THE SKIN EFFECT AND ITS INFLUENCE ON THE PRODUCTIVE CAPACITY OF A WELL

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ABSTRACT

The pressure drop in a well per unit rate of flow is controlled by the resistance of the formation, the viscosity of the fluid, and the additional resistance concentrated around the well bore resulting from the drilling and completion technique employed and, perhaps, from the production practices used. The pressure drop caused by this additional resistance is defined in this paper as the skin effect, denoted by the symbol S. This skin effect considerably detracts from a well's capacity to produce. Methods are given to determine quantitatively (a) the value of S, (b) the final build-up pressure, and (c) the product of average permeability times the thickness of the producing formation.

INTRODUCTION

Equations which relate the pressure in a well producing from a homogeneous formation with pressures existing at various distances around the well are generally used within the industry. The relation is quite simple when the fluid flowing is assumed to be incompressible. It becomes somewhat more complicated when the flowing fluid is considered compressible so that the duration of the flow can be considered. In each case the major portion of the pressure drop occurs close to the well bore. However analyses of pressure build-up curves indicate that the pressure drop in the vicinity of the well bore is greater than that computed from these equations using the known, physical characteristics of the formation and the fluids. In order to explain these excessive drops it is necessary to assume that permeability of the formation at and near the well bore is substantially reduced as a result of drilling, completion and, perhaps, production practice. This possibility has been recognized in the literature.[1,2,3]

A method to compute the pressure drop due to a reduction of the permeability of the formation near the well bore, which is designated as the skin effect, S, is given in the following paragraphs. To start, equations normally used to describe flow in the vicinity of a well are given without considering this effect. These equations then are modified to include the effect of a skin on the pressure behavior. Finally a method is given to estimate the effect of the skin on the pressure and production behavior of a well.

PRESSURE EQUATIONS

Incompressible Fluid Flow

If p_f is defined as the flowing pressure in a well of radius r_w, the pressure at distance r from the well has been shown to be:[4]

$$p_{(r)} = p_{(f)} + \frac{q\mu}{2\pi kh}\, ln\, \frac{r}{r_w}$$

The total pressure drop between the drainage boundary, r_b, and the well bore is given by

$$p(r_b) - p_f = \frac{q\mu}{2\pi kh}\, ln\ (r_b/r_w) \quad . \quad . \quad . \quad . \quad (1)$$

[1]References given at end of paper.

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These equations are valid only if the flow towards the well occurs in a horizontal homogeneous medium and the fluids are incompressible. The assumptions imply that all fluid taken from the well enters the system at r_b, a condition rarely encountered in practice.

Compressible Fluid Flow, Steady State

A more realistic equation is obtained if it is assumed that the compressibility, c, of the flowing fluids is small and has a constant value over the pressure range encountered. After the well has been producing for some time so that its rate has become constant and steady state is reached, the pressures throughout the drainage area are falling by the same amount per unit of time, and the pressure differences between a point in the drainage area and the well are constant. When these conditions are met, the rate of production, q, from a well is equal to $\pi h r_b^2 c f (dp/dt)$, where dp/dt is the pressure drop per unit time. The fluid flowing at a distance r from the center of the well is equal to $q(r_b^2 - r^2)/r_b^2$. From the last equation and from Darcy's law it can be shown that

$$p(r_b) - p_f = \frac{q\mu}{2\pi kh}\ (ln(r_b/r_w) - 1/2) \quad . \quad . \quad . \quad (2)$$

The equation holds for a depletion-type reservoir of radius r_b, drained by a well located in its center, provided the compressibility of the fluid per unit pressure drop is small and constant, and no fluid moves across the boundary r_b.

Compressible Fluid Flow — Nonsteady State

Table III of reference (5) shows the relationship between the pressure at the well bore and the reduced time, $T = kt/f\mu c r_w^2$. The pressure-drop function, $p_{(T)}$, represents the drop below the original reservoir pressure, p_R, caused by unit rate of production ($q_{(T)} = q\mu/2\pi kh = 1$) for several values of R, the ratio of drainage boundary radius, r_b, to well radius, r_w. In most reservoirs the values of r_b/r_w approach infinity, and under these conditions the values of $p_{(T)}$ shown in Table I of reference (5) can be used where $p_{(T)}$ then signifies the difference between the pressure in the well and the prevailing reservoir pressure per unit rate of flow. The total pressure drop below prevailing reservoir pressure amounts to $p_R - p_f = (q\mu/2\pi kh)p_{(T)}$, where the factor $q\mu/2\pi kh$ converts the cumulative pressure drop per unit rate of production to cumulative pressure drop for actual rate, q. For values of $T > 100$ the $p_{(T)}$ function may be written (equation VI-15 of reference 5) as

$$p_{(T)} = 1/2\ (ln\, T + 0.809),$$

Using the time conversion $T = kt/f\mu c r_w^2$, the difference in pressure between reservoir and well becomes

$$p_R - p_f = \frac{q\mu}{4\pi kh}[ln\ (kt/f\mu c r_w^2) + 0.809] \quad . \quad . \quad . \quad (3)$$

