Hodíme 100-krát mincí

X … počet líců … $Bi(n=100,\ q=\frac{1}{2})$ $\quad P[X=x]=\binom{n}{x}\left(\frac{1}{2}\right)^x\left(\frac{1}{2}\right)^{n-x}=\binom{100}{x}\frac{1}{2^x}\cdot\frac{1}{2^{100-x}}=\binom{100}{x}\frac{1}{2^{100}}$

$$P[49\le X\le 51]=\binom{100}{49}\frac{1}{2^{100}}+\binom{100}{50}\frac{1}{2^{100}}+\binom{100}{51}\frac{1}{2^{100}}$$
$$\doteq 0{,}2356$$

jednotlivé hody jsou i.i.d.

$EX=n\cdot q=50$

$DX=nq(1-q)=25$

CLV
$$P[49\le X\le 51]=P\left[\frac{49-50}{\sqrt{25}}\le\frac{X-50}{\sqrt{25}}\le\frac{51-50}{\sqrt{25}}\right]$$
$$=P\left[\frac{-1}{5}\le \text{norm}X\le\frac{1}{5}\right]=\Phi(0{,}2)-\Phi(-0{,}2)$$
$$=\Phi(0{,}2)-(1-\Phi(0{,}2))$$
$$=2\Phi(0{,}2)-1$$
$$=2\cdot 0{,}5793-1=\underline{0{,}1586}$$

$n=10\,000$: $\quad EX=5000\quad DX=2500\quad P[X=x]=\binom{10000}{x}\frac{1}{2^x}\frac{1}{2^{10000-x}}=\binom{10000}{x}\frac{1}{2^{10\,000}}$

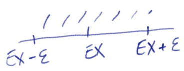
49% z 10 000 je 4900

$$P[4900\le X\le 5100]=\frac{1}{2^{10^4}}\sum_{x=4900}^{5100}\binom{10000}{x}\doteq\underline{0{,}9556}$$

CLV
$$P\left[\frac{-100}{50}\le\frac{X-5000}{\sqrt{2500}}\le\frac{100}{50}\right]=\Phi(2)-\Phi(-2)$$
$$=2\Phi(2)-1$$
$$=2\cdot 0{,}9772-1=\underline{0{,}9544}$$

Čebyševova nerovnost: pro každou náh. vel. X platí:

$$\forall\delta>0:\ P[|\text{norm}X|<\delta]\ge 1-\frac{1}{\delta^2}$$
$$P\left[\frac{|X-EX|}{\sqrt{DX}}<\delta\right]\ge 1-\frac{1}{\delta^2}\cdot\frac{\sqrt{DX}^2}{\sqrt{DX}^2}$$
$$P[|X-EX|<\underbrace{\delta\sqrt{DX}}_{\varepsilon>0}]\ge 1-\frac{DX}{(\underbrace{\delta\sqrt{DX}}_{\varepsilon})^2}$$



$$\boxed{\forall\varepsilon>0:\ P[|X-EX|<\varepsilon]\ge 1-\frac{DX}{\varepsilon^2}}$$

$$P[4900\le X\le 5100]$$
$$=P[-100\le X-EX\le 100]=P[|X-EX|\le 100]=P[|X-EX|<101]\ge 1-\frac{2500}{101^2}$$
$$=1-\frac{2500}{10\,201}$$
$$\doteq\frac{3}{4}=0{,}75$$

$P[X\le 100]$

$P[X<100]$

počet hodů je m

jaké musí být m, aby: $P[0{,}49m \le X \le 0{,}51m] \ge 0{,}99$

$EX = mq = 0{,}5m$

$DX = mq(1-q) = \frac{m}{4}$

$\sqrt{DX} = \frac{\sqrt{m}}{2}$

$P[X=x] = \binom{m}{x}\frac{1}{2^m}$

$$\frac{1}{2^m}\sum_{x=0{,}49m}^{0{,}51m}\binom{m}{x} \ge 0{,}99$$

CLV:

$$P\left[\frac{0{,}49m-0{,}5m}{\frac{\sqrt{m}}{2}} \le \frac{X-0{,}5m}{\frac{\sqrt{m}}{2}} \le \frac{0{,}51m-0{,}5m}{\frac{\sqrt{m}}{2}}\right] \ge 0{,}99$$

$$P\left[-2\frac{0{,}01m}{\sqrt{m}} \le \text{norm}X \le 2\frac{0{,}01m}{\sqrt{m}}\right] = \Phi(0{,}02\cdot\sqrt{m}) - \Phi(-0{,}02\sqrt{m})$$

$$= 2\Phi(0{,}02\sqrt{m}) - 1 \ge 0{,}99$$

$$m \ge \left(\frac{1}{0{,}02}\Phi^{-1}\underbrace{\left[\frac{1{,}99}{2}\right]}_{0{,}995}\right)^2 = \left(\frac{\Phi^{-1}(0{,}995)}{0{,}02}\right)^2$$

$$= \left(\frac{2{,}576}{0{,}02}\right)^2$$

$$\doteq 16589{,}44\ldots$$

Čeb:

$$P[0{,}49m \le X \le 0{,}51m] \ge 0{,}99$$

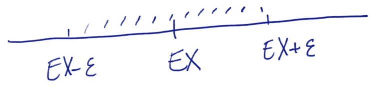
$$P[-0{,}01m \le X-EX \le 0{,}01m]$$

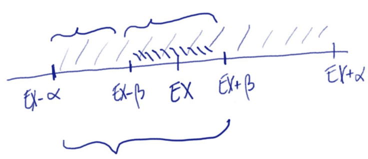
$$P[|X-EX| < 0{,}01m] \ge 1 - \frac{\frac{m}{4}}{(0{,}01m)^2} \ge 0{,}99$$

$$\frac{m}{4\cdot\frac{1}{10\,000}m^2} \le 0{,}01$$

$$\frac{10\,000}{4} \le \frac{m}{100}$$

$$250\,000 \le m$$





Pravidlo 3σ

$$P[-3\sigma \le X - EX \le 3\sigma] \overset{?}{\ge} 1 - 3‰ = 0{,}997$$

X .. $N(\mu, \sigma^2)$: $P[-3\sigma \le X - \mu \le 3\sigma]$

$$P\left[-3 \le \frac{X-\mu}{\sigma} \le 3\right] = \Phi(3) - \Phi(-3)$$
$$= 2\Phi(3) - 1$$
$$= 2 \cdot 0{,}99865 - 1 \doteq 0{,}9973$$

$\ge 0{,}997$

X .. libovolné n $\quad P[-3\sqrt{DX} \le X - EX \le 3\sqrt{DX}] \ge 0{,}89$

$$P[|X - EX| \le 3\sqrt{DX}] \ge 1 - \frac{DX}{(3\sqrt{DX})^2} = 1 - \frac{1}{9} = \frac{8}{9} \doteq 0{,}89$$

X ... rovnoměrné r. na $\langle 0,1\rangle$

Y ... — " —

$f_Y = f_X$

$Z = X + Y$

$$\boxed{f_Z(z) = \int_{-\infty}^{\infty} f_X(x)\cdot f_Y(z-x)\,dx} = \int_0^1 1\cdot f_Y(\underbrace{z-x}_{u})\,dx \quad \Bigg| \begin{array}{l} z-x=u \\ -dx = du \end{array}$$

$$= -\int_z^{z-1} f_Y(u)\,du = \int_{z-1}^{z} f_Y(u)\,du$$