$\Omega = \{ K, HK, HHK, HHH \}$

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{8}$

$\mathcal{A} = \exp\Omega = \{ \emptyset, \{K\}, \dots, \{K, HHH\}, \dots \{K, HK, HHK, HHH\} \}$

$P : \mathcal{A} \to \mathbb{R}^+$

$P(A) =$

$P(\{K\}) = \frac{1}{2}$

$P(\{HK\}) = \frac{1}{4}$

$\vdots$

$\Omega = \{ K, HK, HHK, HHH \}$

$X : \Omega \to \mathbb{R}$

$\forall I \subseteq \mathbb{R} : X^{-1}(I) \in \mathcal{A}$

$X^{-1}(I) = \{ \omega \in \Omega \mid X(\omega) \in I \}$

ϕ 1 2 3

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{4}$

$P[X=1] = \frac{1}{2}$

$P[X=2] = \frac{1}{4}$

$P[X=3] = \frac{1}{4}$

$\Omega = \{ K, HK, HHK, HHH \}$

$\mathcal{A} = \{ \emptyset, \{K, HK, HHK\}, \{HHH\}, \{K, HK, HHK, HHH\} \}$

X 1 2 I 3

$X^{-1}(I) = \{ HK, HHK, HHH \}$

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{4}$

1 2 3

$F_X(x) = P[X \leq x]$

F_X

$\frac{3}{4}$

$P[X \leq 1] = \frac{1}{2}$

1 2 3

$P[X \leq 2] - P[X \leq 1]$

$P[1 \leq X \leq 2] =$

$= P[X \leq 2] - P[X < 1]$

$= F_X(2) - \lim_{x \to 1^-} F_X(x)$

$= \frac{3}{4} - 0 = 3/4$

$EX = \sum x \cdot P[X = x] = 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} + 3 \cdot \frac{1}{4}$

$= \frac{1}{2} + \frac{1}{2} + \frac{3}{4} = \frac{7}{4} = 1{,}75$

$\frac{1}{2}$ $\frac{1}{4}$ $\frac{1}{8}$ $\frac{1}{8}$

$\Omega = \{ K, HK, HHK, HHH \}$

Y 0 $\frac{1}{8}$ 1 $\frac{7}{8}$

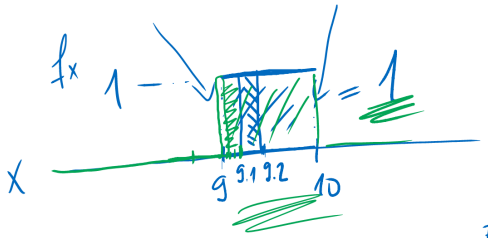
F_Y

1

$\frac{1}{8}$

0 1

$\Omega = \langle 9.00 \ldots 10.00 \rangle$

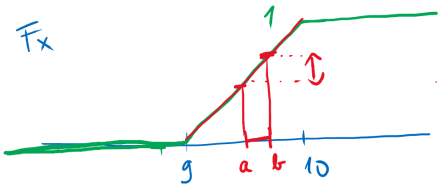


$EX = \sum x \cdot P[X=x]$

$EX = \int_{-\infty}^{\infty} x \cdot f_X(x)\,dx$

$EX = 9{,}5$

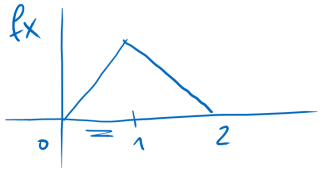
$F_X(x) = \int_{-\infty}^{x} f_X(u)\,du$



$P[a \leq X \leq b] = F_X(b) - F_X(a)$

(4)

$$f_X(x) = \begin{cases} 0 & (-\infty, 0) \\ x & \langle 0,1) \\ 2-x & \langle 1,2) \\ 0 & \langle 2,\infty) \end{cases}$$



