② X, Y $\quad DX = 9 \quad EX = -1 \quad cov(X,Y) = -3$

$\quad DY = 4 \quad EY = 5$

1. kovarianční matice (X,Y)

$$\begin{bmatrix} \overbrace{cov(X,X)}^{=DX} & cov(X,Y) \\ \underbrace{cov(Y,X)}_{=cov(X,Y)} & \underbrace{cov(Y,Y)}_{=DY} \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ -3 & 4 \end{bmatrix}$$

$$\rho(X,Y) = \frac{cov(X,Y)}{\sqrt{DX} \cdot \sqrt{DY}} = \frac{-3}{3 \cdot 2} = -\frac{1}{2}$$

$$\boxed{D(X \pm Y) = DX + DY + 2cov(X,Y)}$$

korelační matice $\begin{bmatrix} 1 & -1/2 \\ -1/2 & 1 \end{bmatrix}$

2. $U = 2X + 3Y - 4$, $\quad V = 4X - 5Y + 1$ $\qquad cov(U,V)$

$EU = E(2X+3Y-4) = 2EX + 3EY - E4 = 2(-1) + 3 \cdot 5 - 4 = -2 + 15 - 4 = 9$

$EV = 4EX - 5EY + 1 = -4 - 25 + 1 = -28$

$DU = D(2X+3Y-4) = \cancel{D(2x) + D(3Y) + D4 = 4 \cdot DX + 9DY + 0 = 4 \cdot 9 + 9 \cdot 4 =}$

$= D(2X) + D(3Y) + 2 \cdot cov(2X, 3Y)$

$= 4DX + 9DY + 2 \cdot 2 \cdot 3\, cov(X,Y)$

$= 4 \cdot 9 + 9 \cdot 4 + 2 \cdot 2 \cdot 3 \cdot (-3) =$

$= 36 + 36 - 36 = 36$

$DV = D(4X - 5Y + 1) = D(4X) + D(5Y) + 2\, cov(4X, -5Y)$

$= 16 \cdot 9 + 25 \cdot 4 + 2 \cdot 4(-5)(-3) = \cancel{364}\ 364$

$cov(U,V) = cov(2X + 3Y - 4, 4X - 5Y + 1)$

$= cov(2X + 3Y, 4X - 5Y)$

$= cov(2X, 4X - 5Y) + cov(3Y, 4X - 5Y)$

$= cov(2X, 4X) + cov(2X, -5Y) + cov(3Y, 4X) + cov(3Y, -5Y)$

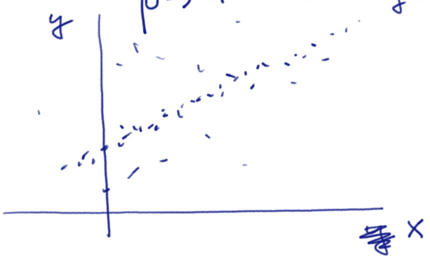
$= 8 \underbrace{cov(X,X)}_{DX} - 10\, cov(X,Y) + 12\, cov(Y,X) - 15 \underbrace{cov(Y,Y)}_{DY}$

$= 8 \cdot 9 - 10(-3) + 12(-3) - 15 \cdot 4$

$= 6$

$$\rho(U,V) = \frac{\cancel{6}}{\cancel{\sqrt{36}} \sqrt{364}} = \frac{1}{\sqrt{364}} \doteq 0{,}05$$

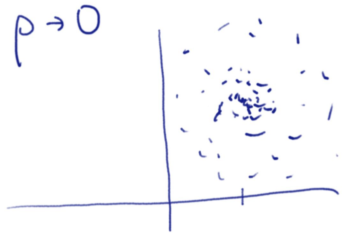
$\rho \to 1$ "$y \approx kx + q$"



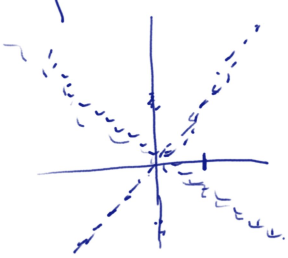
$\rho \to 0$

$\rho \to -1$

$\rho \to 0$



$\rho \to 0$



④ (X,Y)

$$P_{(X,Y)}(x,y) = P[X=x, Y=y]$$

$x \backslash y$	$y=0$	$y=1$	$P[X=x]$
2	$1/6$	$1/12$	$3/12$
4	$1/2$	$1/4$	$3/4$
$P[Y=y]$	$4/6$	$4/12$	$12/12 = 12/12 = 1$

