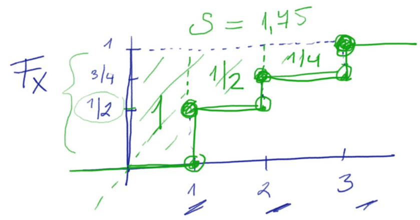
② X .. počet dětí

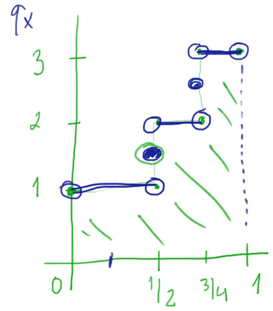
x	1	2	3
$P[X=x]$	1/2	1/4	1/4

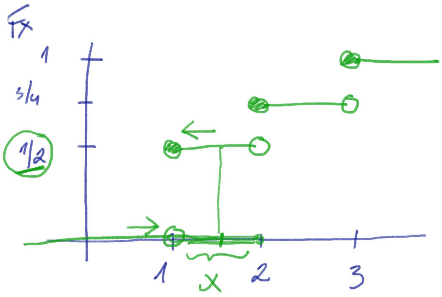


$$EX = \sum_x x P[X=x]$$
$$= 1\cdot\frac{1}{2} + 2\cdot\frac{1}{4} + 3\,\frac{1}{4}$$
$$= \frac{2}{4} + \frac{2}{4} + \frac{3}{4} = \frac{7}{4} = 1{,}75$$

$$DX = E(X^2) - (EX)^2$$
$$E(X^2) = 1^2\frac{1}{2} + 2^2\frac{1}{4} + 3^2\frac{1}{4}$$
$$= \frac{2}{4} + \frac{4}{4} + \frac{9}{4} = \frac{15}{4} = 3{,}75$$
$$\sigma_X^2 = DX = 3{,}75 - (1{,}75)^2 \doteq 0{,}69$$
$$\sigma_X = \sqrt{DX} = \sqrt{0{,}69} \doteq 0{,}83$$

kvantil $q_X(\alpha)$





$$P[X<x] \le 1/2 \le P[X\le x]$$
0 nebo 1/2 — 1/2 nebo 3/4

$$EX = \int_0^1 q_X(\alpha)\,dx$$

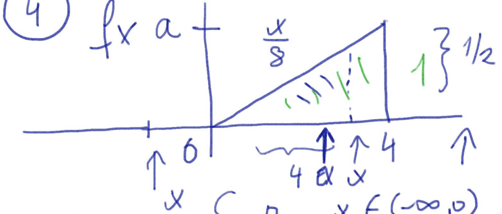
$$q_X(\alpha) = x$$
$$\underbrace{P[X<x]}_{\lim\limits_{x\to u_-} F_X(u)} \le \alpha \le \underbrace{P[X\le x]}_{F_X(x)}$$

dolní kvantil $q_X\left(\frac{1}{4}\right) = 1$

horní kvantil $q_X\left(\frac{3}{4}\right) = 2{,}5$

medián $q_X\left(\frac{1}{2}\right) = 1{,}5$

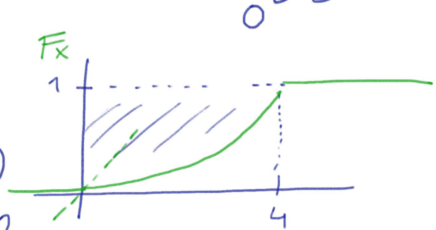
④ f_X a



$$S = \frac{4\cdot a}{2} = 2a = 1$$
$$a = 1/2$$
$$F_X(x) = \int_0^x \frac{u}{8}\,du = \frac{1}{8}\frac{1}{2}\left[u^2\right]_0^x$$

$$f_X(x) = \begin{cases} 0 & x\in(-\infty,0) \\ x/8 & 0,4 \\ 0 & 4,\infty \end{cases}$$

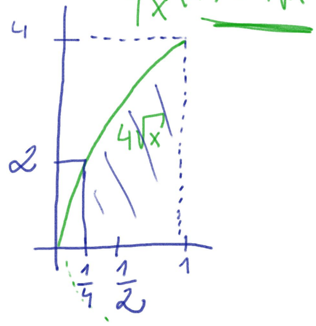
$$F_X(x) = \begin{cases} 0 & x\in(-\infty,0) \\ x^2/16 & \langle 0,4) \\ 1 & \langle 4,\infty) \end{cases}$$

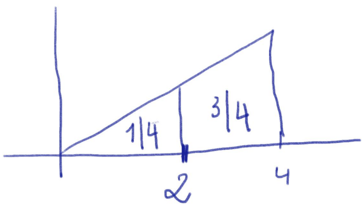


$$EX = \int_{-\infty}^{\infty} x f_X(x)\,dx = \int_0^4 x\cdot\frac{x}{8}\,dx = \frac{1}{8}\frac{1}{3}\left[x^3\right]_0^4 = \frac{\cancel{4}\cdot4\cdot4\cdot2}{\cancel{4}\cdot\cancel{2}\cdot3} = \frac{8}{3}$$

