/T_{4D}$, in which $\coth(x)$ is the hyperbolic cotangent function; p is the Laplace variable; $I_v(x)$ and $K_v(x)$ are the first and second kinds of the modified Bessel functions with the order v. As stated in the Appendix, the solutions for case B and case C can be easily obtained and they have the same patterns as Eqs. (39) and (40). For case B, one needs to change the coth function to tanh function for λ_1^2 and λ_2^2. For case C, one needs to change the coth function in the last term of λ_1^2 and λ_2^2 to tanh function. The results for cases B and C are not listed here considering the space.

Up to now, we have obtained the solutions of drawdown in the Laplace domain. However, for a constant-head test, it is also important to analyze the well discharge which can be expressed as:

$$Q(t) = -\lim_{r \to r_w} 2\pi b r k_1 \frac{ds_1(r, t)}{dr}, \quad (44)$$

where $Q(t)$ is the well discharge. Eq. (44) can be rewritten in a dimensionless format as:

$$Q_D(t_D) = -T_D \lim_{r_D \to 1} r_D \frac{ds_{1D}(r_D, t_D)}{dr_D}, \quad (45)$$

in which $Q_D(t_D) = \frac{Q(t)}{2\pi b k_2 s_w}$ is the dimensionless well discharge. Applying the Laplace transform to Eq. (45) and considering Eq. (39), one obtains the dimensionless well discharge in the Laplace domain as:

$$\bar{Q}_D(p) = -T_D \lim_{r_D \to 1} r_D \frac{d\bar{s}_{1D}(r_D, p)}{dr_D} = \frac{T_D}{p}\left[\frac{\omega_1 \lambda_1 I_1(\lambda_1) + \omega_2 \lambda_1 K_1(\lambda_1)}{-\omega_1 I_0(\lambda_1) + \omega_2 K_0(\lambda_1)}\right]. \quad (46)$$

Eq. (46) can be inverted to yield the solution in the real time domain. It is difficult, if not impossible, to invert such Laplace-domain solutions analytically in this study. However, it is rather convenient to invert these solutions numerically. There are several possible ways to do such numerical inversion, such as the Stehfest method (Stehfest, 1970a,b), the Crump method (Crump, 1976), and the de Hoog method (de Hoog et al., 1982). In the following computation, the Stehfest method was used because of its simplicity. The accuracy of the Stehfest method was tested later as well.

4. Results and discussion

For a constant-head test, it is important to analyze the well discharge. In the following, we analyzed the impact of the leakage and the skin on the well discharge. A sensitivity analysis was also conducted to see the dependence of the well discharge on different parameters. We used the dimensionless variables in the following analysis. One benefit of using the dimensionless rather than the dimensional variables is that the number of independent variables is usually smaller (Park and Zhan, 2002, 2003; Zhan et al., 2009a, Zhan et al., 2009b). However, in order to help the readers to understand the range of those dimensionless variables, several realistically possible values of the real parameters are given in Table 2. For the sake of simplicity of illustration, we assumed that the storage coefficients are all the same: $S_1 = S_2 = S_3 = S_4 = 10^{-3}$. The radius of the well is chosen to be 0.2 m. The thickness of the main aquifer is chosen to be 4 m, and the thickness of each aquitard is chosen to be 1 m. The transmissivity of the formation zone is chosen to be 40 m^2/day (the hydraulic conductivity is 10 m/day). The transmissivities of the skin zone are 4, 20, 40, 80 and 200 m^2/day (the hydraulic conductivities are 1, 5, 10, 20 and 50 m/day), corresponding to T_D of 0.1, 0.5, 1, 2 and 5, respectively. The transmissivities of the aquitards are 0.04, 0.2 and 0.32 m^2/day (the hydraulic conductivities are 0.04, 0.2 and 0.32 m/day), corresponding to T_{3D} and T_{4D} of 0.001, 0.005 and 0.008, respectively. The thickness of the skin are

Table 2
The default values used in this study.

Parameter name	Symbol	Default value
Storage coefficient of the formation zone	S_2	1×10^{-3}
Storage coefficient of the skin zone	S_1	1×10^{-3}
Storage coefficient of the aquitard	S_3, S_4	1×10^{-3}
Radius of the well	r_w	0.2 m
Thickness of the main aquifer	b_1, b_2	4 m
Thickness of the aquitard	b_3, b_4	1 m
Transmissivity of the formation zone	T_2	40 m^2/day
Transmissivity of the skin zone	T_1	4, 20, 40, 80, 200 m^2/day
Transmissivity of the aquitard	T_3, T_4	0.04, 0.2, 0.32 m^2/day
Thickness of the skin	r_s	0.4, 0.6, 1, 2 m

