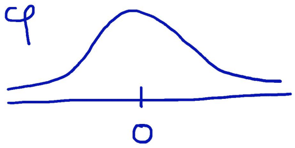
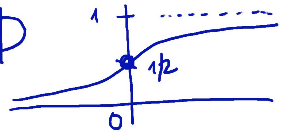
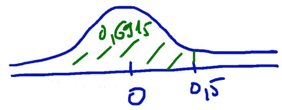
1) $X = N(\mu = 0, \sigma^2 = 1)$ φ



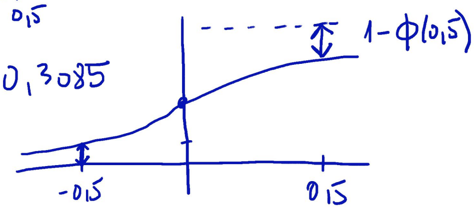
Φ



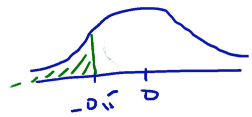
$P[X \leq 0{,}5] = \Phi(0{,}5) \doteq 0{,}6915$



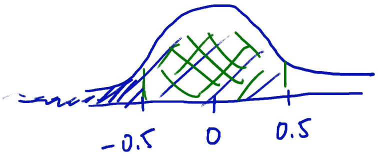
$P[X \leq -0{,}5] = \Phi(-0{,}5) = 1 - \Phi(0{,}5) \doteq 1 - 0{,}6915 = 0{,}3085$



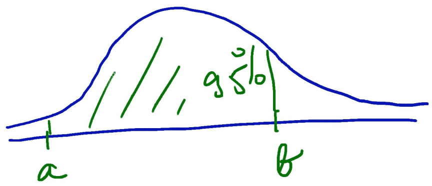
$\Phi(-x) = 1 - \Phi(x)$



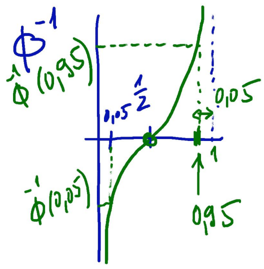
$P[-0{,}5 \leq X \leq 0{,}5] = \Phi(0{,}5) - \Phi(-0{,}5) \doteq 0{,}6915 - 0{,}3085 = 0{,}383$



95%



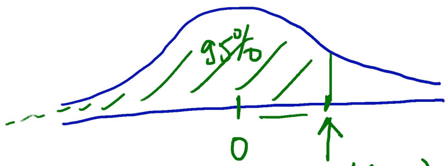
JEDNOSTRANNÉ



$\Phi^{-1}(0{,}05) = -\Phi^{-1}(0{,}95)$

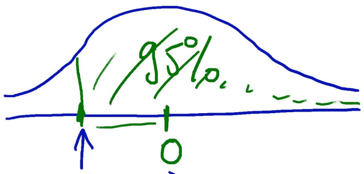
$\Phi^{-1}(x) = -\Phi^{-1}(1-x)$

OBOUSTRANNÝ

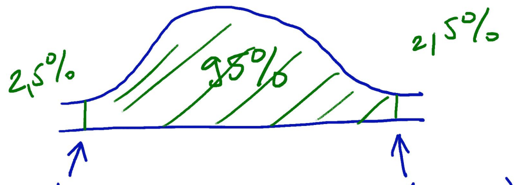


$\Phi^{-1}(0{,}95) = 1{,}645$

$\Phi(1{,}645) = 0{,}95$



$\Phi^{-1}(0{,}05) = -\Phi^{-1}(0{,}95) = -1{,}645$

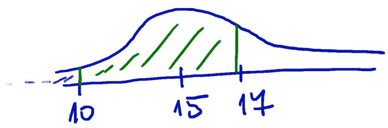


$\Phi^{-1}(0{,}025) = -\Phi^{-1}(0{,}975) = -1{,}96$

$\Phi^{-1}(0{,}975) = 1{,}96$

$(-\infty; 1{,}645\rangle$ $\langle -1{,}645; \infty)$ $\langle -1{,}96; 1{,}96\rangle$

3) $X \sim N(\mu = 15, \sigma^2 = 16)$



$P[10 \leq X \leq 17] = P[X \leq 17] - P[X < 10] = 0{,}6915 - \overset{0{,}10565}{\cancel{0{,}89435}} = 0{,}58585$