If values for the physical constants of the formation and the fluids are inserted, it is found that T exceeds 100 after a few seconds of production (or closed-in time), so that the approximation becomes valid almost at once.

A simple relation between the pressure in the well and in the reservoir can also be derived by considering the well as a point source[1,5,6] instead of a unit circle source, that is, by using Lord Kelvin's solution instead of the unit circle source solution.[5] The difference in pressure at the well bore and the prevailing reservoir pressure then is

$$p_R - p_t = -\frac{q\mu}{4\pi kh} Ei\,(-f\mu c r^2_w/4kt) \quad . \quad . \quad . \quad (4)$$

and this expression also approaches Equation (3) closely whenever the value of $(f\mu c r^2_w/4kt)$ is smaller than 0.01 which as stated before is the case a few seconds after the start of production (or shut-in).

In all of the equations given so far it is assumed that unit rate of production is obtained immediately upon opening the well and, alternatively, that upon shutting-in the well the rate of production from the formation ceases abruptly. The storage capacity of the casing and tubing prevents this ideal condition from being obtained immediately in wells. Hence it is normally observed that the pressure builds up gradually so that it takes from a few minutes up to several days (depending on the characteristics of the formation, the contained fluid, and the storage capacity of the casing) before the observed pressure-time relationship assumes the logarithmic relation mentioned above.

Effect of Storage Capacity of Casing and Tubing

Two methods can be used to express the effect of the storage capacity of casing and tubing on the flow equations given above.

In both cases it is assumed that a well has been closed-in sufficiently long for the pressure to attain substantially the prevailing reservoir pressure. The well then is opened and its measured production is corrected for the fluid obtained from casing and tubing to obtain the cumulative production from the formation proper. These corrections are derived from observations of casing-head, tubing-head, and bottom-hole pressures, and from a knowledge of the dimensions of casing and tubing, and the weight of the oil and gas columns.

Method 1: The cumulative production from the formation is plotted versus time, measured from the instant of opening the well, and, in general, a graph is obtained similar to Fig. 1.

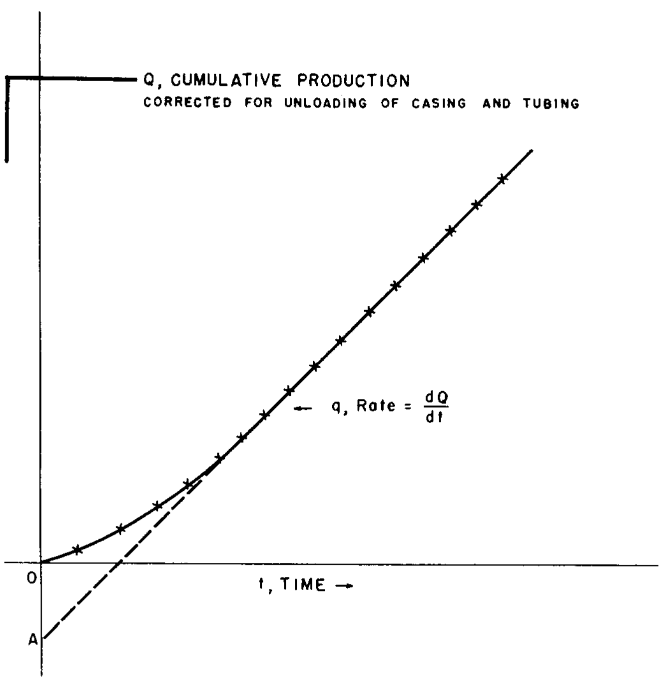

FIG. 1 — DETERMINATION OF RATE OF FLUID ENTRY INTO WELL BORE.