0.4, 0.6, 1 and 2 m, corresponding to r_{sD} of 2, 3, 5 and 10, respectively. It should be pointed out that the values used in Table 2 are reasonable for the real aquifer–aquitard system. As stated in Bear (1979), the hydraulic conductivity of the aquifer with clean sand or sand and gravel is around 10^{-3}–10^{0} cm/s or 8.64×10^{-1}–8.64×10^{2} m/day. The hydraulic conductivity of the semipervious aquifer (aquitard) with very fine sand, silt, loess, or loam is around 10^{-6}–10^{-3} cm/s or 8.64×10^{-4}–8.64×10^{-1} m/day. The chosen hydraulic conductivity of 10 m/day for the formation and 0.04, 0.2 and 0.32 m/day for the aquitards are within those ranges. It is also notable that the hydraulic conductivities of the aquitards are at least two orders of magnitude smaller than that of the formation, so the assumption of horizontal flow in the formation and vertical flow in the aquitard is valid. The hydraulic conductivities of the aquitards are about one order of magnitude smaller than the hydraulic conductivities of the skin zone for a few cases. However, since the thickness of the skin zone is small (less than 2 m), the assumption of horizontal flow in the skin zone and vertical flow in the aquitard will only be slightly affected within small regions near the skin-aquitard contacts. Therefore, along the same line with previous studies (e.g., Yeh and Yang, 2006; Yang and Yeh, 2009), flow in the skin zone is also assumed to be horizontal.

In order to test the accuracy of the Stehfest method, we first considered a simple case in which a closed-form analytical solution is available. Jacob and Lohman (1952) provided such a closed-form analytical solution for constant-head test in a confined aquifer without the leakage and skin. We used this analytical solution to test the accuracy of the numerical Laplace inversion method. If the leakage and the skin are not considered, the solution in the Laplace domain can be easily obtained from Eq. (46). The comparison of the closed-form analytical solution with that obtained by the Stehfest method is shown in Fig. 2. N is the number of terms used in the Stehfest method, which is suggested to be an even number ranging from 4 to 24. As shown in Fig. 2, different N values can yield solutions in very good agreement with the analytical solution except that a slight difference has been found for the case of N of 24. In this study, N is chosen to be 12.

4.1. Effect of leakage and skin

For the purpose of illustration, we used the same values for the parameters of the upper and lower aquitards. First, we compared the well discharges for three different cases, as shown in Fig. 3. The parameters are given as $T_D = 0.1$ (positive skin) and $T_D = 5$ (negative skin) with $b_{3D} = b_{4D} = 5$, $T_{3D} = T_{4D} = 0.005$ and $r_{sD} = 5$, respectively, for (a) $S_D = S_{3D} = S_{4D} = 1$ and (b) $S_D = S_{3D} = S_{4D} = 5$. It can be seen that the features of the curves are similar for the positive skin and the negative skin cases as well as for different dimensionless storage coefficients. That is, the well discharges for three different cases approach the same asymptotic values at early times, this is because the leakage has not arrived at the aquifer and consequently has little effect on the flow. The well discharge is the largest for case A and the smallest for case B. This is understandable. A check of these three cases (Fig. 1) indicates that the leakage of case A is the largest at late times, which will result in the largest well discharge at late times. Comparing Fig. 3a with b, one can see that larger storage coefficients results in greater well discharge at early times, this is because a larger storage coefficient means the aquifer (or aquitard) can release more water under a same drawdown. As the case discussed in this study is a constant-head test, thus it is no wonder to see such features at early times as shown in Fig. 3a and b. We also find that the impact of the parameters on the well discharge for case A is similar to that for case B and case C. Therefore, we only consider the results of case A in the following analysis.

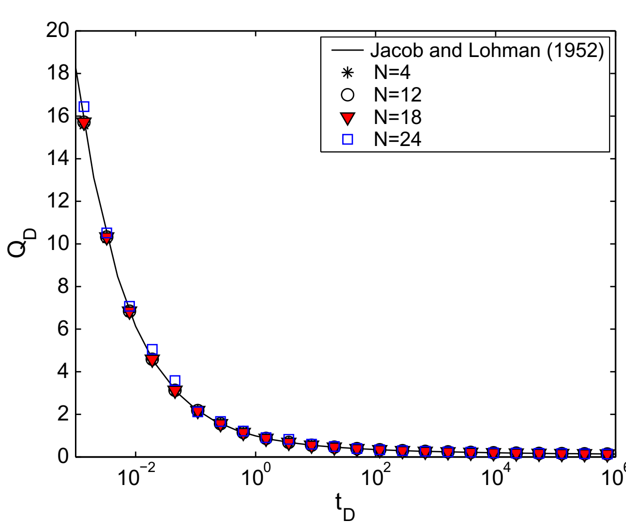

Fig. 2. Test the accuracy of the Stehfest method used in this study.

