② Hodíme 100x mincí 1 hod mincí X_i ... 0 nebo 1 s prav. 1/2

Y... kolikrát padnul líc

jaká je prav. že $49\% \leq Y \leq 51\%$ hodů

$$P[Y=x] = \binom{100}{x}\cdot\left(\tfrac{1}{2}\right)^{x}\cdot\left(\tfrac{1}{2}\right)^{100-x} = \binom{100}{x}\frac{1}{2^{100}}$$

$$EX_i = 0\tfrac{1}{2}+1\tfrac{1}{2} = \tfrac{1}{2}$$

$$P[49\leq Y\leq 51] = \binom{100}{49}\frac{1}{2^{100}} + \binom{100}{50}\frac{1}{2^{100}} + \binom{100}{51}\frac{1}{2^{100}} \doteq \underline{0{,}2356}$$

DX_i

CLV:

X_i ... i.i.d.

$$P[49\leq Y\leq 51] = P\left[\frac{49-50}{\sqrt{25}} \leq \underbrace{\frac{Y-50}{\sqrt{25}}}_{\text{norm } Y} \leq \frac{51-50}{\sqrt{25}}\right] = P\left[-\tfrac{1}{5}\leq \text{norm } Y \leq \tfrac{1}{5}\right]$$

$$EY = E\left(\sum_{i=1}^{100} X_i\right) = \sum EX_i = 100\cdot\tfrac{1}{2} = 50$$

$$DY = nq(1-q) = 100\cdot\tfrac{1}{2}\left(1-\tfrac{1}{2}\right) = 25$$

$$= \Phi\left(\tfrac{1}{5}\right) - \Phi\left(-\tfrac{1}{5}\right) = \Phi(0.2) - \Phi(-0.2)$$

$$= \Phi(0.2) - [1-\Phi(0.2)] = 2\cdot\Phi(0.2) - 1$$

$$= 2\cdot 0{,}5793 - 1 = \underline{0.1586}$$

10 000 x hodíme mincí Y... počet líců ... $Bi\left(n=10\,000,\ q=\tfrac{1}{2}\right)$

EY ... 5000 $DY = 2500$

$$P[Y=x] = \binom{10\,000}{x}\frac{1}{2^x}\cdot\frac{1}{2^{10\,000-x}} = \binom{10000}{x}\frac{1}{2^{10000}}$$

$$P[4900\leq Y\leq 5100] = \sum_{x=4900}^{5100}\binom{10\,000}{x}\frac{1}{2^{10\,000}} = \frac{1}{2^{10000}}\sum\binom{10\,000}{x} \doteq \underline{0{,}9556}$$

CLV:

$$\underline{P[4900\leq Y\leq 5100]} = P\left[\underset{-2}{\frac{-100}{50}} \leq \underbrace{\frac{Y-5000}{\sqrt{2500}}}_{\text{norm } Y} \leq \underset{2}{\frac{100}{50}}\right] = \Phi(+2)-\Phi(-2) = 2\cdot\Phi(2)-1$$

$$= 2\cdot 0{,}9772 - 1 = \underline{0{,}9544}$$

Čebyševova nerovnost

$$\boxed{\text{pro každou náh. vel. } X:\ \forall\delta>0:\ P[|\text{norm } X| < \delta] \geq 1-\frac{1}{\delta^2}}$$

$$P\left[\left|\frac{X-EX}{\sqrt{DX}}\right| < \delta\right] \geq 1-\frac{1}{\delta^2}\frac{\sqrt{DX}^2}{\sqrt{DX}^2}$$

$$P\left[|X-EX| < \underset{>0}{\delta\sqrt{DX}}\right] \geq 1-\frac{DX}{(\delta\sqrt{DX})^2}$$

$$\boxed{\forall\varepsilon>0:\ P[|X-EX| < \varepsilon] \geq 1-\frac{DX}{\varepsilon^2}}$$

$$\underline{P[|Y-EY|\leq 100]} = P[|Y-EY| < 101] \geq 1-\frac{2500}{101^2} \doteq \underline{\underline{0{,}75493}}$$

---

n = počet hodů

jaké je n, aby $P[0{,}49n\leq Y\leq 0{,}51n] \geq 0{,}99$?