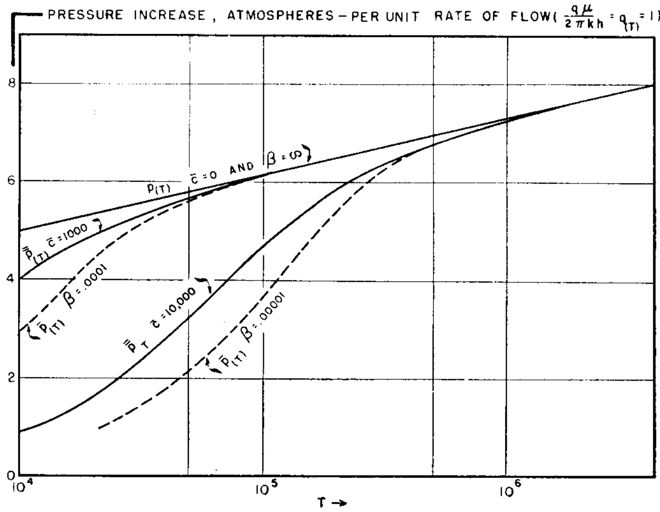

FIG. 2 — $\overline{p}_{(T)}$ AND $\overline{\overline{p}}_{(T)}$ CURVES VS. LOG OF TIME.

which shows that the rate of production from the formation can be closely approximated by a formula of the type $q(1-e^{\quad})$, where both q and α are constants evaluated from the observations.

The rate of production, q, in cc/second at reservoir conditions equals $q\mu/2\pi kh$ reduced rates. The dimension of the factor α, appearing in the exponent, is T^{-1}. Expressing the time in reduced units $T = kt/f\mu c r^2_w$ causes the value of α to change to $\alpha f\mu c r^2_w/k = \beta$. The numerical value of the product βT remains equal to αt.

From reference (5) it is clear that if a unit rate of production ($q_{(T)} = q\mu/2\pi kh = 1$) gives a pressure drawdown of $p_{(T)}$ atmospheres, then the rate $(1 - e^{-\beta T})$ during the pressure drawdown will result in a pressure drop, $\overline{p}_{(T)}$, which is given by the relation

$$\overline{p}_{(T)} = \int_o^T (1 - e^{-\beta T'})\, p'_{(T-T')} dT' \quad . \quad . \quad . \quad . \quad (5)$$

where $p'_{(T)}$ is the differential of the unit function $p_{(T)}$ with respect to time.

Using $\frac{1}{2}(-Ei(-\frac{1}{4}T)$ for the pressure drop caused by unit rate of production, it is found that Equation (5) has as its explicit solution

$$\overline{p}_{(T)} = p_{(T)} - \tfrac{1}{2} e^{-\beta T} [-\ln\beta - 2\gamma + \ln 4 + Ei(\beta T)] \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (6)$$

where $\gamma = 0.57722 =$ Euler's contant and $Ei\,(\beta T) = \int_{-\infty}^{\beta T} (e^u/u)\,du$, whose values are given in reference (7).

To analyze the complications encountered in the pressure build-up curves of wells the point source solution is used. The adoption of this solution instead of the correct unit circle source solution (which is more difficult to handle) is considered permissible as shown by the following table.

$\overline{p}_{(T)}$ computed from:		Point Source	Unit Circle Source
$\beta = 0.001$	T = 50	0.09354	0.09499
	100	0.21247	0.21608
	200	0.46799	0.47336
	500	1.20915	1.21437
	1000	2.19670	2.20112
	2000	3.38670	3.38844

Method 2: Observations on wells have also shown that the amount of fluid, C, which can be withdrawn from (or stored into) casing and tubing per atmosphere pressure difference is a constant whose value can be determined with reasonable accuracy. These values of C are expressed in cc/atmosphere at reservoir conditions. In the system of reduced units used throughout this work $C = 2\pi f h c r_w^2 \overline{C}$, as previously discussed.[5]

If we denote the pressure-drop — time relationship which would result from the relation of rate of production and unloading of casing as $\overline{\overline{p}}_{(T)}$, the rate of production in reduced units from the formation will approach unity according to

$$(1-\overline{C}\, d\overline{\overline{p}}_{(T)}/dT) \quad . \; . \; . \; . \; . \; . \quad (7)$$

and by the superposition theorem[5] the drawdown in a well after opening will be equal to

$$\overline{\overline{p}}_{(T)} = \int_0^T [1-\overline{C}(d\overline{\overline{p}}_{(T')}/dT']p'_{(T-T')}dT' \quad . \; . \quad (8)$$

which is Equation VIII-3 appearing in reference (5). $\overline{\overline{p}}_{(T)}$ curves are shown graphically as Fig. 8 of the same reference.

As shown on Fig. 2, both $\overline{p}_{(T)}$ and $\overline{\overline{p}}_{(T)}$ when plotted versus the log of time show some of the lag in pressure build-up so characteristic in all pressure observations.

