(2) $\Omega = \{K, HK, HHK, HHH\}$

$\frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{8} \quad \frac{1}{8}$

$\mathcal{A} = \exp \Omega$

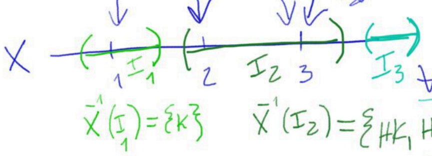
$\mu_A(x) = \begin{cases} 0 & x \notin A \\ 1 & x \in A \end{cases}$

$A \in \mathcal{A}$: $P(A) = \frac{1}{2}\mu_A(K) + \frac{1}{4}\mu_A(HK) + \frac{1}{8}\mu_A(HHK) + \frac{1}{8}\mu_A(HHH)$

$P(\underbrace{\{K, HHH\}}_{A}) = \frac{1}{2} + \frac{1}{8} = \frac{5}{8}$

$X: \Omega \to \mathbb{R}$

$X^{-1}(I_3) = \emptyset$

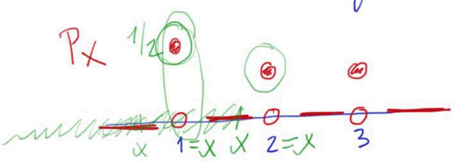


$\forall I \subseteq \mathbb{R}: X^{-1}(I) \in \mathcal{A}$

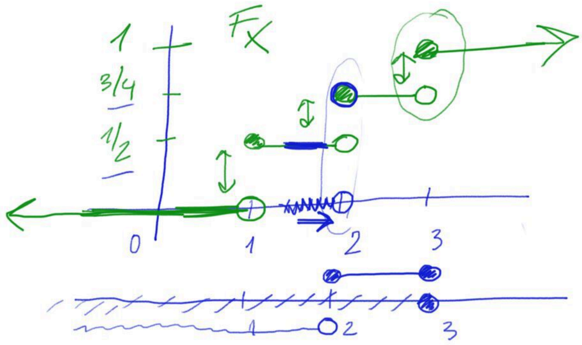
$X^{-1}(I_1) = \{K\}$ $\quad X^{-1}(I_2) = \{HK, HHK, HHH\}$

$\mathcal{A} = \{\emptyset, \{K, HK, HHK, HHH\}\}$

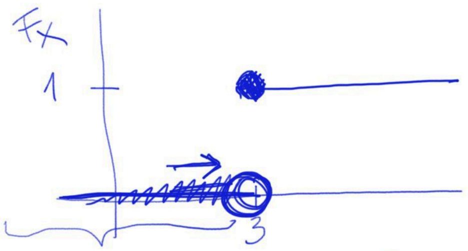
$P[X=x] = \begin{cases} 1/2 & x = 1 \\ 1/4 & x = 2 \\ 1/4 & x = 3 \\ 0 & \text{jinak} \end{cases}$

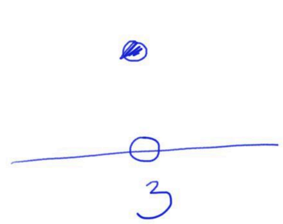


$F_X(x) = P[X \leq x]$



$P[2 \leq X \leq 3] =$
$= P[X \leq 3] - P[X < 2]$
$= F_X(3) - \lim_{x \to 2_-} F_X(x)$
$= 1 - \frac{1}{2} = \frac{1}{2}$

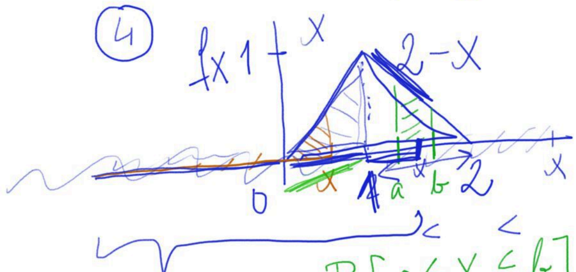




$F_X(3) = P[X \leq 3] = 1$

$\lim_{x \to 3_-} F_X(x) = P[X < 3] = 0 \neq F_X(3)$

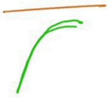
(4)



$\int_{-\infty}^{\infty} f_X(x)\,dx = 1$

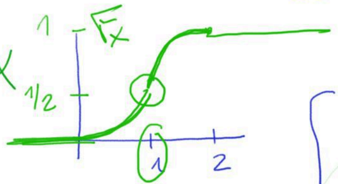
$f_X(x) \geq 0$

$F_X(x) = P[X \leq x]$



$= \int_{-\infty}^{x} f_X(u)\,du$

$P[a \leq X \leq b] = \int_a^b f_X(x)\,dx$



$f_X(x) = \begin{cases} 0 & x \in (-\infty, 0) \\ x & \langle 0, 1) \\ 2-x & \langle 1, 2) \\ 0 & \langle 2, \infty) \end{cases}$