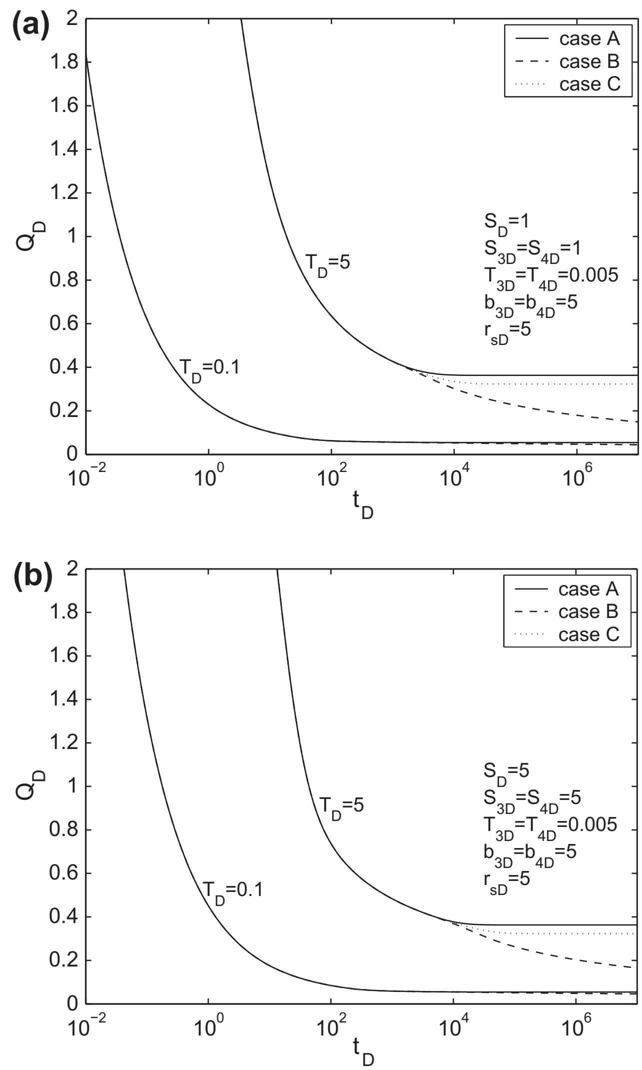

Fig. 3. The well discharge versus time for three different cases for $T_D = 0.1$ (positive skin) and $T_D = 5$ (negative skin) with $b_{3D} = b_{4D} = 5$, $T_{3D} = T_{4D} = 0.005$ and $r_{sD} = 5$, respectively. (a) $S_D = S_{3D} = S_{4D} = 1$; (b) $S_D = S_{3D} = S_{4D} = 5$.

Fig. 4 is about the well discharge versus time for case A for $T_D = 0.1$ (positive skin) and $T_D = 5$ (negative skin) with $S_D = 1$,

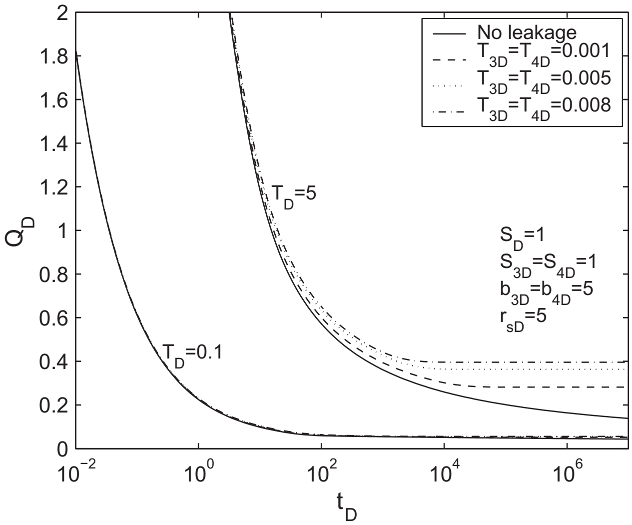

Fig. 4. The well discharge versus time for $T_D = 0.1$ (positive skin) and $T_D = 5$ (negative skin) with $S_D = 1$, $S_{3D} = S_{4D} = 1$, $b_{3D} = b_{4D} = 5$, $r_{sD} = 5$, $T_{3D} = T_{4D} = 0.001$, 0.005 and 0.008, respectively.

$S_{3D} = S_{4D} = 1$, $b_{3D} = b_{4D} = 5$, $r_{sD} = 5$, $T_{3D} = T_{4D} = 0.001$, 0.005 and 0.008, respectively. The case without leakage is also presented in this figure as a reference. As shown in Fig. 4, the dimensionless transmissivity of the aquitard has little effect on the well discharge at early times; while a larger transmissivity of the aquitard leads to a larger well discharge at late times. This is because the leakage of the aquitard has not reached the main aquifer at early times. While at late times, the well discharge is partially from the leakage. A larger transmissivity means a greater leakage, consequently yields a greater well discharge, as shown in Fig. 4. It can also be found that the well discharge for the positive skin case is smaller than that of the negative skin case.

