1

i	x_i	$x_i-\bar{x}$	$(x_i-\bar{x})^2$
1.	6,10	0,6	0,36
2.	5,95	0,45	0,2025
3.	5,90	0,4	0,16
4.	5,88	0,38	⋮
5.	5,85	0,35	⋮
6.	5,84	0,34	⋮
7.	5,80	0,3	⋮
8.	5,78	0,28	⋮
9.	5,42	−0,08	0,0064
10.	2,50	−3	9
$\sum$	55,00		10,265
$\frac{1}{n}\sum$	5,50		1,0265
$\frac{1}{n-1}\sum$			1,1406

$\bar{x} = 5{,}5\,m$

X ... střední hodnota
$\bar{X}$ ... výběrový průměr

$E\bar{X} = EX$

"uřezaný" výb. průměr
$\bar{X}_t$ t ... trimmed

$$\bar{x}_t = \frac{5{,}95+5{,}90+\dots+5{,}42}{8} = \frac{46{,}4}{8} = 5{,}8\,m$$

výběrový medián

$$\frac{5{,}85+5{,}84}{2} = 5{,}845$$

výběrový rozptyl

$$S_X^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i-\bar{x})^2$$

$$s_x^2 = 1{,}1406\,m^2$$

výběrová směrodatná odchylka

$$s_x = \sqrt{s_x^2} = \sqrt{1{,}1406}\,m \doteq 1{,}068\,m$$

$$x = 5{,}5 \pm 1{,}068\,m$$

vychýlený výběrový rozptyl

$$\Sigma_X^2 = \frac{1}{n}\sum(x_i-\bar{x})^2$$

$$ES_X^2 = DX \;;\; E\Sigma_X^2 \neq DX$$

$$DS_X^2 \geq D\Sigma_X^2$$

②

i	x_i	$x_i - \bar{x}$	$(x_i-\bar{x})^2$
1	8,1	⋮	⋮
2	7,4		
3	5,2		
4	8,7		
5	6,5		
6	5,7		
7	9,1		
8	6,8		
9	4,6		
Σ	62,1		19,56

$\frac{1}{n}\Sigma \quad \bar{x} = 6{,}9$

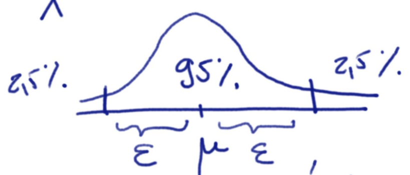
$\frac{1}{n-1}\Sigma \qquad s_X^2 = 2{,}445$

Určete intervalový odhad
1) střední hodnoty μ
2) rozptylu σ^2
se spolehlivostí 95 %

1) int. odh. μ se spolehlivostí $1-\alpha = 0{,}95$

hledáme $\varepsilon > 0$ tak, aby:

X symetrický



jednostranný horní

$$P\left[\mu-\varepsilon \le \bar{X} \le \mu+\varepsilon\right] \ge 1-\alpha$$

$$P\left[\frac{-\varepsilon}{\sqrt{D\bar{X}}} \le \frac{\bar{X}-\mu}{\sqrt{D\bar{X}}} \le \frac{\varepsilon}{\sqrt{D\bar{X}}}\right] \ge 1-\alpha$$

$D\bar{X} = \frac{1}{n} DX$

$$P\left[\frac{-\varepsilon}{\sqrt{\frac{DX}{n}}} \le \frac{\bar{X}-\mu}{\sqrt{\frac{DX}{n}}} \le \frac{\varepsilon}{\sqrt{\frac{DX}{n}}}\right] \ge 1-\alpha$$

norm $\bar{X}$

pokud známe DX:

$$\Phi\left(\frac{\varepsilon}{\sqrt{\frac{DX}{n}}}\right) - \left(1-\Phi\left(\frac{+\varepsilon}{\sqrt{\frac{DX}{n}}}\right)\right) \ge 1-\alpha$$

$$2\Phi\left(\frac{\varepsilon}{\sqrt{\frac{DX}{n}}}\right) - 1 \ge 1-\alpha$$

$$\varepsilon \ge \sqrt{\frac{DX}{n}}\,\Phi^{-1}\left(1-\frac{\alpha}{2}\right)$$

$$\varepsilon \ge \sqrt{\frac{DX}{10}} \cdot 1{,}96$$

$\Phi^{-1}\left(1-\frac{0{,}05}{2}\right)$

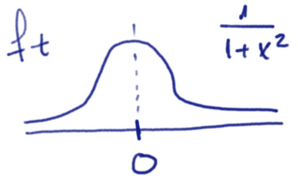
$\Phi^{-1}(0{,}975) = 1{,}96$

$$P\left[\frac{-\varepsilon}{\sqrt{\frac{S_X^2}{n}}} \le \frac{\bar{X}-\mu}{\sqrt{\frac{S_X^2}{n}}} \le \frac{\varepsilon}{\sqrt{\frac{S_X^2}{n}}}\right] \ge 1-\alpha$$