$F_X(x) = \begin{cases} 0 & x \in (-\infty, 0) \\ \frac{x^2}{2} & \langle 0, 1) \\ 2x - \frac{x^2}{2} - 1 & \langle 1, 2) \\ 1 & \langle 2, \infty) \end{cases}$

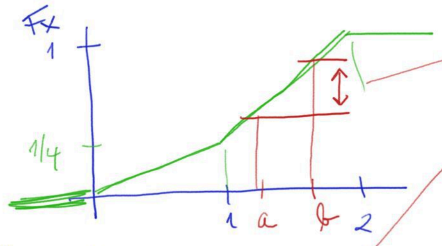
$x \in \langle 0, 1)$: $\int_{-\infty}^{x} f_X(u)\,du = \int_{-\infty}^{x} u\,du = \int_0^x u\,du = \left[\frac{u^2}{2}\right]_0^x = \frac{x^2}{2}$

$x \in \langle 1, 2)$: $\int_{-\infty}^{x} f_X(u)\,du = 0 + \frac{1}{2} + \int_1^x (2-u)\,du = \frac{1}{2} + \left[2u - \frac{u^2}{2}\right]_1^x = \frac{1}{2} + 2x - \frac{x^2}{2} - 2 + \frac{1}{2}$

$\int (2-x)\,dx = 2x - \frac{x^2}{2} + C$ $\quad \frac{1^2}{2} = 2 - \frac{1^2}{2} + C \Rightarrow C = -1$ $\quad = 2x - \frac{x^2}{2} - 1$

(5)

$$F_X(x) = \begin{cases} 0 & (-\infty, 0) \\ x/4 & \langle 0, 1) \\ 3/4\,x - 1/2 & \langle 1, 2) \\ 1 & \langle 2, \infty) \end{cases}$$

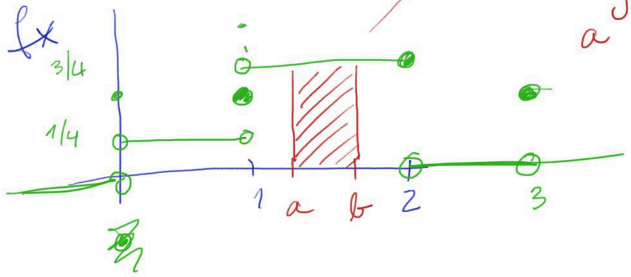


$P[a \leq X \leq b]$

$= F_X(b) - F_X(a)$

$= \int_a^b f_X(x)\,dx$

$$f_X(x) = \begin{cases} 0 & (-\infty, 0) \\ 1/4 & \langle 0, 1\rangle \\ 3/4 & (1, 2\rangle \\ 0 & (2, \infty) \end{cases}$$



(6) 20x  5x 

$$\underbrace{\underbrace{20\times}_{\check{C}\ \frac{20}{25}=\frac{4}{5}} \quad \underbrace{5\times}_{\check{S}}}_{X = Mix_{(\frac{4}{5}, \frac{1}{5})}(\check{C}, \check{S})}$$

x	1	2	3	4	5	6	
$P[\check{S}=x]$	1/6	1/6	1/6	1/6	1/6	1/6	
$P[\check{C}=x]$	1/4	1/4	1/4	1/4	0	0	
$P[X=x]$	7/30	7/30	7/30	7/30	1/30	1/30	$= 1$

$$P[X=x] = \frac{4}{5} P[\check{C}=x] + \frac{1}{5} P[\check{S}=x]$$

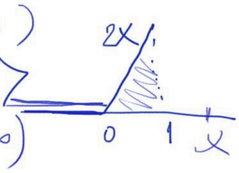
$$P[X=1] = \frac{4}{5} \cdot \frac{1}{4} + \frac{1}{5} \cdot \frac{1}{6} = \frac{4}{20} + \frac{1}{30} = \frac{2}{10} + \frac{1}{30} = \frac{7}{30}$$

$$P[X=5] = \frac{4}{5} \cdot 0 + \frac{1}{5} \cdot \frac{1}{6} = \frac{1}{30}$$

(8) D:

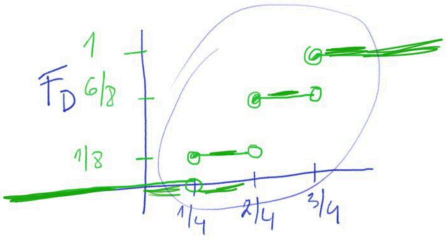
x	$\frac{1}{4}$	$\frac{2}{4}$	$\frac{3}{4}$
$P[X=x]$	$\frac{1}{8}$	$\frac{5}{8}$	$\frac{2}{8}$

