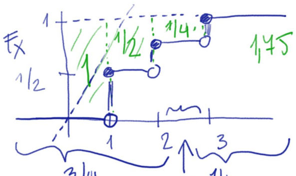
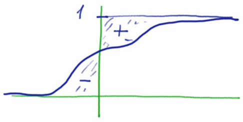
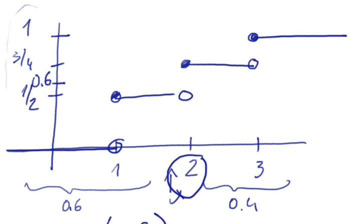
②

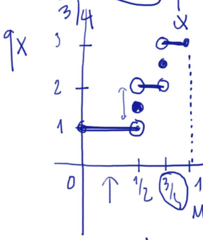
X	1	2	3
$P[X=x]$	1/2	1/4	1/4

$$EX = \frac{1}{2}\cdot 1 + \frac{1}{4}\cdot 2 + \frac{1}{4}\cdot 3 = \frac{1}{2} + \frac{1}{2} + \frac{3}{4} = \frac{7}{4} = \cancel{1{,}25}\ 1{,}75$$









$q_X(\alpha) = x$, $\alpha \in \langle 0,1\rangle$

$(-\infty, x\rangle$ → α $\qquad$ $\langle x, \infty)$ → $1-\alpha$

$q_X(0.6) = x$

$P[X \in (-\infty, 2\rangle] =$

MEDIÁN $q_X\left(\frac{1}{2}\right) = 1{,}5$

DOLNÍ KVARTIL $q_X\left(\frac{1}{4}\right) = 1$

HORNÍ KVARTIL $q_X\left(\frac{3}{4}\right) = 2{,}5$

DOLNÍ DECIL $q_X(0.1)$

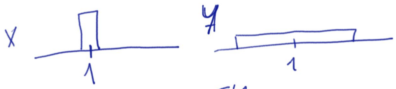
X	1	2	3
P	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

X^2	1	4	9
P	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{4}$

$EX = 1{,}75$

$DX = E(X^2) - (EX)^2 = 3{,}75 - 1{,}75^2 \doteq 0{,}69$

$$E(X^2) = 1\cdot\frac{1}{2} + 4\cdot\frac{1}{4} + 9\cdot\frac{1}{4} = \frac{1}{2} + 1 + \frac{9}{4} = \frac{15}{4} = 3{,}75$$



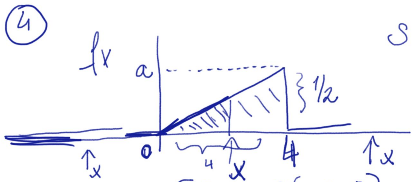
$EX = EY$

$DX < DY$

$\sigma_X = \sqrt{DX} = \sqrt{0{,}69} \doteq 0{,}83$

$$\int_0^x \frac{1}{8} u\, du = \frac{1}{8}\cdot\frac{1}{2}\left[u^2\right]_0^x = \frac{x^2}{16}$$

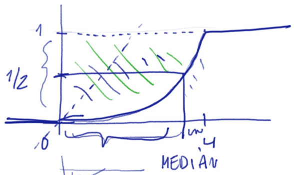
④

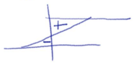


$$S = \frac{4\cdot a}{2} = 2a = 1 \qquad a = 1/2$$

$$f_X(x) = \begin{cases} 0 & x \in (-\infty, 0) \\ 1/8 \cdot x & x \in \langle 0, 4) \\ 0 & x \in \langle 4, \infty) \end{cases} \qquad F_X(x) = \begin{cases} 0 & x \in (-\infty, 0) \\ x^2/16 & x \in \langle 0, 4) \\ 1 & x \in \langle 4, \infty) \end{cases}$$

$$EX = \int_{-\infty}^{\infty} x f_X(x)\, dx = \int_0^4 x\cdot\frac{1}{8}x\, dx = \frac{1}{8}\frac{1}{3}\left[x^3\right]_0^4 = \frac{4\cdot 4\cdot 4\cdot 2}{4\cdot 2\cdot 3} = \frac{8}{3}$$

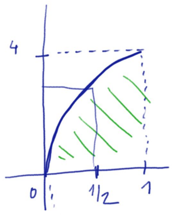




$$EX = \int_0^4 1\, dx - \int_0^4 \frac{x^2}{16}\, dx = \left[x\right]_0^4 - \frac{1}{16}\cdot\frac{1}{3}\left[x^3\right]_0^4 = 4 - \frac{4\cdot 4\cdot 4}{16\cdot 3} = \frac{12-4}{3} = \frac{8}{3}$$

$$DX = E(X^2) - (EX)^2 = 8 - \left(\frac{8}{3}\right)^2 = \frac{72-64}{9} = \frac{8}{9}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\, dx = \int_0^4 x^2 \frac{1}{8} x\, dx = \frac{1}{8}\frac{1}{4}\left[x^4\right]_0^4 = \frac{4\cdot 4\cdot 4\cdot 4\cdot 2}{8\cdot 4\cdot 2} = 8$$



