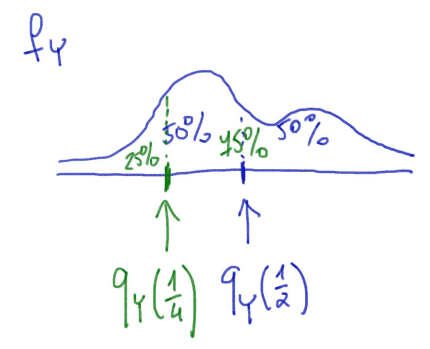
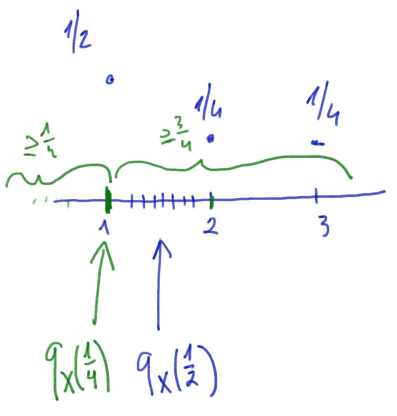
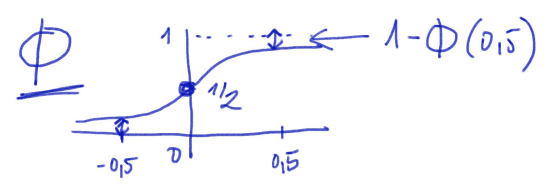
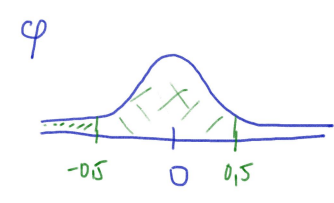


$$\begin{aligned}
 F_X\left(\frac{1}{4}\right) &= 1 \\
 F_X\left(\frac{1}{2}\right) &= 1.5 \\
 F_X\left(\frac{3}{4}\right) &= 2.5
 \end{aligned}$$



① $X \dots N(0, 1)$
 str. h. μ σ^2 σ

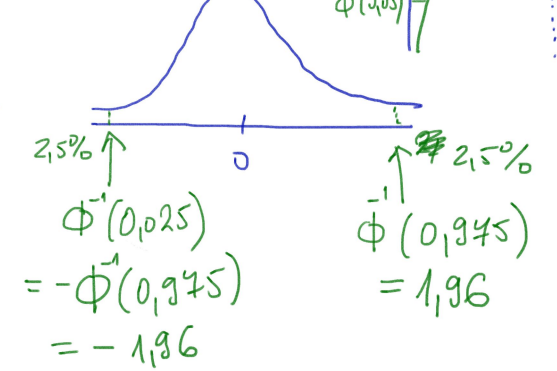
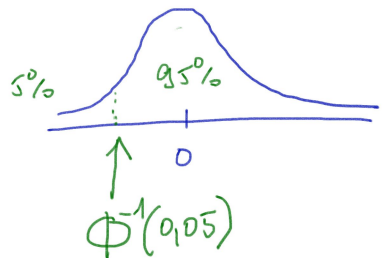
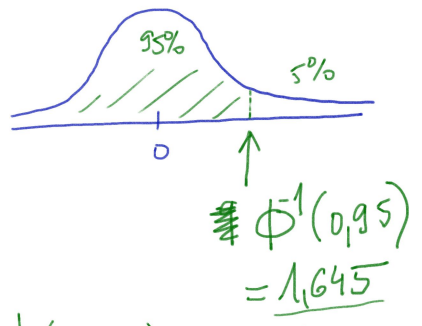
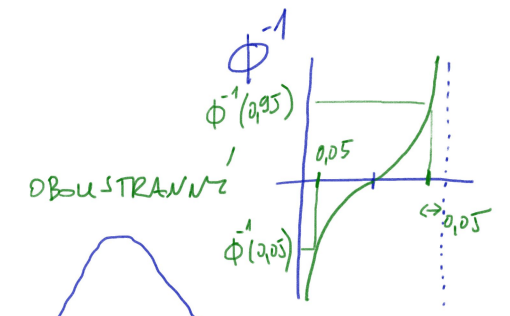
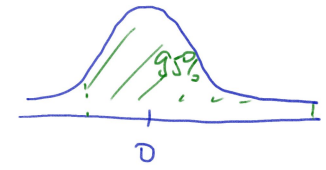
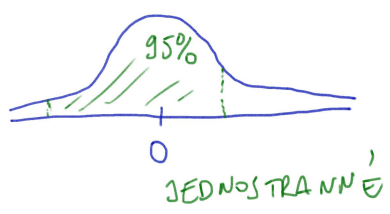


$$\Phi(-x) = 1 - \Phi(x)$$

$$P[X \leq 0.5] = \Phi(0.5) \doteq 0.6915$$

$$P[X \leq -0.5] = \Phi(-0.5) = 1 - \Phi(0.5) \doteq 1 - 0.6915 = 0.3085$$

$$P[-0.5 \leq X \leq 0.5] = P[X \leq 0.5] - P[X \leq -0.5] = 0.6915 - 0.3085 = 0.383$$