The Skin Effect

Although the pressure-drop function modified for the variable rate of production prevailing immediately after closing-in or opening-up of a well shows some of the characteristics of the observed pressure build-up or drawdown curves encountered in practice, agreement between these modified functions and factual data leaves much to be desired. In general, the pressure difference between prevailing reservoir pressure and flowing pressure is larger than can be accounted for by allowing for the variable rate in the manners explained above. Better agreement can be obtained if it is assumed that the permeability of the formation at and near the well bore is substantially reduced as a result of drilling, completion and, perhaps, production practices. Whatever may be the cause for this reduction no reason can be found to assume that this reduction is present beyond 20 ft around the bore hole and probably not that far. The volume of the fluids contained in such a cylinder is small compared to the volume of fluids within the drainage area of a well. It may therefore be concluded that any transient conditions set up in this cylinder are of short duration and can be neglected in the analysis. Hence the effect of a reduction in permeability in this cylinder can be taken into account as an additional pressure drop, proportional at all times to the rate of production from the formation. For this reason the additional pressure drop (per unit rate of flow) near the well bore is considered to be caused by a skin and denoted by S.

Under these conditions Equation (5) is modified further to give

$$\overline{p}_{(T)} = \int_0^T (1-e^{-\beta T'})\, p'_{(T-T')}dT' + (1-e^{-\beta T})S \quad . \; . \; . \quad (9)$$

which has the explicit solution

$$\overline{p}_{(T)} = p_{(T)} + S - \tfrac{1}{2}e^{-\beta T}[-\ln\beta - 2\gamma + \ln 4 + Ei(\beta T) + 2S] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (10)$$

It is easy to see that for large times when $e^{-\beta T}$ becomes zero, Equation (10) gives

$$p_{(T)} = p_{(T)} + S,$$

so that the entire pressure drop equals

$$\Delta p = \frac{q\mu}{2\pi kh}[p_{(T)} + S] = \frac{q\mu}{4\pi kh}[\ln(kt/f\mu c r_w^2) + 0.809 + 2S] \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (11)$$

which is the same as Equation (3) after allowing for the pressure drop caused by the skin.

Furthermore, the presence of a skin causes Equation (8) to be modified to

$$\overline{\overline{p}}_{(T)} = \int_0^T [1-\overline{C}(d\overline{\overline{p}}_{(T)}/dT')]\, p'_{(T-T')}dT' + (1-\overline{C}d\overline{\overline{p}}_{(T)})/dT)S \quad . \; . \; . \quad (12)$$

With the help of LaPlace transformations $\overline{\overline{p}}_{(T)}$ can be expressed as the infinite integral

$$\overline{\overline{p}}_{(T)} = \int_0^\infty \frac{(1-e^{-u^2T})\, J_o(u)\,du}{u\{[1-u^2\overline{C}S + \frac{1}{2}\pi u^2\overline{C}Y_o(u)]^2 + [\frac{1}{2}\pi\overline{C}u^2 J_o(u)]^2\}} \quad . \; . \; . \; . \; . \; . \; . \; . \; . \; . \quad (13)$$

which becomes identical with Equation VIII-11 of reference (5) when $S = 0$.

It can be shown that for large times, Equation (12) also gives $\overline{\overline{p}}_{(T)} = p_{(T)} + S$.

Both Equations (10) and (13) can be used to represent with reasonable accuracy the entire pressure build-up in a well, so that it is felt that all factors influencing the pressure rise (or drop) have been taken into consideration. However, it is not possible to determine the numerical value of the various parameters entering the equations; to be precise, it is not possible to determine from a pressure build-up curve, even if it fits a theoretical curve neatly, the value of S and of the time conversion $k/f\mu c r_w^2$. Equation (11) gives, in a simpler manner, all information useful in field operations which to date has been extracted from the more complex Equations (10) and (13).

ANALYSIS OF BUILD-UP CURVES

The pressure build-up curve of a well is obtained by measuring the bottom-hole pressure in a flowing well, p_f, together with the subsequent pressure increases during a period of sufficient duration following the shutting-in. It is assumed that the well has been producing at a constant rate, q, during a considerable time, t. The pressure increase upon closing-in is recorded as a function of the closed-in time, δ, and only those pressure increases are used after the effects of storage