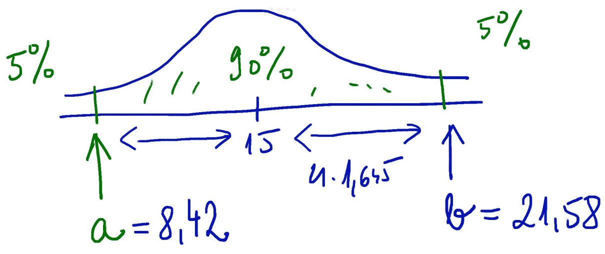
$P[X \leq 17] = P[X - 15 \leq 17 - 15] = P\Big[\underbrace{\frac{X-15}{\sqrt{16}}}_{\text{norm } X} \leq \frac{17-15}{\sqrt{16}}\Big] = \Phi\left(\frac{2}{4}\right) = 0{,}6915$

$P[X < 10] = P\left[\frac{X-15}{4} < \frac{10-15}{4}\right] = P\left[\text{norm } X < -\frac{5}{4}\right] = \lim_{x \to -\frac{5}{4}^-} \Phi(x)$

$= \Phi(-5/4) = 1 - \Phi\left(\frac{5}{4}\right) \doteq 1 - \frac{0{,}8925 + 0{,}8962}{2} = 1 - 0{,}89435$

$= 0{,}10565$

90%

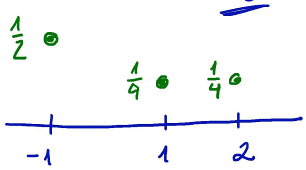


$\langle 8{,}42 ; 21{,}58 \rangle$

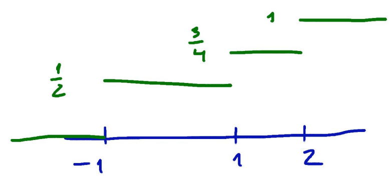
$$P[X \le a] = 0{,}05$$
$$P\left[\frac{X-15}{4} \le \frac{a-15}{4}\right] = 0{,}05$$
$$\Phi\left(\frac{a-15}{4}\right) = 0{,}05$$
$$\frac{a-15}{4} = \Phi^{-1}(0{,}05)$$
$$a = 4\Phi^{-1}(0{,}05) + 15 = -4\Phi^{-1}(0{,}95) + 15$$
$$= -4 \cdot 1{,}645 + 15 = \underline{\underline{8{,}42}}$$
$$\underline{15} - 4 \cdot 1{,}645$$

$$P[X \le b] = 0{,}95$$
$$P\left[\frac{X-15}{4} \le \frac{b-15}{4}\right] = 0{,}95$$
$$\Phi\left(\frac{b-15}{4}\right) = 0{,}95$$
$$\frac{b-15}{4} = \Phi^{-1}(0{,}95)$$
$$b = 4\Phi^{-1}(0{,}95) + 15$$
$$= 4 \cdot 1{,}645 + 15$$
$$= \underline{\underline{21{,}58}}$$
$$\underline{15} + 4 \cdot 1{,}645$$

8) X .. P_X



F_X



$$EX = -1 \cdot \frac{1}{2} + 1 \cdot \frac{1}{4} + 2 \cdot \frac{1}{4} = \frac{1}{4}$$

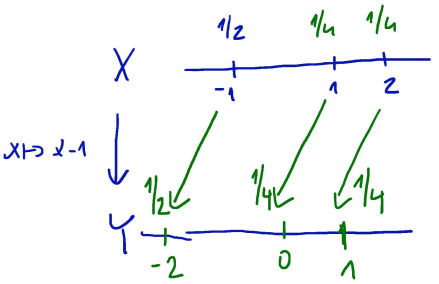
$$F_X(x) = \begin{cases} 0 & (-\infty, -1) \\ 1/2 & \langle -1, 1) \\ 3/4 & \langle 1, 2) \\ 1 & \langle 2, \infty) \end{cases}$$

$$DX = E(X^2) - (EX)^2 = \frac{4}{4} - \frac{1}{16} = \frac{27}{16}$$

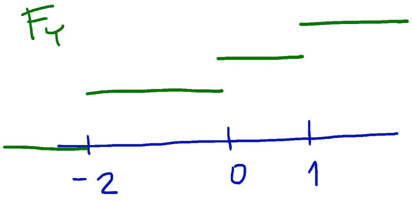
$$E(X^2) = (-1)^2 \frac{1}{2} + 1^2 \cdot \frac{1}{4} + 2^2 \frac{1}{4} = \frac{1}{2} + \frac{1}{4} + 1 = \frac{4}{4}$$