$\sigma_X = \sqrt{\frac{8}{9}}$

$y = \frac{x^2}{16}$

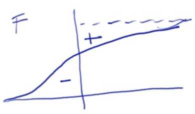
$16y = x^2$

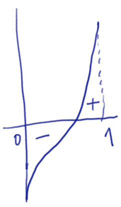
$\pm 4\sqrt{y} = x$

$q_X(x) = 4\sqrt{x}$

$q_X(1/2) = 4\sqrt{\frac{1}{2}}$

$$EX = \int_0^1 q_X(x)\, dx = \int_0^1 4\sqrt{x}\, dx = \cancel{4\left[\frac{1}{2\sqrt{x}}\right]_0^1} = 4\left[\frac{2x^{3/2}}{3}\right]_0^1 = \frac{8}{3}$$





PETROHRADSKÝ PARADOX

líc padne v k-tém hodu : výhra je 2^k CZK

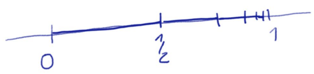
$A_k \dots (\underbrace{R, R, \dots, R}_{k-1}, L)$

$|\Omega| = \infty$ (spočetná)

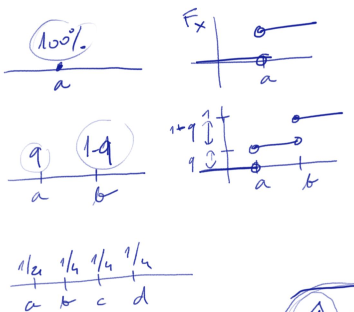
k	1	2	3	4	...	k
$P(A_k)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$		$\frac{1}{2^k}$

x	2	4	8	16	2^k
$P[X=x]$	1/2	1/4	1/8	1/16	$1/2^k$

$$\sum_{k=1}^{\infty} P(A_k) = 1 = \sum_{k=1}^{\infty} \frac{1}{2^k} = 1$$



$$EX = \sum x P[X=x]$$
$$= \sum_{k=1}^{\infty} 2^k \cdot \frac{1}{2^k} = \sum_{k=1}^{\infty} 1 = \infty$$



(4) 4x △   1/4 3/4 3/4 3/4 $\binom{4}{2} = \frac{4 \cdot 3}{2 \cdot 1} = 6$

X... počet jedniček, které padly

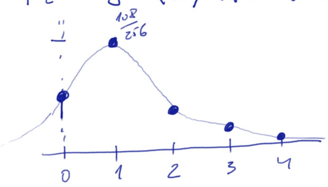
$P[X=0] = 1 \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{81}{256}$

$P[X=1] = 4 \cdot \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{27}{64} = \frac{108}{256}$ $P[X=x] = \binom{4}{x}\left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{4-x}$

$P[X=2] = \binom{4}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{54}{256}$

$P[X=3] = \binom{4}{3} \frac{1}{4} \frac{1}{4} \frac{1}{4} \frac{3}{4} = \frac{12}{256}$

$P[X=4] = \binom{4}{4} \frac{1}{4} \frac{1}{4} \frac{1}{4} \frac{1}{4} = \frac{1}{256}$



$\sum x P[X=x]$

$EX = nq = 4 \cdot \frac{1}{4} = 1$

(10) 200 slepic
5 vajec za 1 den
1,5% pprav. že se vejce rozbije

$n = 1000$ vajec
$q = 1{,}5\% = 0{,}015$

$P[X=x] = \binom{1000}{x} 0{,}015^x \, 0{,}985^{1000-x}$ $P[X=10] = \binom{1000}{10} 0{,}015^{10} \cdot 0{,}985^{990} \doteq 0{,}0482$

$EX = n \cdot q = 1000 \cdot 0{,}015 = 15$

$DX = nq(1-q) = 1000 \cdot 0{,}015 \cdot 0{,}985 \doteq 14{,}8$

Poissonovo r. λ ... stř. h.

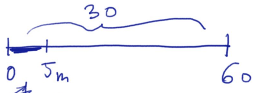
$$P[X=x] = \frac{\lambda^x}{x!} e^{-\lambda}$$

(11) v průměru se každý den rozbije 15 vajec

$P[X=x] = \frac{15^x}{x!} e^{-15}$ $P[X=x] = \frac{15^{10}}{10!} e^{-15} \doteq 0{,}0486$

$EX = \lambda$
$DX = \lambda$

12 30 host/hod



12 $\frac{30}{\cancel{20}}$ host/5 min

$\lambda = \cancel{\frac{3}{2}}\ \frac{30}{12} = \frac{5}{2}$ $\quad \frac{1}{12}$

$$P[X=x] = \frac{(\cancel{3/2}\ 5/2)^x}{x!} e^{-\cancel{3/2}\ 5/2}$$

$$P[X=0] = \frac{(\cancel{3/2}\ 5/2)^0}{0!} e^{-\cancel{3/2}\ 5/2} = e^{-\cancel{3/2}\ 5/2} \doteq 0{,}082$$

$$P[X=2] = \frac{(\cancel{3/2}\ 5/2)^2}{2!} e^{-\cancel{3/2}\ 5/2} = \frac{9}{8} e^{-\cancel{3/2}\ 5/2} \doteq 0{,}257$$