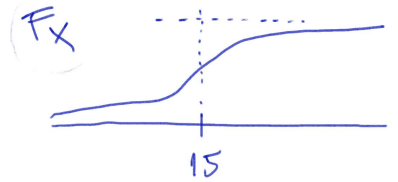
$$\Phi(1.645) = 0.95$$

$$(-\infty; 1.645)$$

$$\begin{aligned}
 &= -\Phi^{-1}(1 - 0.05) \\
 &= -\Phi^{-1}(0.95) \\
 &= -1.645 \\
 &(-1.645; \infty)
 \end{aligned}$$

$$(-1.96; 1.96)$$

③ $X \dots N(\mu=15, \sigma^2=16)$



$$P[X \leq 17] = P[X-15 \leq 17-15] = P\left[\frac{X-15}{\sqrt{16}} \leq \frac{17-15}{\sqrt{16}}\right] =$$

norm X ... N(μ=0, σ²=1)

$$= P\left[\text{norm } X \leq \frac{2}{4}\right] = \Phi(0,5) = \underline{0,6915} \quad P[X \geq x] = 1 - P[X < x]$$

$$P[X \geq 10] = P\left[\frac{X-15}{4} \geq \frac{10-15}{4}\right] = P\left[\text{norm } X \geq \frac{-5}{4}\right]$$

$$= 1 - P\left[\text{norm } X < \frac{-5}{4}\right] = 1 - \lim_{x \rightarrow -\frac{5}{4}^-} \Phi(x) = 1 - \Phi\left(-\frac{5}{4}\right)$$

(protože Φ je spojitá)

$$= 1 - \Phi(-1,25) = 1 - (1 - \Phi(1,25))$$

$$= \Phi(1,25) = \frac{0,8925 + 0,18962}{2} = \underline{0,8944}$$

$$P[10 \leq X \leq 17] = P[X \leq 17] - P[X < 10] = 0,6915 - 0,1056 = \underline{\underline{0,5859}}$$

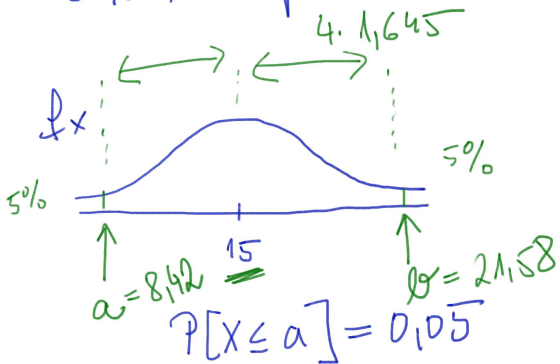


~~$\lim_{x \rightarrow 10^-} \Phi(x) = \Phi(10)$~~

$$P[X < 10] = P\left[\frac{X-15}{4} < \frac{10-15}{4}\right] = \Phi\left(-\frac{5}{4}\right) =$$

$$= \Phi(-1,25) = 1 - \Phi(1,25) = 1 - 0,8944 = \underline{\underline{0,1056}}$$

90% int. spolehlivosti



$$P\left[\frac{X-15}{4} \leq \frac{a-15}{4}\right] = 0,05$$

$$\Phi\left(\frac{a-15}{4}\right) = 0,05$$

$$\frac{a-15}{4} = \Phi^{-1}(0,05) \rightarrow a = 4\Phi^{-1}(0,05) + 15 = -4\Phi^{-1}(0,95) + 15$$

$$= -4 \cdot 1,645 + 15 = 8,42$$

$$P[X \leq b] = 0,95$$

$$P\left[\frac{X-15}{4} \leq \frac{b-15}{4}\right] = 0,95$$

$$\Phi\left(\frac{b-15}{4}\right) = 0,95$$

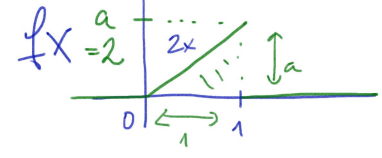
$$b = 4\Phi^{-1}(0,95) + 15$$

$$= 4 \cdot 1,645 + 15 = 21,58$$

$$= \underline{15} + \underline{4 \cdot 1,645}$$

$$\underline{15} - \underline{4 \cdot 1,645}$$

6

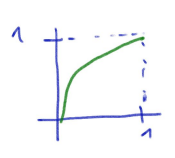
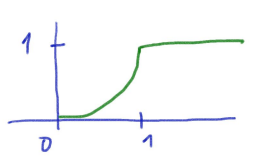