S:
$$f_S(x) = \begin{cases} 0 & (-\infty, 0) \\ 2x & \langle 0, 1\rangle \\ 0 & (1, \infty) \end{cases}$$



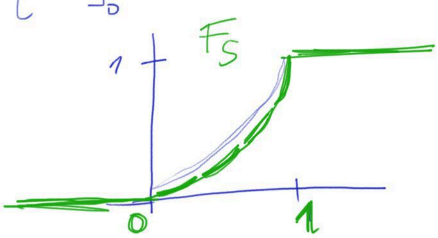
$$X = Mix_{(70\%, 30\%)}(D, S)$$

$$F_X(x) = 0{,}7\, F_D(x) + 0{,}3\, F_S(x)$$



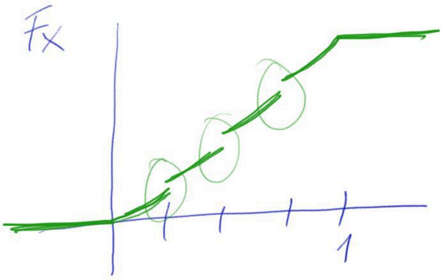
$$F_S(x) = \begin{cases} 0 & (-\infty, 0) \\ x^2 & \langle 0, 1) \\ 1 & (1, \infty) \end{cases}$$

$$\int_0^x 2u\,du = [u^2]_0^x = x^2$$



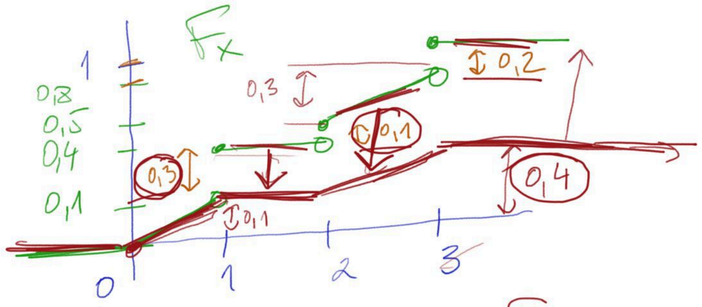
$$F_D(x) = \begin{cases} 0 & (-\infty, 1/4) \\ 1/8 & \langle 1/4, 2/4) \\ 6/8 & \langle 2/4, 3/4) \\ 1 & \langle 3/4, \infty) \end{cases}$$

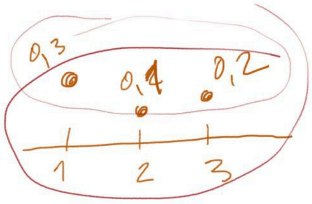
$$F_X(x) = \begin{cases} 0 & (-\infty, 0) \\ 0{,}3x^2 + 0{,}7 \cdot 0 & \langle 0, 1/4) \\ 0{,}3x^2 + 0{,}7 \cdot \frac{1}{8} & \langle 1/4, 2/4) \\ 0{,}3x^2 + 0{,}7 \cdot \frac{6}{8} & \langle 2/4, 3/4) \\ 0{,}3x^2 + 0{,}7 \cdot 1 & \langle 3/4, 1) \\ 0{,}3 \cdot 1 + 0{,}7 \cdot 1 & \langle 1, \infty) \end{cases}$$



9

$$F_X(x) = \begin{cases} 0 & (-\infty, 0) \\ 0.1x & \langle 0, 1) \\ 0.4 \; - 0{,}3 & \langle 1, 2) \\ 0.3x - 0.1 \; - 0{,}4 & \langle 2, 3) \\ 1 \; - 0{,}6 & \langle 3, \infty) \end{cases}$$





$$X = Mix_{(\alpha, 1-\alpha)}(D, S)$$

0,6 → α, 0,4 → $1-\alpha$

$$F_S(x) = \begin{cases} \frac{0}{0.4} & (-\infty, 0) \\ \frac{0.1x}{0.4} & \langle 0, 1) \\ 0{,}1 \div 0{,}4 & \langle 1, 2) \\ \frac{0.3x - 0.5}{0.4} & \langle 2, 3) \\ \frac{0.4}{0.4} & \langle 3, \infty) \end{cases}$$

x	1	2	3	
$P[D=x]$	$\frac{0{,}3}{0{,}6}$	$\frac{0{,}1}{0{,}6}$	$\frac{0{,}2}{0{,}6}$	$= \frac{0{,}6}{0{,}6}$
	$1/2$	$1/6$	$1/3$	1