$$\forall x,y : P[X=x, Y=y] = P[X=x] \cdot P[Y=y]$$

$$\frac{3}{12} \cdot \frac{4}{6} = \frac{1}{6}$$

jsou nezávislé

$$\operatorname{cov}(X,Y) = E(X \cdot Y) - EX \cdot EY$$

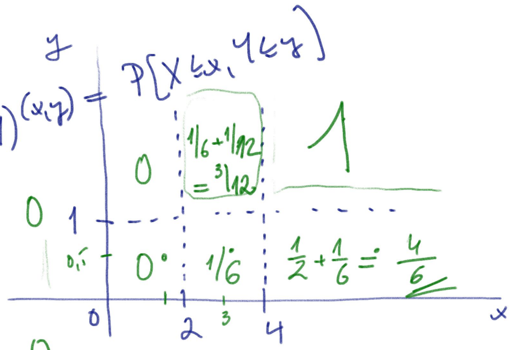
$$= \cancel{0 \cdot 2 \cdot \tfrac{1}{6}} + 1 \cdot 2 \cdot \tfrac{1}{12} + \cancel{0 \cdot 4 \cdot \tfrac{1}{2}} + 1 \cdot 4 \cdot \tfrac{1}{4}$$

$$- \left(2 \cdot \tfrac{3}{12} + 4 \cdot \tfrac{3}{4}\right) \cdot \left(\cancel{0 \cdot \tfrac{4}{6}} + 1 \cdot \tfrac{4}{12}\right) =$$

$$= \frac{1}{6} + 1 - \left(\frac{1}{2} + 3\right) \cdot \left(\frac{1}{3}\right) = \frac{7}{6} - \frac{7}{2} \cdot \frac{1}{3} = 0$$

$x \backslash y$	$y=0$	$y=1$	$P[X=x]$
2	$1/6$	$1/12$	$3/12$
4	$1/2$	$1/4$	$3/4$
$P[Y=y]$	$4/6$	$4/12$	$12/12 = 12/12$

$$F_{(X,Y)}(x,y) = P[X \le x, Y \le y]$$



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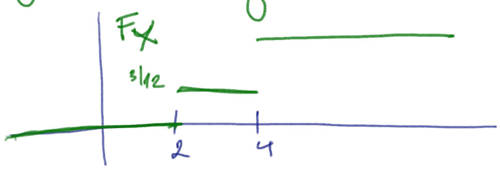
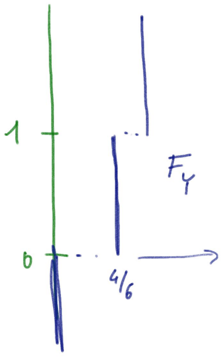




5 (X,Y)

$x \backslash y$	$\boxed{0}$	1	2	$P_X(x)$
0	$1/8$	0	$1/8$	$2/8$
$\boxed{1}$	$\boxed{1/8}$	$1/8$	$1/8$	$3/8$
3	0	$3/8$	0	$3/8$
$P_Y(y)$	$2/8$	$4/8$	$2/8$	$8/8$

$$EX = 0\cdot\frac{2}{8} + 1\cdot\frac{3}{8} + 3\cdot\frac{3}{8} = \frac{3+9}{8} = \frac{12}{8} = \frac{6}{4} = \frac{3}{2}$$

$$EY = 0\frac{2}{8} + 1\frac{4}{8} + 2\frac{2}{8} = \frac{4+4}{8} = 1$$

$$\operatorname{cov}(X,Y) = E(X\cdot Y) - EX\cdot EY = \frac{3}{2} - \frac{3}{2}\cdot 1 = 0$$

$$E(X\cdot Y) = 0\cdot0\cdot\frac{1}{8} + \overset{0\cdot1}{1\cdot0\cdot0} + 0\cdot2\cdot\frac{1}{8} + \boxed{1\cdot0\cdot\frac{1}{8}} + 1\cdot1\cdot\frac{1}{8} + 1\cdot2\cdot\frac{1}{8} + 3\cdot0\cdot0 + 3\cdot1\cdot\frac{3}{8} + 3\cdot2\cdot0$$
$$= \frac{1}{8} + \frac{2}{8} + \frac{9}{8} = \frac{12}{8} = \frac{3}{2}$$

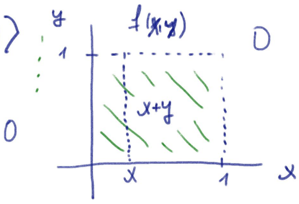
X,Y nejsou nezávislé, protože
např. $P[X=0, Y=1] = 0$
$\neq P[X=0]\cdot P[Y=1] = 2/8 \cdot 4/8$

~~DXZ Cov(X,Z)~~

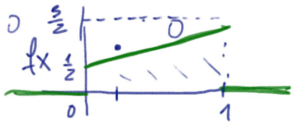
$$E(X,Y) = (EX, EY) = \left(\frac{3}{2}, 1\right)$$
$$D(X,Y) = (DX, DY) = \left(\frac{3}{2}, \frac{1}{2}\right)$$

10 (X,Y)

$$f_{(X,Y)}(x,y) = \begin{cases} a(x+y) & \text{pokud } x,y \in \langle 0,1\rangle \\ 0 & \text{jinak} \end{cases}$$