$x \in \langle 0,1) :$

$F_X(x) = \int x\,dx = \frac{x^2}{2}$

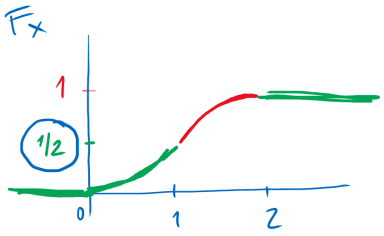
$x \in \langle 1,2) :$

$F_X(x) = \int (2-x)\,dx + c$

$= 2x - \frac{x^2}{2} - 1$

$x=1 : \; 2 - \frac{1}{2} = \frac{3}{2}$

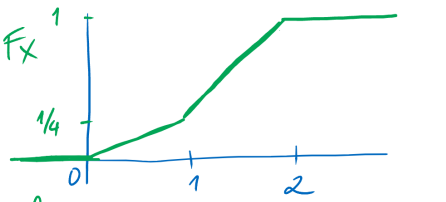
$x=2 : \; 4 - 2 - 1 = 1$

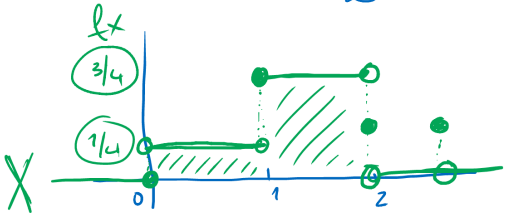


(5)

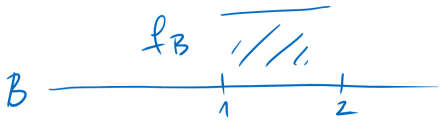
$$F_X(x) = \begin{cases} 0 & (-\infty, 0) \\ \frac{1}{4}x & \langle 0,1) \\ \frac{3}{4}x - \frac{1}{2} & \langle 1,2) \\ 1 & \langle 2,\infty) \end{cases}$$

$$f_X(x) = \begin{cases} 0 & \ldots \\ 1/4 & \ldots \\ 3/4 & \ldots \\ 0 & \ldots \end{cases}$$





$X = \text{Mix}_{(\frac{1}{4}, \frac{3}{4})}(A, B)$



5× ⚀ 20× ◇ $Z = Mix_{(\frac{5}{25}, \frac{20}{25})}(\check{S}, \check{C})$

x	1	2	3	4	5	6
$P[\check{S}=x]$	1/6	1/6	1/6	1/6	1/6	1/6
$P[\check{C}=x]$	1/4	1/4	1/4	1/4	0	0
$P[Z=x]$	7/30	7/30	7/30	7/30	1/30	1/30

$(\frac{1}{5}, \frac{4}{5})$

$\frac{1}{5}\cdot\frac{1}{6} + \frac{4}{5}\cdot 0 \neq 1$

$\frac{1}{5}\cdot\frac{1}{6} + \frac{4}{5}\cdot\frac{1}{4} = \frac{1}{30} + \frac{4}{20} = \frac{1}{30} + \frac{2}{10} = \frac{7}{30}$

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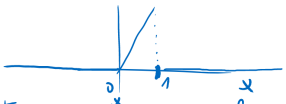
(8) D:

x	1/4	2/4	3/4
$P[D=x]$	1/8	5/8	2/8

a $f_S(x) = \begin{cases} 0 & \\ 2x & \langle 0,1\rangle \\ 0 & \end{cases}$

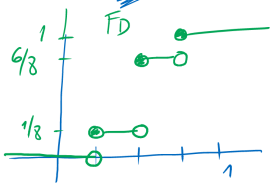
$Z = Mix_{(0{,}7;0{,}3)}(D, S)$

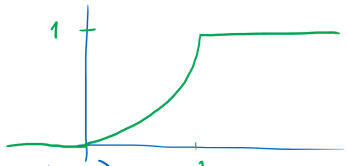


$F_S(x) = \int_0^x f(u)\,du = \int_0^x 2u\,du = [u^2]_0^x = x^2$

$F_S(x) = \begin{cases} 0 & \\ x^2 & \langle 0,1\rangle \\ 1 & \end{cases}$

$F_D(x) = \begin{cases} 0 & -\infty, 1/4 \\ 1/8 & 1/4, 2/4 \\ 6/8 & 2/4, 3/4 \\ 1 & 3/4, \infty \end{cases}$



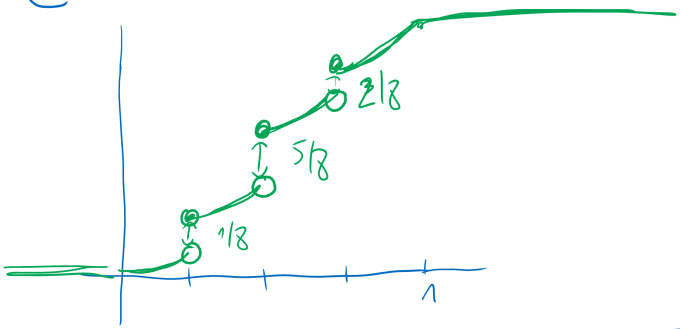


$Z = Mix_{(0{,}7;0{,}3)}(D, S)$

$F_Z(x) = 0{,}7\cdot F_D(x) + 0{,}3\cdot F_S(x)$

$F_Z(x) = \begin{cases} 0 & (-\infty, 0) \\ 0{,}3x^2 & \langle 0, 1/4) \\ 0{,}3x^2 + \frac{7}{80} & \langle 1/4, 1/2) \\ 0{,}3x^2 + \frac{42}{80} & \langle 1/2, 3/4) \\ 0{,}3x^2 + 0{,}7 & \langle 3/4, 1) \\ 1 & \langle 1, \infty) \end{cases}$

