$100\times$ hodíme mincí

$X \dots$ počet líců $\dots Bi(n=100,\ q=\frac{1}{2})$ $\quad q=\frac{1}{2}$ $\quad P[X=x]=\binom{100}{x}\cdot\left(\frac{1}{2}\right)^{x}\cdot\left(\frac{1}{2}\right)^{100-x}=\binom{100}{x}\frac{1}{2^{100}}$

$$P[49\le X\le 51]=\binom{100}{49}\frac{1}{2^{100}}+\binom{100}{50}\frac{1}{2^{100}}+\binom{100}{51}\frac{1}{2^{100}}\doteq 0{,}2356$$

$EX=nq=50$

$DX=nq(1-q)=25$

CLV:

$$P\left[\frac{49-50}{\sqrt{25}}\le\frac{X-50}{\sqrt{25}}\le\frac{51-50}{\sqrt{25}}\right]=P\left[-\frac{1}{5}\le \text{norm}\,X\le\frac{1}{5}\right]=\Phi(0{,}2)-\Phi(-0{,}2)$$

$$\Phi(0{,}2)-\left(1-\Phi(0{,}2)\right)=2\cdot\Phi(0{,}2)-1=2\cdot 0{,}5793-1=0{,}1586$$

$10\,000\times$ hodíme mincí

$X \dots Bi(n=10\,000,\ q=1/2)$, $\quad EX=5000,\quad DX=2500,\quad P[X=x]=\binom{10\,000}{x}\frac{1}{2^{10\,000}}$

$$P[4900\le X\le 5100]=\frac{1}{2^{10000}}\sum_{x=4900}^{5100}\binom{10\,000}{x}\doteq 0{,}9556$$

CLV:

$$P\left[\frac{-100}{50}\le\frac{X-5000}{\sqrt{2500}}\le\frac{100}{50}\right]=\Phi(2)-\Phi(-2)=2\Phi(2)-1$$

$$=2\cdot 0{,}9772-1=0{,}9544$$

Čebyševova nerovnost:

$$\boxed{\text{pro každou náh. vel. } X:\quad \forall\delta>0:\quad P[|\text{norm}\,X|<\delta]\ge 1-\frac{1}{\delta^2}}$$

$$P\left[\frac{|X-EX|}{\sqrt{DX}}<\delta\right]\ge 1-\frac{1}{\delta^2}\cdot\frac{\sqrt{DX}^2}{\sqrt{DX}^2}$$

$$P\left[|X-EX|<\delta\sqrt{DX}\right]\ge 1-\frac{DX}{(\delta\sqrt{DX})^2}\qquad \delta\sqrt{DX}=\varepsilon$$

$$\boxed{\forall\varepsilon>0:\quad P[|X-EX|<\varepsilon]\ge 1-\frac{DX}{\varepsilon^2}}$$

$$P[4900\le X\le 5100]$$

$$=P[-100\le X-EX\le 100]=P[|X-EX|\le 100]=P[|X-EX|<101]\ge 1-\frac{2500}{101^2}$$

$$\doteq 0{,}75493$$

m - krát

X … počet lící

$$EX = mq = 0{,}5m \qquad DX = mq(1-q) = \frac{m}{4}$$

$$P[X=x] = \binom{m}{x}\frac{1}{2^x}\frac{1}{2^{m-x}} = \binom{m}{x}\frac{1}{2^m}$$

$$P[0{,}49m \le X \le 0{,}51m] \ge 0{,}99$$

$$\frac{1}{2^m}\sum_{x\in 0{,}49m}^{0{,}51m}\binom{m}{x} \ge 0{,}99$$

CLV:

$$P\left[2\cdot\frac{-0{,}01m}{\sqrt{m}} \le \underbrace{\frac{X-0{,}5m}{\sqrt{\frac{m}{4}}}}_{} \le 2\cdot\frac{0{,}01m}{\sqrt{m}}\right] = \Phi(0{,}02\cdot\sqrt{m}) - \Phi(-0{,}02\sqrt{m})$$

$$\| $$

$$2\Phi(0{,}02\sqrt{m}) - 1 \ge 0{,}99$$

$$\Phi(0{,}02\sqrt{m}) \ge \frac{1{,}99}{2}$$

$$m \ge \left[\frac{\Phi^{-1}(0{,}995)}{0{,}02}\right]^2 = \left(\frac{2{,}576}{0{,}02}\right)^2 = \underline{16\,589{,}4}$$

Čeb:

$$P[0{,}49m \le X \le 0{,}51m] \ge 0{,}99$$

$$P[-0{,}01m < X - 0{,}5m < 0{,}01m]$$

$$P[\underline{|X-0{,}5m| < 0{,}01m}] \overset{\text{Čeb}}{\ge} 1 - \frac{\frac{m}{4}}{(0{,}01m)^2} \ge 0{,}99$$

$$\frac{m}{4\cdot 0{,}0001\cdot m^2} \le 1-0{,}99 = 0{,}01$$

$$\frac{10000}{4\cdot m} \le \frac{1}{100}$$

$$\frac{1\,000\,000}{4} \le m$$

$$m \ge \underline{250\,000}$$

Pravidlo 3σ ... ve vzdálenosti 3σ od stř. h. μ je méně než 3‰ pozorování!

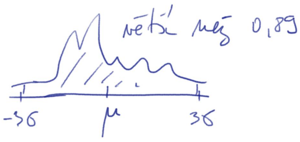
$X = N(\mu, \sigma^2)$

$$P[-3\sigma \le X-\mu \le 3\sigma] \ge 0{,}997$$

$$P\left[-3 \le \frac{X-\mu}{\sigma} \le 3\right] = \Phi(3)-\Phi(-3) = 2\Phi(3)-1$$

$$= 2\cdot 0{,}99865 - 1 = 0{,}997\underline{\underline{3}}$$

$$\ge 0{,}997$$



X ... libovolné rozdělení, $EX=\mu$, $DX=\sigma^2$

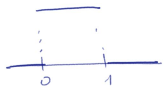
$$P[-3\sigma \le X-\mu \le 3\sigma]$$

$$P[|X-\mu| < 3\sigma] \overset{\text{Čeb}}{\ge} 1-\frac{\sigma^2}{(3\sigma)^2} = 1-\frac{1}{9} = \frac{8}{9} \doteq 0{,}89\ldots$$

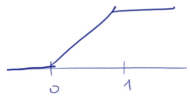
X ... rovnoměrné rozd. na $\langle 0,1)$

Y .. — " —

$f_Y = f_X$



F_X



$Z = X+Y$

konvoluce

$$f_Z(z) = \int_{-\infty}^{\infty} f_X(x) \cdot f_Y(z-x)\,dx = \int_0^1 1 \cdot f_Y(z-x)\,dx$$

$$\left|\ \begin{aligned} z-x &= u \\ -dx &= du \\ 1 &\to z-1 \\ 0 &\to z \end{aligned}\right.$$

$$= -\int_z^{z-1} f_Y(u)\,du = \int_{z-1}^{z} f_Y(u)\,du$$