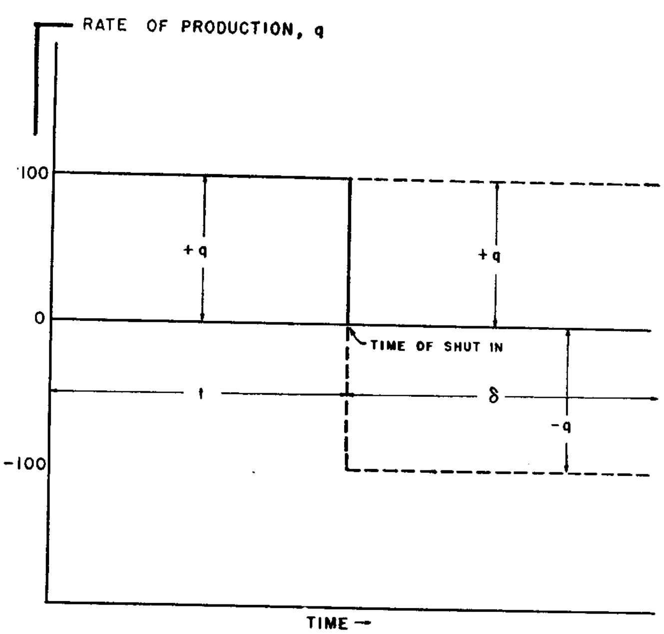

FIG. 3. — SUPERPOSITION OF FLOW RATE FOR CLOSING IN A WELL.

in casing and tubing have died down. In the formulas the closing-in of a well is taken into account by superposing a negative rate, q, so that the rate of withdrawing fluids from the formation becomes zero, as shown on Fig. 3.

From Equation (11), and as shown in reference (6), the pressure drop at any time $(t+\delta)$ caused by the production is found to be

$$\frac{q\mu}{4\pi kh}\left[ln\ (t+\ \delta)\ + ln\ (k/f\mu c r^2_w) + 0.809 + 2S\right],$$

and the pressure increase caused by the superposed negative rate q is

$$-\frac{q\mu}{4\pi kh}\left[ln\ \delta + ln\ (k/f\mu c r^2_w) + 0.809 + 2S\right].$$

The actual pressure decrease at time δ after closing-in is given by the sum of these two expressions

$$p_R - p_\delta = \Delta p_\delta = \frac{q\mu}{4\pi kh}\left[ln\ (t+\delta) - ln\ \delta\right] = \frac{q\mu}{4\pi kh}\ ln\left[(t+\delta)/\delta\right] \quad . \ . \ . \ . \ . \ . \ (14)$$

Equation (14) indicates that the pressure change is proportional to a ln-function of $(t+\delta)/\delta$ and therefore forms a straight-line relationship when plotted on semilog paper. Using Equation (14) for the determination of the prevailing reservoir pressure requires that r_b/r_w be essentially infinite. For the determination of the average permeability and the skin factor it is only necessary that the pressure build-up curves contain a straight-line portion.

Some of this information has been presented in somewhat different form[2,6] and is included here to present a complete analysis of a pressure curve, including the skin effect.

Determination of Prevailing Reservoir Pressure

For infinite closed-in time $(\delta = \infty)$, $ln\left[(t+\delta)/\delta\right]$ and Δp become zero. Therefore the pressure read at $ln\left[t+\delta)/\delta\right] = 0$ reflects the reservoir pressure, p_R, which would prevail at that point had no fluids ever been taken from the reservoir at that particular location. The relation affords a simple means to determine reservoir pressures from build-up surveys where the actual measured pressures are still considerably below final shut-in pressures. It requires, however, that continuous measurements be made. So-called spot readings are likely to be misleading, since the errors in such measurements (especially when made with different recorders) are apt to exceed the relatively small pressure increases actually occurring and thereby give an erroneous slope to the straight-line portion of the build-up curves.

Table 1 — Reperforating Job

	Before	After
Perforated interval	6258-70 ft	6258-70 ft
h, producing interval	548 cm	548 cm
Cumulative production	3956 B/D	5500 B/D
Production rate	96 B/D	60 B/D
Production time	41.21 days	91.67 days
t, production time	3.561,000 sec	7,920,000 sec
Shrinkage factor	0.795	0.795
q at reservoir conditions	222 cc/sec	139 cc/sec
Pressure increase per cycle	6 psi	4 psi
p_f, flowing pressure	2060 psi	2502 psi
μ, viscosity	0.65 cp	0.65 cp
f, porosity	0.219	0.219
c, fluid compressibility	0.00017/atm	0.00017/atm
r_w, well radius	6.3 cm	6.3 cm

Determination of Average Reservoir Permeability

The slope of the straight-line portion of the build-up curve according to (14) is equal to $q\mu/4\pi kh$. When a plot on semilog-log paper is used, the pressure increase per unit rate of flow $(q_{(T)} = q\mu/2\pi kh = 1)$, per 10-fold increase in $(t+\delta)/\delta$ is equal to $\frac{1}{2}\ ln\ 10 = 1.1513$ atmospheres $= 16.924$ lb. Hence if the pressure increases 40 lb per cycle, the well has produced at $40/16.924 = q_{(T)} = 2.36$ unit rates $= q\mu/2\pi kh$. When q and μ are known from other sources the average value of kh can be found. If a reliable value of h is also available an average value of the permeability can be obtained.