The skin effect on the well discharge is shown in Fig. 5. The parameters are given as $S_D = 1$, $T_{3D} = T_{4D} = 0.005$, $S_{3D} = S_{4D} = 1$, $b_{3D} = b_{4D} = 5$, $r_{sD} = 5$, $T_D = 0.1$, 0.5, 1, 2 and 5, respectively. It is notable that $T_D = 1$ means that the transmissivity of the skin zone is equal to that of the formation zone (i.e., the skin is absent). It is evident that the well discharge for the positive skin is smaller than that without the skin, while the well discharge for the negative skin is larger than that without the skin.

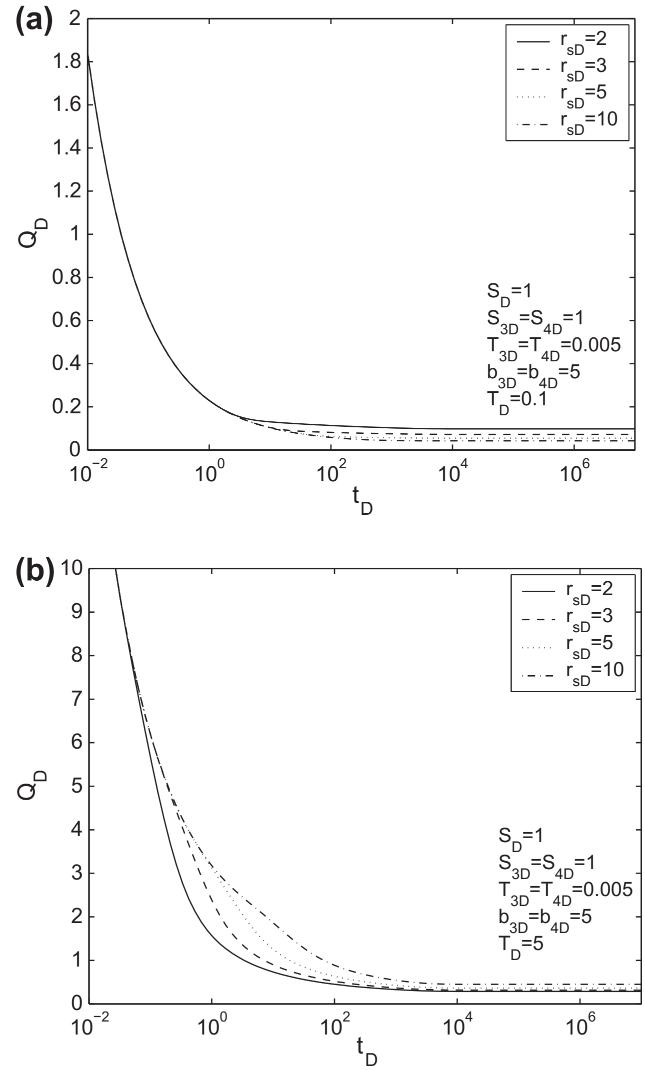

Fig. 6. The well discharge versus time with respect to $S_D = 1$, $T_{3D} = T_{4D} = 0.005$, $S_{3D} = S_{4D} = 1$, $b_{3D} = b_{4D} = 5$, $r_{sD} = 2$, 3, 5 and 10, respectively. (a) $T_D = 0.1$ (positive skin); (b) $T_D = 5$ (negative skin).

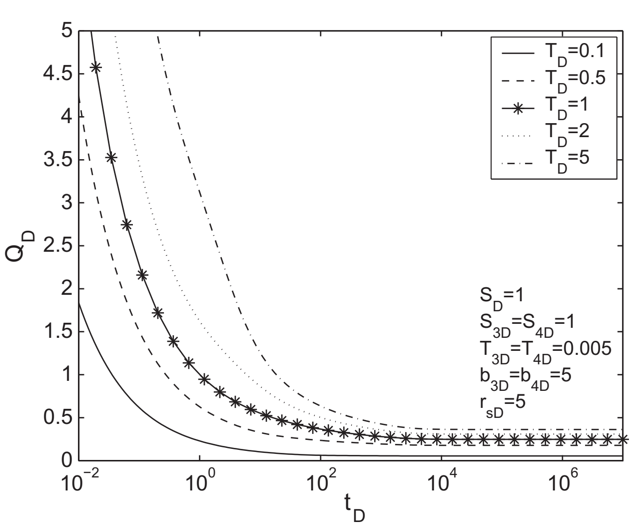

Fig. 5. The well discharge versus time for $S_D = 1$, $T_{3D} = T_{4D} = 0.005$, $S_{3D} = S_{4D} = 1$, $b_{3D} = b_{4D} = 5$, $r_{sD} = 5$, $T_D = 0.1$, 0.5, 1, 2 and 5, respectively.