$$\frac{1 \cdot a}{2} = 1 \Rightarrow a = 2$$

$$f_X(x) = \begin{cases} 0 & (-\infty, 0) \\ 2x & x \in (0, 1) \\ 0 & (1, \infty) \end{cases}$$

$$F_X = \begin{cases} 0 & (-\infty, 0) \\ x^2 & (0, 1) \\ 1 & (1, \infty) \end{cases}$$

$$q_X = \sqrt{x}$$

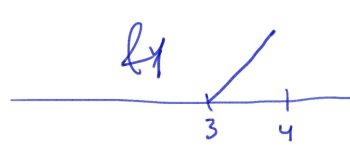
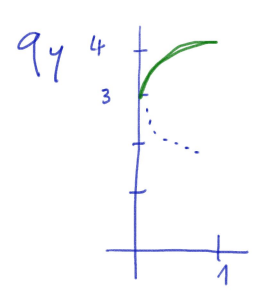
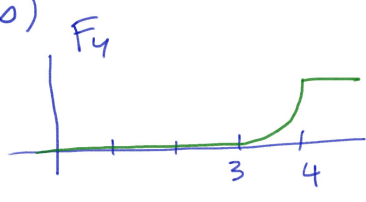


2. $Y = X + 3$

$$F_Y(x) = P[Y \leq x] = P[X + 3 \leq x] = P[X \leq x - 3] = F_X(x - 3)$$

$$F_Y(x) = \begin{cases} 0 & (-\infty, 3) \\ (x-3)^2 & (3, 4) \\ 1 & (4, \infty) \end{cases}$$

$q_{Y^*} \quad y = (x-3)^2$
 $\pm \sqrt{y} = x - 3$
 $x = 3 \pm \sqrt{y}$
 $q_Y(x) = 3 + \sqrt{x}$

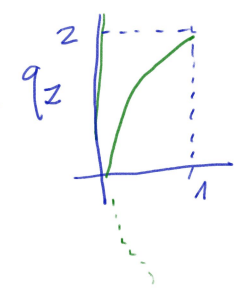
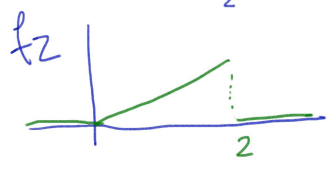
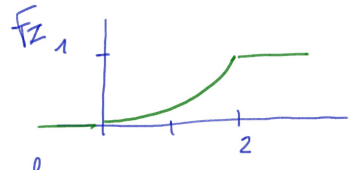


3. $Z = 2 \cdot X$

$$F_Z(x) = P[Z \leq x] = P[2 \cdot X \leq x] = P[X \leq \frac{x}{2}] = F_X(\frac{x}{2})$$

$$F_Z(x) = \begin{cases} 0 & (-\infty, 0) \\ (\frac{x}{2})^2 & (0, 2) \\ 1 & (2, \infty) \end{cases}$$

$$q_Z(x) = \sqrt{2x}$$



4. $U = -X$

$$F_U(x) = P[U \leq x] = P[-X \leq x] = P[X \geq -x] = 1 - P[X < -x]$$

$$= 1 - \lim_{u \rightarrow -x^-} F_X(u) = 1 - F_X(-x)$$

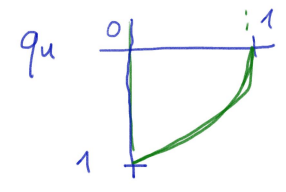
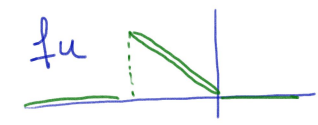
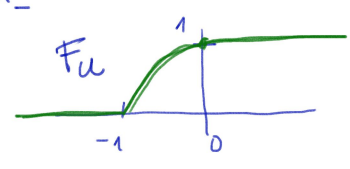
$$q_X(x) = \sqrt{x}$$

$$F_U(x) = \begin{cases} 0 & (-\infty, -1) \\ 1 - (-x)^2 & (-1, 0) \\ 1 & (0, \infty) \end{cases}$$

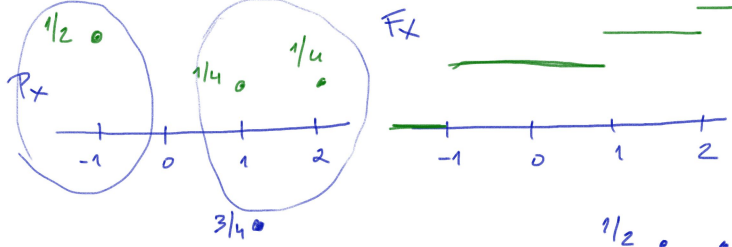
$$q_U(x) = \sqrt{1-x}$$

$$y = 1 - x^2$$

$$x = \sqrt{1-y}$$



8) $X \sim \begin{matrix} x & -1 & 1 & 2 \\ P[X=x] & 1/2 & 1/4 & 1/4 \end{matrix}$