$$EY = n\cdot q = \tfrac{1}{2}n$$

$$DY = n\cdot q\cdot(1-q) = \tfrac{1}{4}n$$

CLV

$$P\left[\frac{0{,}49n-\frac{n}{2}}{\sqrt{\frac{n}{4}}} \leq \frac{Y-\frac{n}{2}}{\sqrt{\frac{n}{4}}} \leq \frac{0{,}51n-\frac{n}{2}}{\sqrt{\frac{n}{4}}}\right] \geq 0{,}99$$

$$P\left[2\cdot\frac{-0{,}01n}{\sqrt{n}} \leq \text{norm } Y \leq 2\cdot\frac{0{,}01n}{\sqrt{n}}\right] \geq 0{,}99$$

$$P\left[-0{,}02\cdot\sqrt{n} \leq \text{norm } Y \leq 0.02\sqrt{n}\right] \geq 0{,}99$$

$$\Phi(0{,}02\sqrt{n}) - \Phi(-0{,}02\sqrt{n}) \geq 0{,}99$$

$$2\Phi(0{,}02\sqrt{n}) - 1 \geq 0{,}99$$

$$0{,}02\sqrt{n} \geq \Phi^{-1}\left(\frac{1{,}99}{2}\right)$$

$$n \geq \left(\frac{\Phi^{-1}(0{,}995)}{0{,}02}\right)^2 = \left(\frac{2{,}576}{0{,}02}\right)^2 = 16\,589{,}44$$

alespoň 16 590 krát

Čeb.

$$P[0{,}49m \le Y \le 0{,}51m] \ge 0{,}99$$
$$P[-0{,}01m \le Y-0{,}5m \le 0{,}01m] \ge 0{,}99$$
$$P[|Y-0{,}5m| \le 0{,}01m] \ge 0{,}99$$
$$P[|Y-0{,}5m| < 0{,}01m] \ge 0{,}99$$

$$DY = \frac{m}{4} = 0{,}25m$$

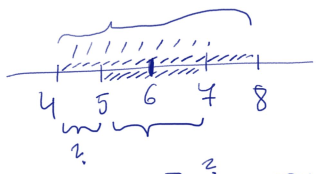
$$P[|Y-0{,}5m| < 0{,}01m] \ge 1-\frac{0{,}25m}{(0{,}01m)^2} \ge 0{,}99$$

$$\frac{0{,}25}{0{,}0001\cdot m} \le 1-0{,}99 = 0{,}01$$

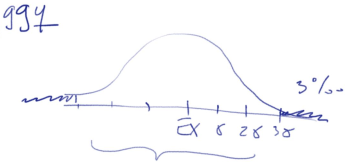
$$0{,}25 \le 0{,}000001m$$

$$\boxed{250\,000 \le m}$$

$$P[4 \le X \le 7] \qquad EX = 6$$



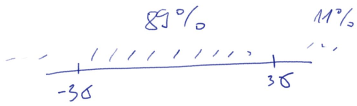
Pravidlo 3σ $\quad P[-3\sigma \le X-\mu \le 3\sigma] \overset{?}{\ge} 0{,}997$



$X \ldots N(\mu, \sigma^2)$

$$P[-3\sigma \le X-\mu \le 3\sigma]$$
$$P\left[\frac{-3\sigma}{\sigma} \le \frac{X-\mu}{\sigma} \le \frac{3\sigma}{\sigma}\right]$$
$$P[-3 \le normX \le 3] = \Phi(3)-\Phi(-3) = 2\Phi(3)-1$$
$$\doteq 2\cdot 0{,}99865-1 = 0{,}9973$$

$X \ldots$ libovolné rozdělení s EX, DX

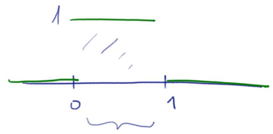


$$P[-3\sqrt{DX} \le X-EX \le 3\sqrt{DX}]$$
$$P[|X-EX| \le 3\sqrt{DX}]$$
$$P[|X-EX| < 3\sqrt{DX}] \ge 1-\frac{DX}{(3\sqrt{DX})^2} = 1-\frac{DX}{9DX} = 1-\frac{1}{9} = \frac{8}{9} \doteq 0{,}89$$

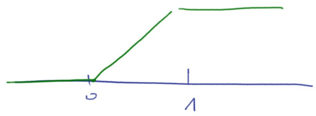
X ... rovnoměrné rozdělení na $\langle 0,1 \rangle$

Y ... ——— " ———

$f_X = f_Y$



$F_X = F_Y$



$Z = X + Y$

konvoluce

$$\boxed{f_Z(z) = \int_{-\infty}^{\infty} f_X(x) \cdot f_Y(z-x)\,dx} = \int_0^1 1 \cdot f_Y(z-x)\,dx \qquad \left| \begin{array}{l} z-x = u \\ -dx = du \\ 1 \to z-1 \\ 0 \to z \end{array} \right|$$

$$= -\int_z^{z-1} f_Y(u)\,du = \int_{z-1}^{z} f_Y(u)\,du$$