Determination of Skin Factor

Equation (11) shows the pressure drop at the time of shut-in to be

$$\Delta p_{(\delta=0)} = \frac{q\mu}{4\pi kh}\left[ln\ t + ln\ (k/f\mu c r^2_w) + 0.809 + 2S\right] \ . \quad (11)$$

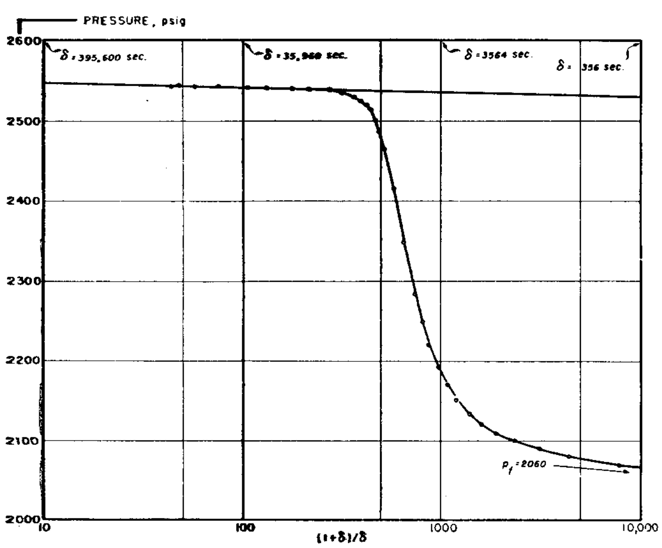

FIG. 4 — PRESSURE BUILD-UP CURVE BEFORE REPERFORATING.

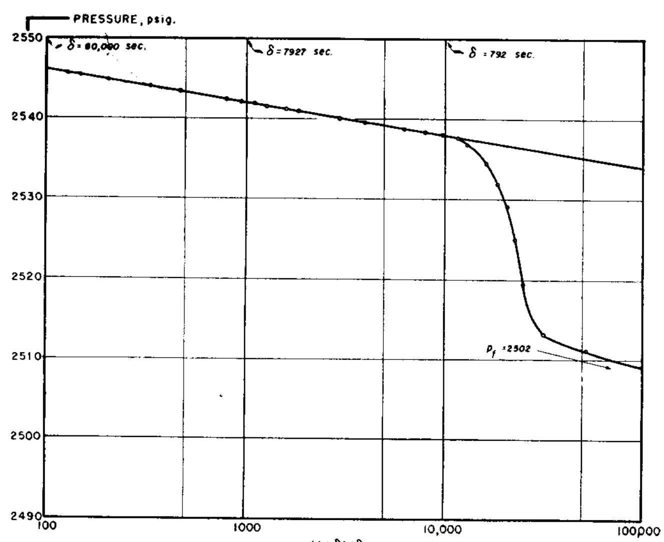

FIG. 5 — PRESSURE BUILD-UP CURVE AFTER REPERFORATING.

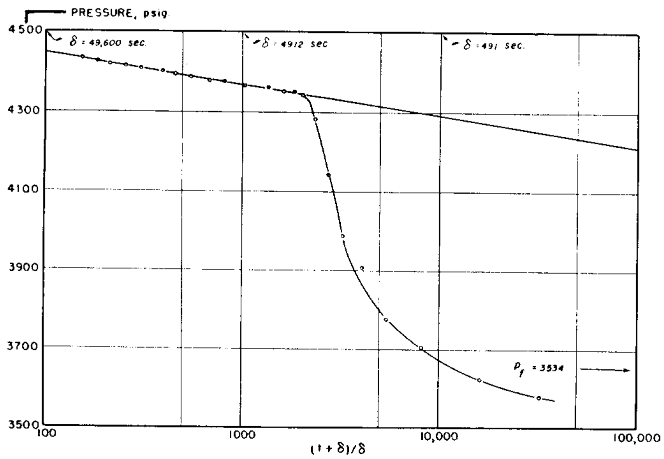

FIG. 6 — PRESSURE BUILD-UP CURVE BEFORE ACIDIZING.