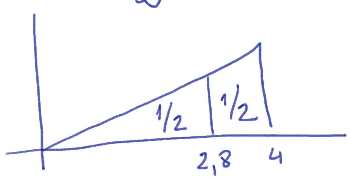
$$y = \frac{x^2}{16}$$
$$16y = x^2$$
$$\oplus\ 4\sqrt{y} = x$$
$$q_X(\alpha) = 4\sqrt{\alpha}$$

$$DX = E(X^2) - (EX)^2$$
$$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_0^4 x^2\frac{x}{8}\,dx = \frac{1}{8}\frac{1}{4}\left[x^4\right]_0^4 = \frac{\cancel{4}\cdot\cancel{4}\cdot\cancel{4}\cdot4\cdot2}{\cancel{4}\cdot\cancel{2}\cdot\cancel{4}} = 8$$
$$DX = 8 - \left(\frac{8}{3}\right)^2 = \frac{72-64}{9} = \frac{8}{9} \qquad \sigma_X = \sqrt{\frac{8}{9}}$$
$$EX = \int_0^4 1\,dx - \int_0^4 \frac{x^2}{16}\,dx = 4 - \frac{1}{16}\frac{1}{3}\left[x^3\right]_0^4 = 4 - \frac{\cancel{4}\cdot4\cdot4}{\cancel{16}\cdot3} = \frac{8}{3}$$





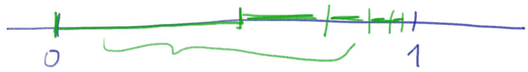
$$q_X\left(\frac{1}{4}\right) = 4\sqrt{\frac{1}{4}} = 2$$
$$q_X\left(\frac{1}{2}\right) = 4\sqrt{\frac{1}{2}} = 2{,}8$$



$$\int_0^1 q_X(\alpha)\,d\alpha = 4\int_0^1 \sqrt{\alpha}\,d\alpha = 4\cdot\frac{2}{3}\left[\alpha^{3/2}\right]_0^1$$
$$= 4\cdot\frac{2}{3} = \frac{8}{3}$$

⑤ A_k ... líc padne v k-tém hodu

$(\underbrace{R, R, \dots, R}_{k-1}, L)$ .... ~~...~~ výhra 2^k CZK



$$\sum_{k=1}^{\infty} \frac{1}{2^k} = 1$$

$|\Omega| = \infty$

k	1	2	3	...	k	
$P(A_k)$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$		$\frac{1}{2^k}$	 1

X ... velikost výhry

x	2	4	8	....	2^k
$P[X=x]$	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$		$\frac{1}{2^k}$

$$EX = \sum P[X=x] \cdot x = \sum_{k=1}^{\infty} \frac{1}{2^k} \cdot 2^k = \sum_{k=1}^{\infty} 1 = \infty$$

⑦ $4x$  X ... počet jedniček

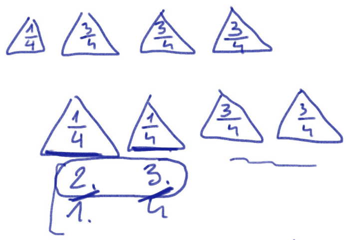
$$P[X=0] = \binom{4}{0} \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{81}{256}$$

$$P[X=1] = \binom{4}{1} \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{108}{256}$$

$$P[X=2] = \binom{4}{2} \frac{1}{4} \cdot \frac{1}{4} \cdot \frac{3}{4} \cdot \frac{3}{4} = \frac{54}{256}$$

$$P[X=3] = \binom{4}{3} \underbrace{\frac{1}{4} \frac{1}{4} \frac{1}{4}}_{x} \frac{3}{4} = \frac{12}{256}$$

$$P[X=4] = \binom{4}{4} \frac{1}{4} \frac{1}{4} \frac{1}{4} \frac{1}{4} = \frac{1}{256}$$



$$\binom{4}{2} = \frac{4 \cdot 3}{2 \cdot 1} = 6$$

$$P[X=x] = \binom{4}{x} \left(\frac{1}{4}\right)^x \left(\frac{3}{4}\right)^{4-x}$$

$$EX = mq = 4 \cdot \frac{1}{4} = 1$$

$$DX = m \cdot q \cdot (1-q) = 4 \cdot \frac{1}{4} \cdot \frac{3}{4} = \frac{3}{4}$$

m kostek

q prav. jedničky

$$P[X=x] = \binom{m}{x} q^x (1-q)^{m-x}$$

(10) 200 slepic
5 vajec za týden
1,5% pravd. rozbití vejce

X ... počet rozbitých vajec během 1 týdne

$n = 1000$
$q = 0{,}015$

$P[X=x] = \binom{1000}{x} 0{,}015^{x}\, 0{,}985^{1000-x}$

$P[X=10] = \binom{1000}{10} 0{,}015^{10}\, 0{,}985^{990} = \underline{0{,}0482}$

$EX = nq = 1000 \cdot 0{,}015 = 15$

$DX = 1000 \cdot 0{,}015 \cdot 0{,}985 \doteq 14{,}775$

Poissonovo r. λ .. stř. h.

$P[X=x] = \frac{\lambda^x}{x!} e^{-\lambda}$ $EX = \lambda = 15$, $DX = \lambda = 15$

(11) $\lambda = 15$

$P[X=x] = \frac{15^x}{x!} e^{-15}$

$P[X=10] = \frac{15^{10}}{10!} e^{-15} \doteq \underline{0{,}0486}$

(12) během 1 h. přijde prům. 30 hostů

během 5 min $\lambda = \frac{5}{60} 30 = \frac{5}{2}$

$P[X=x] = \frac{(5/2)^x}{x!} e^{-5/2}$

$P[X=0] = \frac{(5/2)^0}{0!} e^{-5/2} = e^{-5/2} \doteq \underline{0{,}082}$