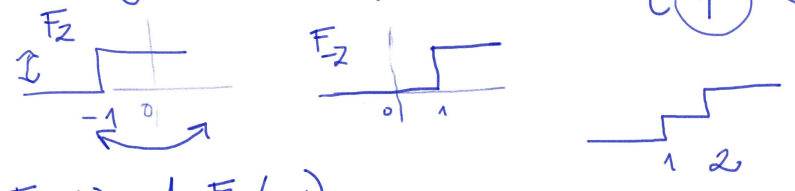


5. $V = |X|$

$X \sim \text{Mix}(Z, K)$
 $(\frac{1}{2}, \frac{1}{2})$

$F_Z(x) = \begin{cases} 0 & (-\infty, -1) \\ 1 & (-1, \infty) \end{cases}$

$F_K(x) = \begin{cases} 0 & (-\infty, 1) \\ 1/2 & [1, 2) \\ 1 & [2, \infty) \end{cases}$

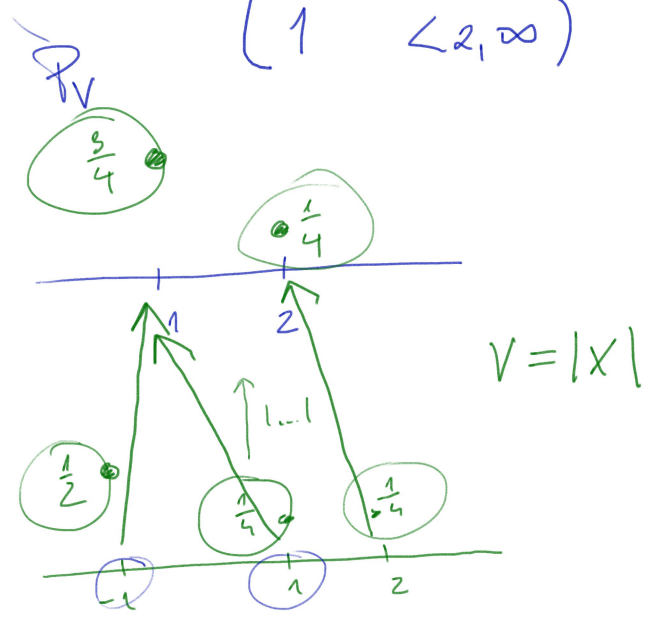
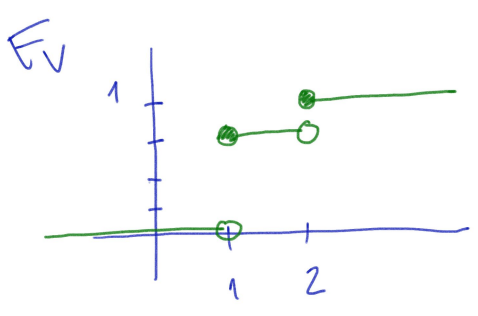


$F_{-Z}(x) = 1 - F_Z(-x)$

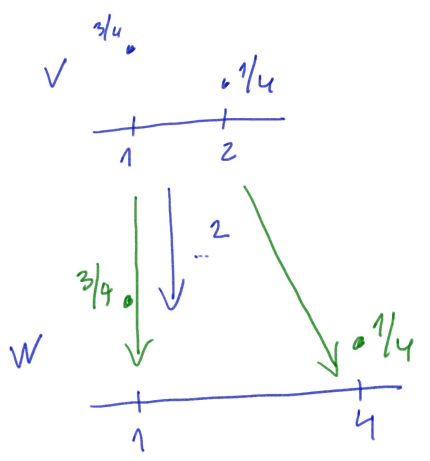
$F_{-Z}(x) = \begin{cases} 0 & (-\infty, 1) \\ 1 & [1, \infty) \end{cases}$

$V = |X| \sim \text{Mix}(-Z, K)$
 $(\frac{1}{2}, \frac{1}{2})$

$F_V(x) = \frac{1}{2} F_{-Z}(x) + \frac{1}{2} F_K(x) = \begin{cases} 0 & (-\infty, 1) \\ 3/4 & [1, 2) \\ 1 & [2, \infty) \end{cases}$



$W = X^2 = |X|^2 = V^2$



$P[W \leq x] = P[V^2 \leq x] = P[V \leq \sqrt{x}] = F_V(\sqrt{x})$