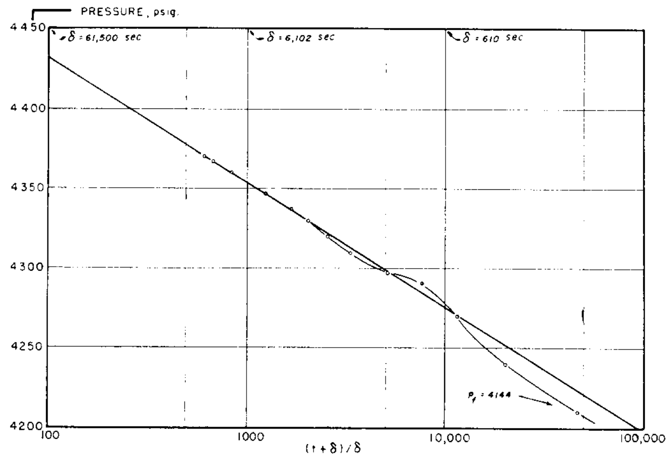

FIG. 7 — PRESSURE BUILD-UP CURVE AFTER ACIDIZING.

Equation (14) shows the pressure drop at closed-in time δ to be

$$\Delta p_{\delta} = \frac{q\mu}{4\pi kh} ln\,[(t+\delta)/\delta] \quad . \; . \; . \; . \; . \quad (14)$$

For values of δ small compared to t, $ln\,[(t+\delta)/\delta]$ is essentially equal to $ln\; t/\delta$, so that Equation (14) can be modified to

$$\Delta p_{\delta} = \frac{q\mu}{4\pi kh}[ln\; t - ln\; \delta] \quad . \; . \; . \; . \; . \; . \; . \; . \quad (14\text{-a})$$

By subtracting Equation (14-a) from (11)

$$\Delta p_{(\delta=0)} - \Delta p_{(\delta)} = \frac{q\mu}{4\pi kh}[ln\; \delta + ln\; (k/f\mu c r_w^2) + 0.809 + 2S] \quad . \; . \; . \; . \; . \quad (15)$$

Since the left-hand side of this equation is the difference between flowing pressure and the pressure on the straight line for time δ, and $q\mu/2\pi kh$ is the slope, a value of

$$ln\; (k/f\mu c r_w^2) + 0.809 + 2S \quad . \; . \; . \; . \quad (16)$$

can be found. If the radius of the well bore, the compressibility of the fluid, and the porosity are inserted in the equation, a value for S is obtained.

To obtain the above-mentioned objectives a definite straight-line portion of the build-up curve should be available. This is further shown by a consideration of Fig. 2 which shows that the $\overline{\overline{p}}_{(T)}$ functions have a tendency to show a linear relationship with $ln\; T$ before actually coinciding with the $p_{(T)}$ functions. Therefore the duration of the pressure survey should be considerably longer than the time required for the effects of storage in casing and tubing to die down.

FIELD APPLICATIONS

Field experience shows that the productive capacity of a well can be increased considerably by reducing the value of $(\frac{1}{2}\, ln\; k/f\mu c r_w^2 + S)$. Two examples are given to illustrate this statement.

Reperforating

The effect of reperforating a well on the value of S is given in Figs. 4 and 5; production, well, and PVT data are summarized in Table 1.

As a well is seldom produced at a constant rate, the time, t, before closing-in is approximated by dividing the cumulative production before closing-in, by the rate prevailing at that time. This approximation of t becomes more reliable the longer the well is produced at the constant rate, q. The following information can be derived from the table and Figs. 4 and 5.

	Before Reperforating	After Reperforating
p_R	2554 psi	2554 psi
$p_R - p_f$	494 psi	52 psi
$\frac{14.7\, q\mu}{2\pi kh}[\frac{1}{2}\, ln\; (k/f\mu c r_w^2) + S]$	454 psi	24 psi
$\frac{14.7\, q\mu}{2\pi kh}[\frac{1}{2}\, ln\; (k/f\mu c r_w^2)]$	13 psi	8 psi
$\frac{14.7\, q\mu}{2\pi kh} S$	441 psi	16 psi
S	84.8	4.6
k	118 md	111 md

The curve shown is typical for a formation having high values of $2\pi kh/\mu$, as usually found in the Miocene sands around the Gulf Coast. Under these conditions the pressure increases per cycle are small, which stresses the necessity of obtaining accurate data for determining the straight-line portion of the build-up curve. The determination of S, also, is highly sensitive to variations in the small values of $q\mu/2\pi kh$. In extreme cases the straight-line portion of the build-up curve approaches a horizontal line, and the entire pressure increase after shut-in of the well is then an indication of the