2. $Y = X - 1$

$$F_Y(x) = P[Y \le x] = P[X-1 \le x] = P[X \le x+1] = F_X(x+1)$$

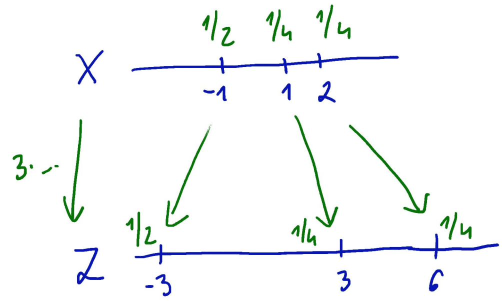


F_Y

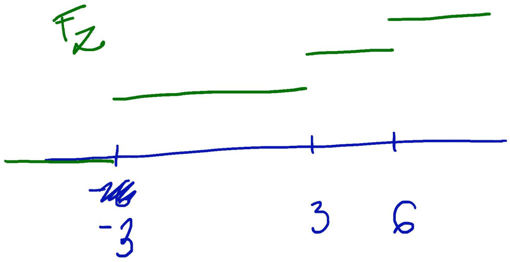


3. $Z = 3 \cdot X$

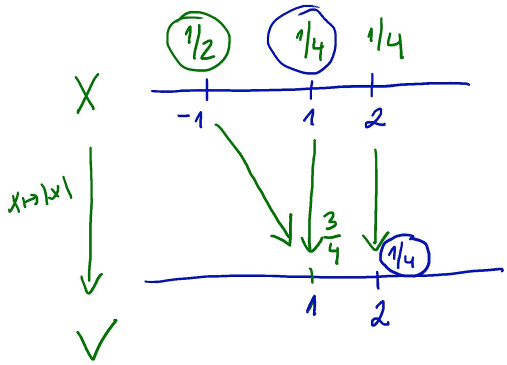
$$F_Z(x) = P[Z \le x] = P[3 \cdot X \le x] = P\left[X \le \frac{x}{3}\right] = F_X\left(\frac{x}{3}\right)$$

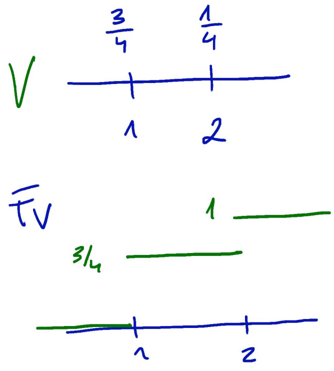


F_Z

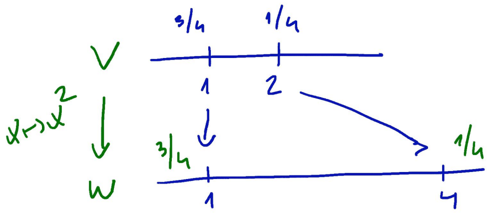


5. $V = |X|$

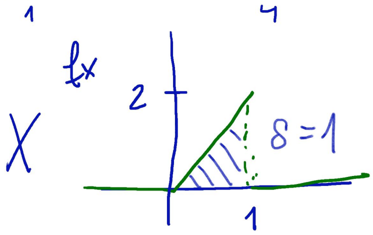




6. $W = X^2 = |X|^2 = V^2$

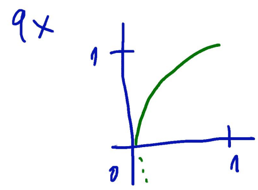


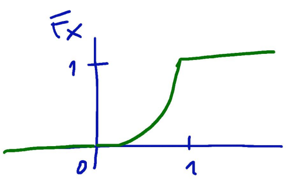
6



$$f_X(x) = \begin{cases} 0 & (-\infty, 0) \\ 2x & \langle 0, 1) \\ 0 & \langle 1, \infty) \end{cases} \qquad F_X(x) = \begin{cases} 0 & (-\infty, 0) \\ x^2 & \langle 0, 1) \\ 1 & \langle 1, \infty) \end{cases}$$

$g_X(x) = \sqrt{x}$





2. $Y = X + 3$

$F_Y(x) = P[Y \le x] = P[X+3 \le x] = P[X \le x-3] = F_X(x-3)$

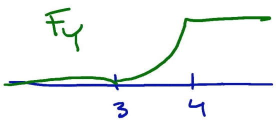
$g_Y(x) = \cancel{\sqrt{x+3}}\ \sqrt{x} + 3$

$$F_Y(x) = \begin{cases} 0 & (-\infty, 3) \\ (x-3)^2 & \langle 3, 4) \\ 1 & \langle 4, \infty) \end{cases}$$

$y = (x-3)^2$

$\pm\sqrt{y} = x - 3$

$x = \cancel{3} \pm \sqrt{y} + 3$



3. $Z = 2 \cdot X$ $\qquad F_Z(x) = P[Z \le x] = P[2 \cdot X \le x] = P[X \le \frac{x}{2}] = F_X(\frac{x}{2})$

