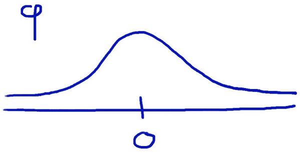
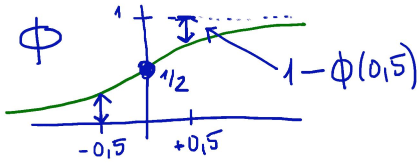
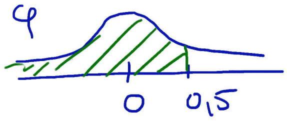
1. $X = N(0,1)$

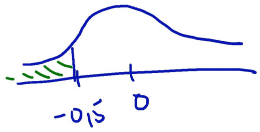




$P[X \leq 0{,}5] = \Phi(0{,}5) \doteq 0{,}6915$

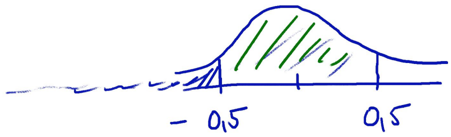


$P[X \leq -0{,}5] = \Phi(-0{,}5) = 1-\Phi(0{,}5) \doteq 1-0{,}6915 = 0{,}3085$

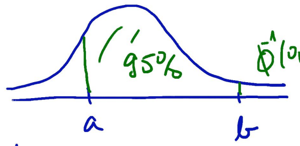


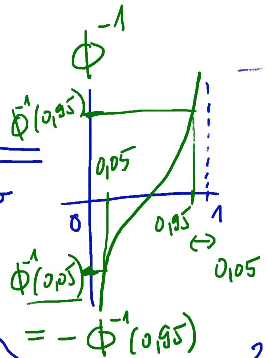
$\Phi(-x) = 1-\Phi(x)$

$P[-0{,}5 \leq X \leq 0{,}5] = \underline{\Phi(0{,}5)} - \underline{\Phi(-0{,}5)} = 0{,}6915 - 0{,}3085 = 0{,}383$



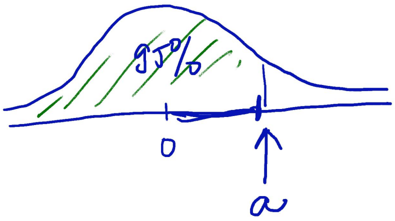
95% interval spolehlivosti

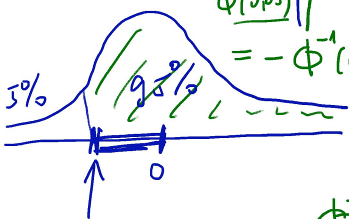


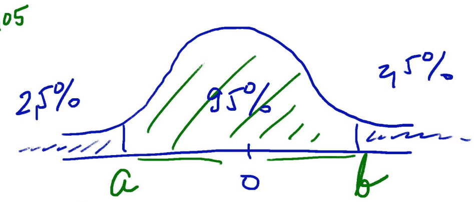


JEDNOSTRANNÉ

OBOUSTRANNÍ







$\Phi(a) = 0{,}95$

$a = \underline{\Phi^{-1}}(0{,}95)$

$= 1{,}645$

$\Phi^{-1}(0{,}05)$

$= -\Phi^{-1}(0{,}95)$

$= -1{,}645$

$\Phi^{-1}(x) = -\Phi^{-1}(1-x)$

$b = \Phi^{-1}(0{,}975)$

$= 1{,}96$

$\Phi(a) = 0{,}025$

$a = \Phi^{-1}(0{,}025)$

$= -\Phi^{-1}(0{,}975)$

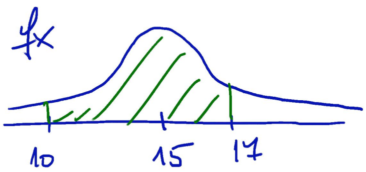
$= -1{,}96$

$\langle -1{,}96 ; 1{,}96 \rangle$

$\langle -1{,}645 ; \infty)$

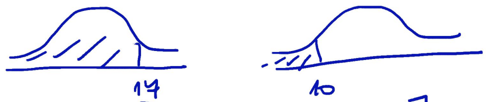
$(-\infty ; 1{,}645\rangle$

(3) $X \,..\, N(\underset{\mu}{15}, \underset{\sigma^2}{16})$



$$P[10 \le X \le 17]$$

$$= P[X \le 17] - P[X \le 10] = 0{,}6915 - 0{,}10565 = 0{,}58585$$



$$P[X \le 17] = P\left[\frac{X-15}{\sqrt{16}} \le \frac{17-15}{\sqrt{16}}\right] = P\left[\text{norm}X \le \frac{2}{4}\right] = \Phi(1/2) = 0{,}6915$$