Table 2 — Acidizing Job

	Before	After
h, producing interval	2103 cm	2103 cm
Production rate	250 B/D	255 B/D
Production time, sec	4,908,000 sec	6,096,000 sec
Shrinkage factor	0.88	0.88
q at reservoir conditions	523 cc/sec	533 cc/sec
Pressure increase per cycle	77.6 psi	78.3 psi
p_f, flowing pressure	3534 psi	4144 psi
μ, viscosity	0.80 cp	0.80 cp
f, porosity	0.039	0.039
c, fluid compressibility	0.0001/atm	0.0001/atm
r_w, well radius	6.03 cm	6.03 cm

presence of a considerable skin effect, whose actual value, however, cannot be determined.

It is evident that in the case discussed above the first perforation job was not efficient and that reperforation essentially removed a large resistance which existed near the well bore. Before reperforating, the entire pressure drop was 494 psi out of which 441 psi or 89 per cent was caused by the skin effect. After reperforating, these figures were 52 psi, 16 psi and 31 per cent respectively. Since the rate before reperforating was 96 B/D against 60 B/D after reperforating, the last set of pressure figures given should be multiplied by 1.6 in order to obtain a valid comparison.

Acidizing

An example of a well before and after acidization is given in Figs. 6 and 7; the production data are assembled in Table 2. Pertinent conclusions follow.

	Before Acidizing	After Acidizing
p_R	4600 psi	4589 psi
$p_R - p_f$	1066 psi	445 psi
$\frac{14.7\, q\mu}{2\pi kh} [\frac{1}{2} \ln (k/f\mu c r^2_w) + S]$	519 psi	−112 psi
$\frac{14.7\, q\mu}{2\pi kh} [\frac{1}{2} \ln (k/f\mu c r^2_w)]$	138 psi	140 psi
$\frac{14.7\, q\mu}{2\pi kh} S$	381 psi	−252 psi
S	5.6	−3.7
k	6.9 md	7.0 md

The presence of a negative skin may be questioned and is shown here merely to illustrate the change resulting from acidizing the well. It is probable that a negative value for S reflects an increased effective well radius, r_w.

The above analysis shows that the value of S, is not the most significant figure that can be obtained from this type of information. It seems reasonable to suppose that acidizing increases the permeability of the formation immediately surrounding the well bore to such an extent that it becomes extremely large compared to its original value. This increase in permeability for some distance around the well bore can be regarded as increasing the effective radius of the well several fold (without increasing the size of the bore hole). Hence, acidization will cause not only the numerical value of S to decrease but also the value of $\frac{1}{2} \ln (k/f\mu c r^2_w)$ to decrease in the same operation, due to an increase in r_w. Hence the effectiveness of acidization can best be judged by comparing the combined values of $\frac{1}{2} \ln (k/f\mu c r^2_w) + S$ before and after acidization, without considering which of these two factors contributes to the improvement. From this reasoning it follows that acidization should be repeated whenever a sizeable decrease in the sum of these two factors can be expected.

NOMENCLATURE

	Used in this Paper	Used in Ref. 5
Production time, seconds	t	T
Production time in reduced units	T	t
Time well has been shut-in, seconds	δ	—
Permeability, darcys	k	K
Porosity, a fraction	f	f
Viscosity of reservoir fluid, centipoises	μ	μ
Compressibility of the reservoir fluid/atm.	c	c
Radius of well, cm.	r_w	R_b
Drainage radius of well, cm.	r_b	RR_b
Rate of fluid flow (cc/sec., reservoir conditions)	q	—
Formation thickness, cm.	h	H
Rate of flow in reduced units	$q_{(T)} = q\mu/2\pi kh$	$q_{(t)} = q\mu/2\pi KH$
Prevailing reservoir pressure, *i.e.*, well pressure after infinite shut-in time	p_R	—
Pressure at drainage boundary of flowing well	$p(r_b)$	—
Flowing bottom-hole pressure in the well at time of shut-in	p_f	—
Increase or decrease in bottom-hole pressure at time δ after shutting-in or opening-up a well	Δp	ΔP
Pressure decrease caused by unit rate of flow	$p_{(T)}$	$P_{(t)}$
Pressure drop per unit rate of flow caused by the skin, dimensionless	S	—
The natural (Naperian) log of a number	$\ln$ ( )	—

NOTE: All terms in the equations used in this paper are in the system of units associated with Darcy's law, and the reader is referred to suitable conversion tables such as shown in reference (6) whenever production data are expressed in a different system.

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