We have also analyzed the effect of skin thickness on the well discharge, as shown in Fig. 6. This figure reflects the well discharge versus time with respect to $S_D = 1$, $T_{3D} = T_{4D} = 0.005$, $S_{3D} = S_{4D} = 1$, $b_{3D} = b_{4D} = 5$, $r_{sD} = 2$, 3, 5 and 10, respectively, for (a) $T_D = 0.1$ (positive skin); (b) $T_D = 5$ (negative skin). Interestingly enough, a thicker skin results in a smaller well discharge for the positive skin case while leads to a larger well discharge for the negative skin case at late times. This feature can be explained as follows. The two-region (skin zone and formation zone) flow model presented in this study may be approximated by an equivalent one-region flow model when analyzing the well discharge. For the positive skin case, the homogenized hydraulic conductivity of the equivalent one-region flow model should be smaller when the skin is thicker, resulting in a smaller well discharge. For the negative skin case, a thicker skin means a greater homogenized hydraulic conductivity of the equivalent one-region flow model, thus a larger well discharge.

Fig. 7 is about the drawdown-distance behavior for $T_D = 0.1$ (positive skin) and $T_D = 5$ (negative skin) with $S_D = 1$, $T_{3D} = T_{4D} = 0.005$, $S_{3D} = S_{4D} = 1$, $b_{3D} = b_{4D} = 5$, $r_{sD} = 5$, $t_D = 1$, 10 and 100, respectively. It can be seen that the drawdown for the positive skin case is less than that of the negative skin at the same time. Another interesting feature is that a positive skin might result in delayed response of the aquifer to pumping. As shown in Fig. 7, one can see that the drawdown in the formation zone is nearly zero for $T_D = 0.1$ with $t_D = 1$ and 10. This is because the hydraulic conductivity of the skin zone is much smaller than that of the formation zone for this positive skin case, thus greater resistance to flow exists across the skin. On the contrary, for the negative skin case ($T_D = 5$), the drawdown increases rapidly after a short period of time as shown in Fig. 7.

4.2. Sensitivity analysis

Sensitivity analysis is a tool to analyze the impact of the input parameters on the results (i.e., well discharge in this study) of a model. In order to see how different parameters would affect the result, the normalized sensitivity method proposed by Kabala (2001) and Huang and Yeh (2007) seems to be a convenient way to do this analysis. The normalized sensitivity of a dependent variable to the relative change of a given parameter can be expressed as follows:

$$X_{i,j} = P_j \frac{\partial R_i}{\partial P_j}, \tag{47}$$

in which $X_{i,j}$ is the normalized sensitivity coefficient for the jth parameter P_j at the ith time step; R_i is the dependent variable at the ith time step and it is the dimensionless well discharge in this study. The partial derivative on the right hand of Eq. (47) can be approximated as follows (Yeh, 1987):

$$\frac{\partial R_i}{\partial P_j} = \frac{R_i(P_j + \Delta P_j) - R_i(P_j)}{\Delta P_j} \tag{48}$$

in which ΔP_j is a small positive increment, which will be chosen as $10^{-2} \times P_j$ (Yang and Yeh, 2009). The sensitivity analysis of the parameters was performed and the result is shown in Fig. 8. Fig. 8a is for a positive skin case ($T_D = 0.1$) while Fig. 8b is for a negative skin case ($T_D = 2$). The other parameters are given as $S_D = 1$, $S_{3D} = S_{4D} = 1$, $T_{3D} = T_{4D} = 0.005$, $b_{3D} = b_{4D} = 5$, and $r_{sD} = 5$. As the flow system is symmetrical vertically, one only needs to analyze the normalized sensitivity of the parameters for the upper aquitard. Fig. 8 shows that a relative increase in S_D and produces a positive effect on well discharge, and a relative increase in T_D produces a positive effect as well on the well discharge at early times. This figure also shows that the normalized sensitivity coefficients on other parameters such as S_{3D}, T_{3D}, b_{3D}, r_{sD} are very small, nearly equal to zero. These features indicate that the well discharge is very sensitive to the change in S_D and T_D, not sensitive to the change of other parameters. Physically speaking, the well discharge is sensitive to the properties of the skin zone and not sensitive to the properties of the aquitards for such an aquifer–aquitard system presented in this study.