normX

$$P[X \le 10] = P\left[\frac{X-15}{4} \le \frac{10-15}{4}\right] = \Phi\left(-\frac{5}{4}\right) = \Phi(-1{,}25) = 1 - \Phi(1{,}25)$$

$$= 1 - \frac{0{,}8925 + 0{,}8962}{2} = 1 - 0{,}89435 = 0{,}10565$$

90% oboustranný int.

$$P[X \le a] = 5\%$$

$$P\left[\frac{X-15}{4} \le \frac{a-15}{4}\right] = 0{,}05$$

$$\Phi\left(\frac{a-15}{4}\right) = 0{,}05$$

$$\frac{a-15}{4} = \Phi^{-1}(0{,}05)$$

$$a = 4\,\Phi^{-1}(0{,}05) + 15$$

$$= -4\cdot\Phi^{-1}(0{,}95) + 15 = -4\cdot 1{,}645 + 15 = 8{,}42$$

$$P[X \le b] = 0{,}95$$

$$P\left[\frac{X-15}{4} \le \frac{b-15}{4}\right] = 0{,}95$$

$$\Phi\left(\frac{b-15}{4}\right) = 0{,}95$$

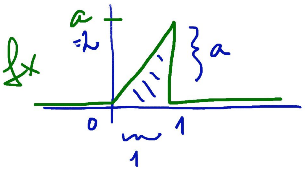
$$b = 4\cdot\Phi^{-1}(0{,}95) + 15$$

$$= 4\cdot 1{,}645 + 15$$

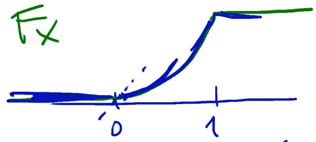
$$= 21{,}58$$

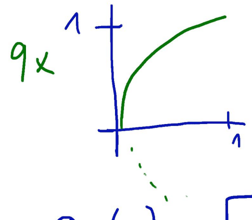
$$\langle 8{,}42 \,;\, 21{,}58 \rangle$$

⑥ X



$$\frac{1 \cdot a}{2} = 1$$
$$a = 2$$





$$f_X(x) = \begin{cases} 0 & (-\infty, 0) \\ 2x & \langle 0, 1) \\ 0 & \langle 1, \infty) \end{cases}$$

$$\int_{-\infty}^{x}$$

$$x^2 + C$$

$$F_X(x) = \begin{cases} 0 & (-\infty, 0) \\ x^2 + \cancel{C} & \langle 0, 1) \\ 1 & \langle 1, \infty) \end{cases}$$

$$q_X(x) = \sqrt{x}$$

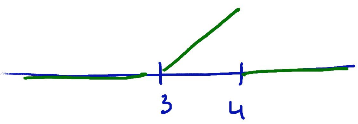
$$y = x^2$$
$$\oplus\sqrt{y} = x$$

$Y = X + 3$ $\quad F_Y(x) = P[Y \leq x] = P[X + 3 \leq x] = P[X \leq x - 3] = F_X(x-3)$

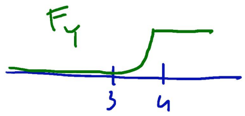
$$f_Y(x) = \begin{cases} 0 \\ 2(x-3) \\ 0 \end{cases}$$

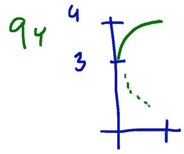
$$F_Y(x) = \begin{cases} 0 & (-\infty, 3) \\ (x-3)^2 & \langle 3, 4) \\ 1 & \langle 4, \infty) \end{cases}$$

$$q_Y(x) = \sqrt{x} + 3$$



$$y = (x-3)^2$$
$$3 \oplus \sqrt{y} = x$$

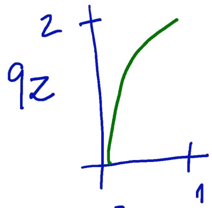


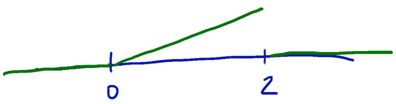


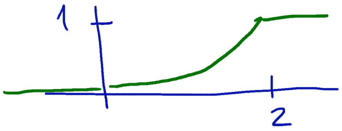
$Z = 2 \cdot X$; $\quad F_Z(x) = P[Z \leq x] = P[2 \cdot X \leq x] = P[X \leq \frac{x}{2}] = F_X(\frac{x}{2})$