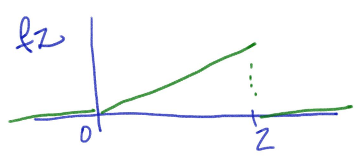
$g_Z(x) = 2\sqrt{x}$

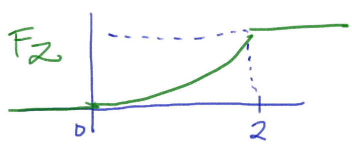
$$f_Z(x) = \begin{cases} 0 & \dots \\ \frac{x}{2} & \dots \langle 0, 2) \\ 0 & \dots \end{cases} \qquad F_Z(x) = \begin{cases} 0 & (-\infty, 0) \\ (\frac{x}{2})^2 & \langle 0, 2 \rangle \\ 1 & \langle 2, \infty) \end{cases}$$

$y = \left(\frac{x}{2}\right)^2$

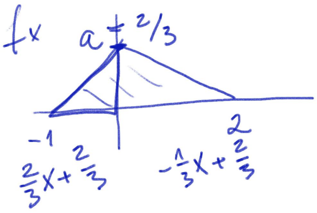
$\pm\sqrt{y} = \frac{x}{2}$

$x = \pm 2\sqrt{y}$

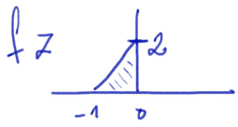


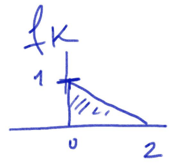


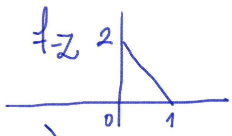
7. f_X $a = 2/3$

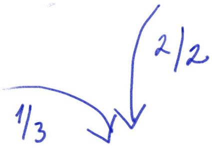


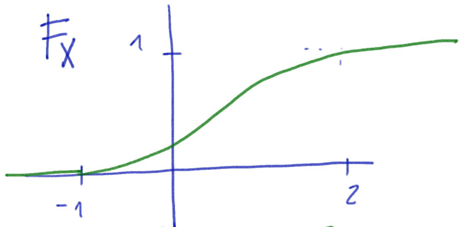
$$1\cdot\frac{a}{2}+\frac{2a}{2}=1$$
$$3a=2$$
$$a=2/3$$







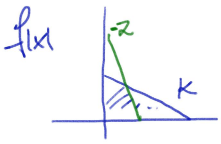




0 ; $\frac{1}{3}(x+1)^2$; $-\frac{1}{6}(x-2)^2+1$; 1

$$V=|X|$$

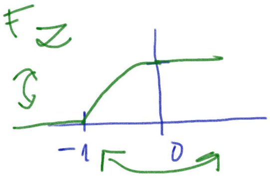
$$X=\text{Mix}\,(Z,K)$$
$$\left(\tfrac{1}{3},\tfrac{2}{3}\right)$$



$$V=|X|=\text{Mix}\,(-Z,K)$$
$$\left(\tfrac{1}{3},\tfrac{2}{3}\right)$$

$$f_Z(x)=\begin{cases}0 & -\infty,-1\\ 2x+2 & \langle -1,0)\\ 0 & 0,\infty\end{cases}$$

$$F_Z(x)=\begin{cases}0 & (-\infty,-1)\\ x^2+2x+1 & \langle -1,0)\\ 1 & \langle 0,\infty)\end{cases}$$



$$F_{-Z}(x)=P[-Z\le x]=P[Z\ge -x]=1-P[Z<-x]=1-\lim_{u\to -x^-}F_Z(u)=1-F_Z(-x)$$

$$F_{-Z}(x)=\begin{cases}0 & (-\infty,0)\\ -x^2+2x & \langle 0,1)\\ 1 & \langle 1,\infty)\end{cases}$$

$$f_K=\begin{cases}0 & (-\infty,0)\\ -\frac{1}{2}x+1 & \langle 0,2)\\ 0 & \langle 2,\infty)\end{cases}\qquad F_K(x)=\begin{cases}0 & (-\infty,0)\\ -\frac{x^2}{4}+x & \langle 0,2)\\ 1 & \langle 2,\infty)\end{cases}$$

$$F_V(x)=\frac{1}{3}F_{-Z}(x)+\frac{2}{3}F_K(x)=\begin{cases}0 & (-\infty,0)\\ \frac{1}{3}(-x^2+2x)+\frac{2}{3}\left(-\frac{x^2}{4}+x\right) & \langle 0,1)\\ \frac{1}{3}\cdot 1+\frac{2}{3}\left(-\frac{x^2}{4}+x\right) & \langle 1,2)\\ 1 & \langle 2,\infty)\end{cases}$$

$F_V(x)=$