$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f_{(X,Y)}(x,y)\,dx\,dy = 1$$



$$\underline{f_X(x)} = \int_{-\infty}^{\infty} f_{(X,Y)}(x,y)\,dy = \int_0^1 (x+y)\,dy$$

$$= \left[xy + \frac{y^2}{2}\right]_0^1 = \begin{cases} x+\frac{1}{2} & x\in\langle 0,1\rangle \\ 0 & \text{jinak} \end{cases} \quad f_Y(y) = \begin{cases} y+\frac{1}{2} & y\in\langle 0,1\rangle \\ 0 & \text{jinak} \end{cases}$$

$$\mathrm{cov}(X,Y) = E(X\cdot Y) - EX\cdot EY = \frac{1}{3} - \frac{7}{12}\cdot\frac{7}{12} = \frac{48-49}{144} = -\frac{1}{144}$$

$$EX = \int_{-\infty}^{\infty} x\cdot f_X(x)\,dx = \int_0^1 \underbrace{x\left(x+\tfrac{1}{2}\right)}_{x^2+\frac{x}{2}}\,dx = \left[\frac{x^3}{3}+\frac{x^2}{4}\right]_0^1 = \frac{1}{3}+\frac{1}{4} = \frac{7}{12}$$

$$EY = \frac{7}{12}$$

$$E(X\cdot Y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} x\cdot y\cdot f_{(X,Y)}(x,y)\,dx\,dy = \int_0^1\int_0^1 \underbrace{xy(x+y)}_{x^2y+xy^2}\,dx\,dy$$

$$= \int_0^1 \left[\frac{x^3y}{3} + \frac{x^2y^2}{2}\right]_0^1 dy = \int_0^1 \left(\frac{y}{3}+\frac{y^2}{2}\right)dy = \left[\frac{y^2}{6}+\frac{y^3}{6}\right]_0^1 = \frac{1}{6}+\frac{1}{6} = \frac{1}{3}$$

$$E(X^2) = \int_{-\infty}^{\infty} x^2 f_X(x)\,dx = \int_0^1 \underbrace{x^2\left(x+\tfrac{1}{2}\right)}_{x^3+\frac{x^2}{2}}\,dx = \left[\frac{x^4}{4}+\frac{x^3}{6}\right]_0^1 = \frac{1}{4}+\frac{1}{6} = \frac{5}{12}$$

$$DX = E(X^2) - (EX)^2 = \frac{5}{12} - \left(\frac{7}{12}\right)^2 = \frac{60-49}{144} = \frac{11}{144}$$

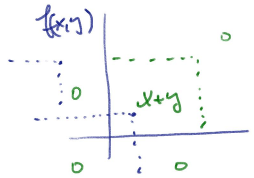
$$DY = \frac{11}{144}$$

kov. mat.
$$\begin{bmatrix} \frac{11}{144} & -\frac{1}{144} \\ -\frac{1}{144} & \frac{11}{144} \end{bmatrix}$$

$$\rho(X,Y) = \frac{-\frac{1}{144}}{\sqrt{\frac{11}{144}}\sqrt{\frac{11}{144}}} = \frac{-\frac{1}{144}}{\frac{11}{144}} = -\frac{1}{11}$$

kor. mat.
$$\begin{bmatrix} 1 & -\frac{1}{11} \\ -\frac{1}{11} & 1 \end{bmatrix}$$

$$f_{(X,Y)}(x,y) = \begin{cases} x+y & (x,y)\in\langle 0,1\rangle^2 \\ 0 & \text{jinak} \end{cases}$$



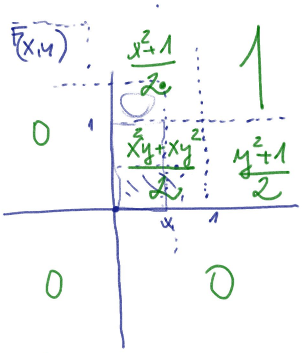
$$F_{(X,Y)}(x,y) = \int_{-\infty}^{x}\int_{-\infty}^{y} f_{(X,Y)}(u,v)\,dv\,du$$

$x,y\in\langle 0,1\rangle^2$:

$$F_{(X,Y)}(x,y) = \int_0^x\int_0^y (u+v)\,dv\,du$$

$$= \int_0^x \left[uv+\frac{v^2}{2}\right]_0^y du = \int_0^x \left(uy+\frac{y^2}{2}\right)du$$

$$= \left[\frac{u^2y}{2}+\frac{uy^2}{2}\right]_0^x = \frac{x^2y+xy^2}{2}$$



$x\in(0,1),\ y\in\langle 1,\infty)$:

$$F_{(X,Y)}(x,y) = \int_0^x \underbrace{\int_0^1 (u+v)\,dv}\,du = \int_0^x \left[uv+\frac{v^2}{2}\right]_0^1 du = \int_0^x \left(u+\frac{1}{2}\right)du = \left[\frac{u^2}{2}+\frac{1}{2}\right]_0^x$$

$$= \frac{x^2+1}{2}$$