$0{,}7\cdot 0 + 0{,}3\cdot x^2$
$0{,}7\cdot 1/8 + 0{,}3\cdot x^2$
$0{,}7\cdot 6/8 + 0{,}3\,x^2$
$0{,}7\cdot 1 + 0{,}3\,x^2$



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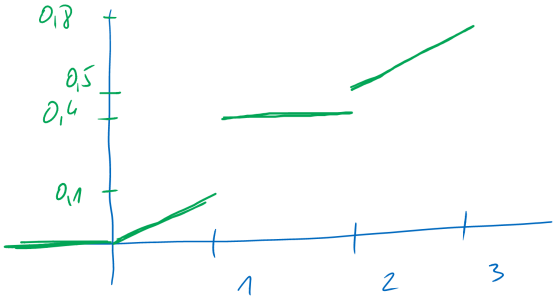
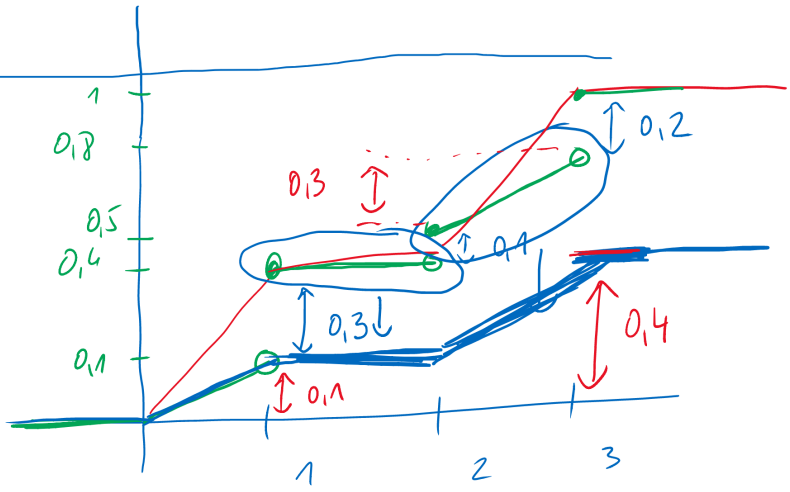
(9)

$F_X(x) = \begin{cases} 0 & (-\infty, 0) \\ 0{,}1x & \langle 0, 1) \\ 0{,}4 & \langle 1, 2) \\ 0{,}3x - 0{,}1 & \langle 2, 3) \\ 1 & \langle 3, \infty) \end{cases}$



9)

$$F_X(x) = \begin{cases} 0 & (-\infty, 0) \\ 0{,}1x & \langle 0, 1) \\ 0{,}4 & \langle 1, 2) \\ 0{,}3x - 0{,}1 & \langle 2, 3) \\ 1 & \langle 3, \infty) \end{cases}$$



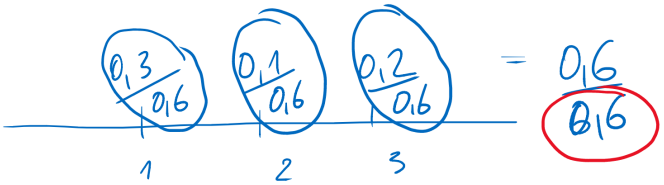
$X = \min_{(\alpha, 1-\alpha)} (D, S)$

$\alpha = 0{,}6$ (↑), $1-\alpha = 0{,}4$ (↖), $(-\infty, \infty)$

$$F_S(x) = \begin{cases} \frac{0}{0{,}4} & \langle 0, 1) \\ \frac{0{,}1x}{0{,}1} & \langle 1, 2) \\ \frac{0{,}1}{} & \langle 2, 3) \\ \frac{0{,}3x - 0{,}5}{0{,}4} & \langle 3, \infty) \\ 0{,}4 & \\ \hline 0{,}4 & \end{cases}$$

0,4

D)



$$P[D = x] = \begin{cases} 1/2 & \dots x = 1 \\ 1/6 & 2 \\ 1/3 & 3 \end{cases}$$

0,6