Generally speaking, the main purpose of a pumping test including the constant-head test is to estimate the hydraulic parameters of the aquifer (or aquitards). For a constant-head test conducted in an aquifer–aquitard system if a finite-thickness skin existed, the drawdown inside the well s_w and the thickness of the aquifer and the aquitards, i.e., b_1, b_2, b_3 and b_4 might be known. The unknown parameters will include S_1, T_1, S_2, T_2, S_3, T_3, S_4, T_4 and r_s. It seems that it is impossible to estimate these parameters with the matching point method associated with some type curves which we usually use for the Theis flow model. In this case, the determination of the aquifer parameters from the constant-head test data might be an optimization problem (Chen and Yeh, 2009). Chen and Yeh (2009) presented an interesting optimization method to estimate the aquifer parameters for a constant-head test in a confined aquifer with a finite-thickness skin, which is similar to the problem

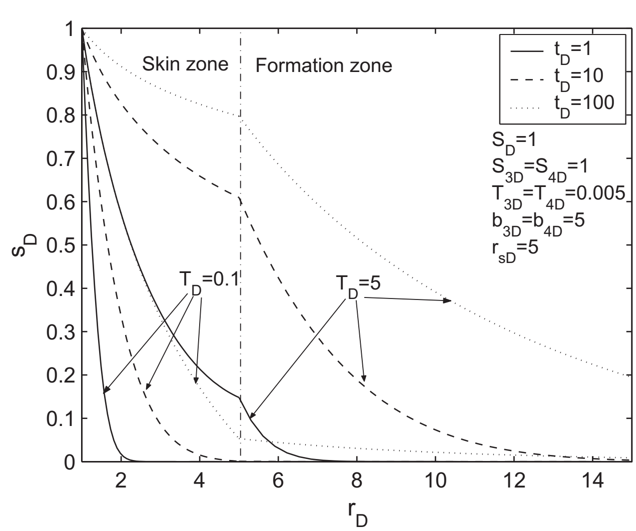

Fig. 7. The drawdown versus distance for $T_D = 0.1$ (positive skin) and $T_D = 5$ (negative skin) with $S_D = 1$, $T_{3D} = T_{4D} = 0.005$, $S_{3D} = S_{4D} = 1$, $b_{3D} = b_{4D} = 5$, $r_{sD} = 5$, $t_D = 1$, 10 and 100, respectively.

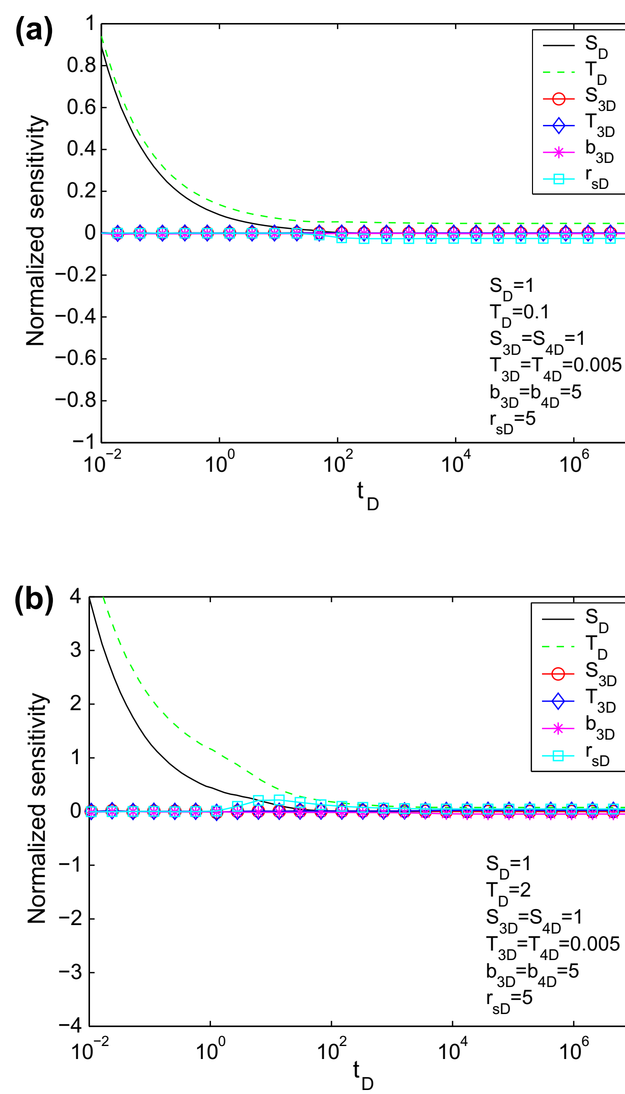

Fig. 8. The normalized sensitivities of dimensionless discharge with respect to $S_D = 1$, $S_{3D} = S_{4D} = 1$, $T_{3D} = T_{4D} = 0.005$, $b_{3D} = b_{4D} = 5$, and $r_{sD} = 5$. (a) $T_D = 0.1$ (positive skin); (b) $T_D = 2$ (negative skin).

described in this study except that the leakage is not considered by Chen and Yeh (2009). This method can also be used to estimate the hydraulic parameters for the aquifer–aquitard system presented in this study if the field data are available. If some of the aquifer parameters are known or one of the aquitards is absent, i.e. only one aquitard bounded the main aquifer, it will make the problem much simpler. Therefore, the study of Chen and Yeh (2009) might be a good reference when using the solutions in this study associated with the pumping test data.