$$f_Z(x) = \begin{cases} 0 \\ \frac{x}{2} \\ 0 \end{cases}$$

$$F_Z(x) = \begin{cases} 0 & (-\infty, 0) \\ \left(\frac{x}{2}\right)^2 & \langle 0, 2) \\ 1 & \langle 2, \infty) \end{cases}$$







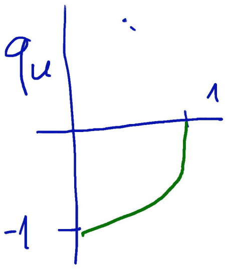
$$y = \left(\frac{x}{2}\right)^2$$
$$\oplus 2\sqrt{y} = x$$
$$q_Z(x) = 2\sqrt{x}$$

$U = -X$; $\quad F_U(x) = P[U \leq x] = P[-X \leq x] = P[X \geq -x]$

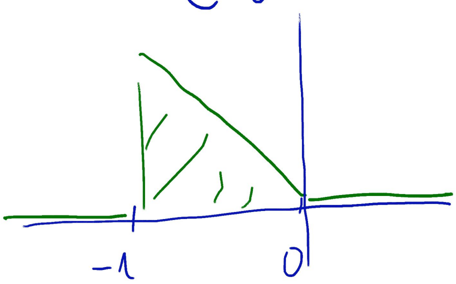
$$= 1 - P[X < -x] = 1 - \lim_{u \to -x_-} F_X(u) = 1 - F_X(-x)$$

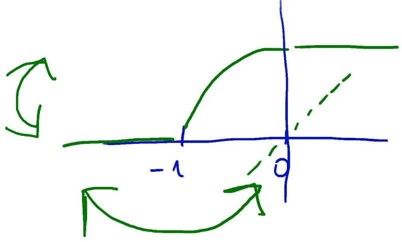
$$f_U(x) = \begin{cases} 0 \\ -2x \\ 0 \end{cases}$$

$$F_U(x) = \begin{cases} 1 - 0 & \langle 0, \infty) \\ 1 - (-x)^2 & \langle -1, 0) \\ 1 - 1 & (-\infty, -1) \end{cases}$$



$$F_U(x) = \begin{cases} 0 & (-\infty, -1) \\ 1 - x^2 & \langle -1, 0) \\ 1 & \langle 0, \infty) \end{cases}$$



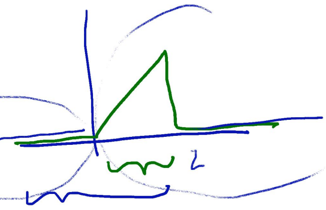


$$y = 1 - x^2$$
$$x = \ominus\sqrt{1 - y}$$

$$q_U(x) = -\sqrt{1 - x} = -q_X(1 - x)$$

$W = X^2$ $\quad F_W(x) = P[W \le x] = P[X^2 \le x] = P[X \le \sqrt{x}] = F_X(\sqrt{x})$

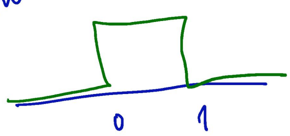
f_X



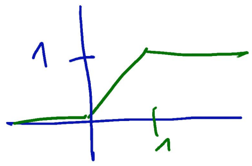
$$F_X(x) = \begin{cases} 0 & (-\infty, 0) \\ x^2 & \langle 0, 1) \\ 1 & \langle 1, \infty) \end{cases}$$

$$F_W(x) = \begin{cases} 0 & (-\infty, 0^2) \\ \sqrt{x}^2 = x & \langle 0^2, 1^2) \\ 1 & \langle 1^2, \infty) \end{cases}$$

f_W

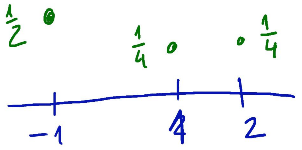


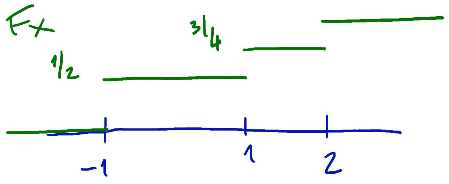
F_W



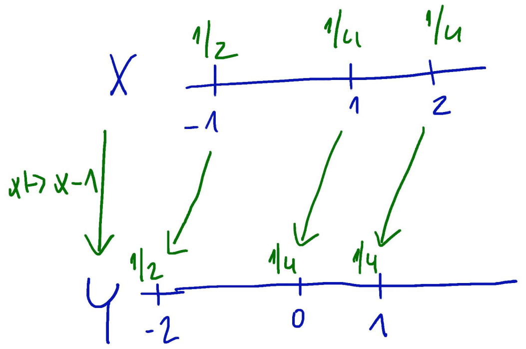
8

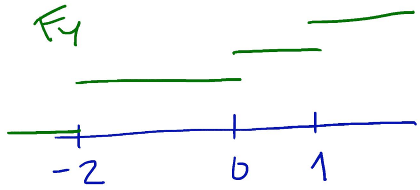
X



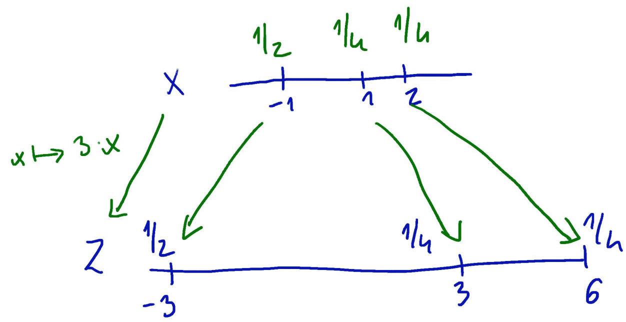


$$\boxed{Y = X-1} \qquad F_Y(x) = P[Y \leq x] = P[X-1 \leq x] = P[X \leq x+1] = F_X(x+1)$$

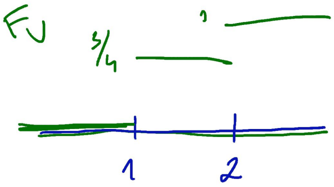


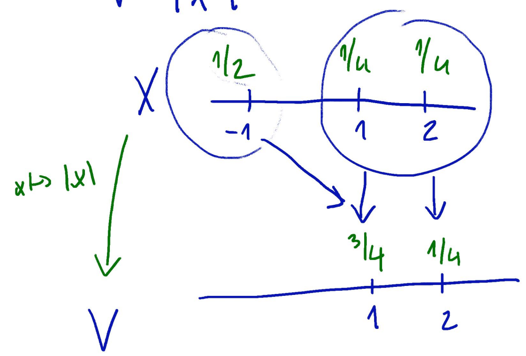


$$Z = 3 \cdot X \qquad F_Z(x) = F_X\left(\frac{x}{3}\right)$$

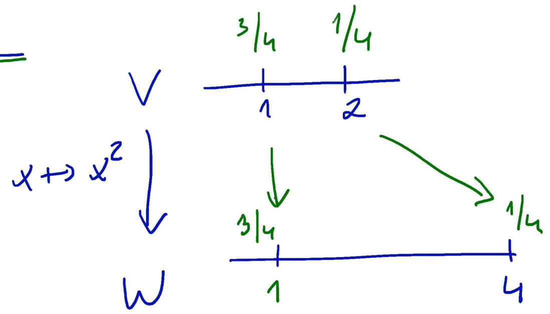


$V = |X|$

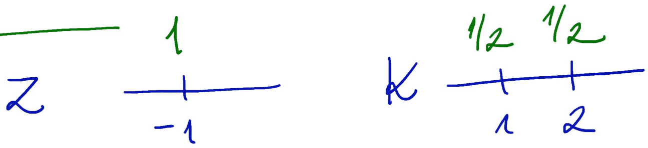




$$W = X^2 = |X|^2 = V^2$$



$V = |X|$ $\qquad X = \underset{(\frac{1}{2},\frac{1}{2})}{\mathrm{Mix}}(Z, K)$



$$V = |X| = \underset{(\frac{1}{2}\,\frac{1}{2})}{\mathrm{Mix}}(-Z, K)$$

$$F_V(x) = \frac{1}{2}F_{-Z}(x) + \frac{1}{2}F_K(x)$$