5. Summary and conclusions

In this study, a mathematical model for radial groundwater flow to a pumping well in an aquifer–aquitard system was presented for a constant-head test considering the finite-thickness skin. Three different cases with different boundary conditions of the aquitards were discussed. The Laplace-domain solutions were obtained and subsequently inverted numerically by using the Stehfest method. We have analyzed the impact of the leakage of the aquitards and the skin effect on the well discharge, and a sensitivity analysis has also been included. Several conclusions can be drawn from this study:

(1) The dimensionless transmissivity of the aquitard has little effect on the well discharge at early times, while a larger transmissivity of the aquitard leads to a larger well discharge at late times.
(2) The well discharge for the positive skin is smaller than that without the skin, while the well discharge for the negative skin is larger than that without the skin. A thicker skin results in a smaller well discharge for the positive skin case while leads to a larger well discharge for the negative skin case at late times.
(3) When the leakage is the same, the drawdown for the positive skin case is less than that of the negative skin case at the same time, and a positive skin results in delayed response of the aquifer to pumping.
(4) The well discharge is sensitive to the properties of the skin zone and not sensitive to the properties of the aquitards for the aquifer–aquitard system presented in this study.

Acknowledgments

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Appendix A

First we will solve the problem of case A. Applying the Laplace transform to Eqs. (27), (29), and (30) with consideration of Eq. (28) will lead to:

$$\frac{d^2\overline{s_{3D}}(z_D,p)}{dz_D^2}=\frac{pS_{3D}}{T_{3D}}\overline{s_{3D}}(z_D,p), \tag{A1}$$

$$\overline{s_{3D}}(z_D,p)|_{z_D=b_D+b_{4D}}=\begin{cases}\overline{s_{1D}}(r_D,p), r_{wD}<r_D<r_{sD}\\ \overline{s_{2D}}(r_D,p), r_{sD}<r_D<\infty\end{cases}, \tag{A2}$$

$$\overline{s_{3D}}(z_D,p)|_{z_D=b_D+b_{3D}+b_{4D}}=0, \tag{A3}$$

in which p is the Laplace variable, and over bar means the variables in the Laplace domain. The solution of Eqs. (A1), (A2), (A3) can be expressed as:

$$\overline{s_{3D}}(z_D,p)=\frac{\sinh[A(b_D+b_{3D}+b_{4D}-z_D)]}{\sinh(Ab_{3D})}\overline{s_i}(r_D,p), \tag{A4}$$

where $A^2=pS_{3D}/T_{3D}$, i = 1 and 2 denotes the drawdown in the skin and formation zones, respectively. sinh(x) and cosh(x) are the hyperbolic sine and cosine functions which are defined as: $\sinh(x)=(e^x-e^{-x})/2$, $\cosh(x)=(e^x+e^{-x})/2$. Similarly, one can have:

$$\overline{s_4}(z_D,p)=\frac{\sinh(Bz_D)}{\sinh(Bb_{4D})}\overline{s_i}(r_D,p), \tag{A5}$$

where $B^2=pS_{4D}/T_{4D}$. In terms of Eqs. (35) and (36), the dimensionless leakage through the upper and lower aquitards in the Laplace domain can be obtained as:

$$\overline{q_{3D}}=\frac{d\overline{s_{3D}}(z_D,p)}{dz_D}|_{z_D=b_D+b_{4D}}=-A\coth(Ab_{3D})\overline{s_i}(r_D,p), \tag{A6}$$

$$\overline{q_{4D}}=\frac{d\overline{s_{4D}}(z_D,p)}{dz_D}|_{z_D=b_{4D}}=B\coth(Bb_{4D})\overline{s_i}(r_D,p), \tag{A7}$$

where coth(x) is the hyperbolic cotangent function. With the Laplace transform, Eqs. (20) and (21) can be transformed to the following equations:

$$\frac{d^2\overline{s_{1D}}(r_D,p)}{dr_D^2}+\frac{1}{r_D}\frac{d\overline{s_{1D}}(r_D,p)}{dr_D}=\lambda_1^2\overline{s_{1D}}(r_D,p),\quad 1\leqslant r_D\leqslant r_{sD}, \tag{A8}$$

$$\frac{d^2\overline{s_{2D}}(r_D,p)}{dr_D^2}+\frac{1}{r_D}\frac{d\overline{s_{2D}}(r_D,p)}{dr_D}=\lambda_2^2\overline{s_{2D}}(r_D,p),\quad r_{sD}\leqslant r_D<\infty, \tag{A9}$$

in which $\lambda_1^2=(S_D/T_D)p+(AT_{3D}/T_Db_{3D})\coth(Ab_{3D})+(BT_{4D}/T_Db_{4D})\times\coth(Bb_{4D})$, and $\lambda_2^2=p+(AT_{3D}/b_{3D})\coth(Ab_{3D})+(BT_{4D}/b_{4D})\coth\times(Bb_{4D})$. The general solutions to Eqs. (A8) and (A9) can be expressed as:

$$\overline{s_{1D}}(r_D,p)=D_1I_0(\lambda_1r_D)+D_2K_0(\lambda_1r_D), \tag{A10}$$

$$\overline{s_{2D}}(r_D,p)=D_3I_0(\lambda_2r_D)+D_4K_0(\lambda_2r_D), \tag{A11}$$

where $I_v(x)$ and $K_v(x)$ are the first and second kinds of the modified Bessel functions with the order v, respectively. D_1, D_2, D_3, D_4 are constants depending on the boundary conditions. Applying the Laplace transform, the boundary conditions can be transformed as:

$$\overline{s_{2D}}(\infty,p)=0, \tag{A12}$$

$$\overline{s_{1D}}(r_{sD},p)=\overline{s_{2D}}(r_{sD},p), \tag{A13}$$

$$T_D\frac{d\overline{s_{1D}}(r_{sD},p)}{dr_D}=\frac{d\overline{s_{2D}}(r_{sD},p)}{dr_D}, \tag{A14}$$

$$\overline{s_{1D}}(1,p)=\frac{1}{p}. \tag{A15}$$

With Eq. (A12), one knows that $D_3=0$. Substituting Eqs. (A10) and (A11) into Eqs. (A13), (A14), and (A15) will yield the following equations:

$$D_1I_0(\lambda_1r_{sD})+D_2K_0(\lambda_1r_{sD})=D_4K_0(\lambda_2r_{sD}), \tag{A16}$$

$$T_D\lambda_1[D_1I_1(\lambda_1r_{sD})-D_2K_1(\lambda_1r_{sD})]=-\lambda_2D_4K_1(\lambda_2r_{sD}), \tag{A17}$$

$$D_1I_0(\lambda_1)+D_2K_0(\lambda_1)=\frac{1}{p}. \tag{A18}$$

Solving above equations leads to:

$$D_1 = \frac{1}{p}\left[\frac{-\omega_1}{-\omega_1 I_0(\lambda_1) + \omega_2 K_0(\lambda_1)}\right], \tag{A19}$$

$$D_2 = \frac{1}{p}\left[\frac{\omega_2}{-\omega_1 I_0(\lambda_1) + \omega_2 K_0(\lambda_1)}\right], \tag{A20}$$

$$D_4 = \frac{1}{p}\left[\frac{\omega_3}{-\omega_1 I_0(\lambda_1) + \omega_2 K_0(\lambda_1)}\right], \tag{A21}$$

in which

$$\omega_1 = \lambda_2 K_0(\lambda_1 r_{sD})K_1(\lambda_2 r_{sD}) - T_D\lambda_1 K_0(\lambda_2 r_{sD})K_1(\lambda_1 r_{sD}), \tag{A22}$$

$$\omega_2 = \lambda_2 I_0(\lambda_1 r_{sD})K_1(\lambda_2 r_{sD}) + T_D\lambda_1 I_1(\lambda_1 r_{sD})K_0(\lambda_2 r_{sD}), \tag{A23}$$

$$\omega_3 = T_D\lambda_1 I_0(\lambda_1 r_{sD})K_1(\lambda_1 r_{sD}) + T_D\lambda_1 I_1(\lambda_1 r_{sD})K_0(\lambda_1 r_{sD}). \tag{A24}$$

After D_1, D_2 and D_4 are obtained the dimensionless drawdowns for the skin and formation zones are:

$$\overline{s_{1D}}(r_D, p) = \frac{1}{p}\left[\frac{-\omega_1 I_0(\lambda_1 r_D) + \omega_2 K_0(\lambda_1 r_D)}{-\omega_1 I_0(\lambda_1) + \omega_2 K_0(\lambda_1)}\right], \tag{A25}$$

$$\overline{s_{2D}}(r_D, p) = \frac{1}{p}\left[\frac{\omega_3 K_0(\lambda_2 r_D)}{-\omega_1 I_0(\lambda_1) + \omega_2 K_0(\lambda_1)}\right]. \tag{A26}$$

For case B and case C, it is notable that the only differences are the leakage through the aquitards. For case B, using the similar derivations above, one can obtain the leakage through the upper and lower aquitards as:

$$\overline{q'_{3D}} = -A\tanh(Ab_{3D})\overline{s_i}(r_{D,p}), \tag{A27}$$

$$\overline{q'_{4D}} = B\tanh(Bb_{4D})\overline{s_i}(r_D, p). \tag{A28}$$

Comparing Eqs. (A27) and (A28) with Eqs. (A6) and (A7), one can find that the equations have the same pattern except that the coth function is replaced by the tanh function. Therefore, the solutions for case B and case C can be easily obtained. For case B, one needs to change the coth function to tanh function for λ_1^2 and λ_2^2. For case C, one needs to change the coth function in the last terms of λ_1^2 and λ_2^2 to tanh function. The final results will not be listed here.

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