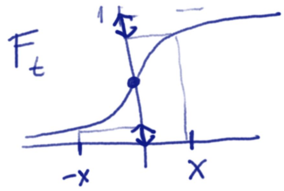
$DX \doteq S_X^2$

$s_X^2 = 2{,}445$

$s_X = 1{,}5636$



Studentovo rozdělení s $n-1$ stupni volnosti ... $t(n-1)$



$F_t(-x) = 1 - F_t(x)$

$$F_{t(n-1)}\left(\frac{\varepsilon}{\sqrt{\frac{s_X^2}{n}}}\right) - \left(1 - F_{t(n-1)}\left(\frac{+\varepsilon}{\sqrt{\frac{s_X^2}{n}}}\right)\right) \ge 1-\alpha$$

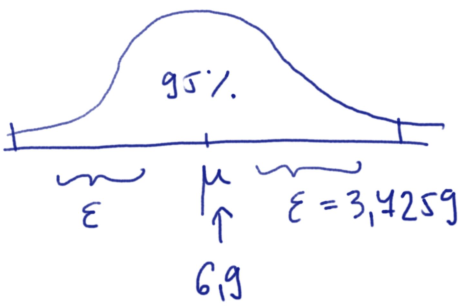
$$2F_{t(n-1)}\left(\frac{\varepsilon\sqrt{n}}{s_X}\right) - 1 \ge 1-\alpha$$

$$\varepsilon \ge \frac{s_X}{\sqrt{n}} F^{-1}_{t(n-1)}\left(1-\frac{\alpha}{2}\right)$$

$$\varepsilon \ge \frac{1{,}5636}{\sqrt{9}} \underbrace{F^{-1}_{t(\cancel{n-1}\,8)}(0{,}975)}_{\underbrace{q_{t(8)}(0{,}975)}_{2{,}31}}$$

$$\varepsilon \ge \frac{1{,}5636 \cdot 2{,}31}{3} = 3{,}4259$$

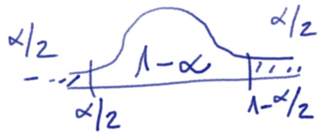
$\bar{x} = 6{,}9$



symetrický int. odhad $\langle 6{,}9 - 3{,}73;\ 6{,}9 + 3{,}73 \rangle$

$\langle 3{,}144;\ \ 10{,}626 \rangle$

$$\frac{\bar{X}-\mu}{\sqrt{\frac{S_X^2}{n}}} \ldots t(n-1)$$



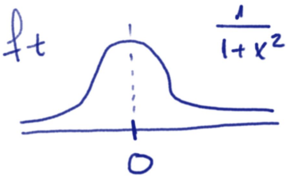
$$P\left[F^{-1}_{t(n-1)}\left(\frac{\alpha}{2}\right) \le \frac{\bar{X}-\mu}{\sqrt{\frac{S_X^2}{n}}} \le F^{-1}_{t(n-1)}\left(1-\frac{\alpha}{2}\right)\right] \ge 1-\alpha$$

$$P\left[\frac{-\varepsilon}{\sqrt{\frac{S_X^2}{n}}} \le \frac{\bar{X}-\mu}{\sqrt{\frac{S_X^2}{n}}} \le \frac{\varepsilon}{\sqrt{\frac{S_X^2}{n}}}\right] \ge 1-\alpha$$

$DX \doteq S_X^2$

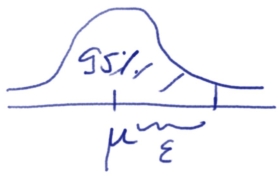
$s_X^2 = 2{,}445$

$s_X = 1{,}5636$



Studentovo rozdělení s $n-1$ stupni volnosti ... $t(n-1)$

horní int. odhad!



$$P[\bar{X} \le \mu+\varepsilon] \ge 1-\alpha$$

$$\frac{\bar{X}-\mu}{\sqrt{D\bar{X}}}$$

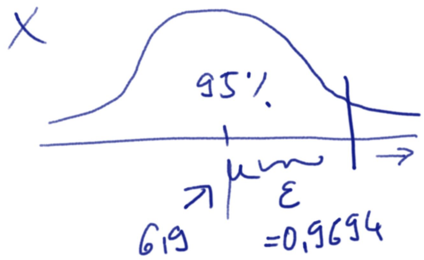
$$\frac{\bar{X}-\mu}{\sqrt{\frac{DX}{n}}}$$

$$P\left[\underbrace{\frac{\bar{X}-\mu}{\sqrt{\frac{S_X^2}{n}}}}_{t(n-1)} \le \frac{\varepsilon}{\sqrt{\frac{S_X^2}{n}}}\right] \ge \overset{0{,}95}{1-\alpha}$$

$$F_{t(n-1)}\left(\frac{\varepsilon\sqrt{n}}{s_X}\right) \ge 1-\alpha$$

$$\varepsilon \ge \frac{s_X}{\sqrt{n}} \cdot F^{-1}_{t(n-1)}(\overset{0{,}95}{1-\alpha})$$

$$\varepsilon \ge \frac{1{,}5636}{\sqrt{9}} F^{-1}_{t(8)}(0{,}95) = \frac{1{,}5636}{3} \cdot 1{,}86 = 0{,}9694$$



horní int. odhad

$(-\infty; 6{,}9+0{,}9694\rangle$

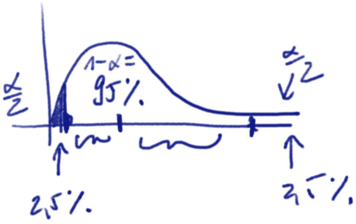
$(-\infty, 7{,}8694\rangle$

2) $\bar{x} = 6{,}9$

$s_x^2 = 2{,}445$

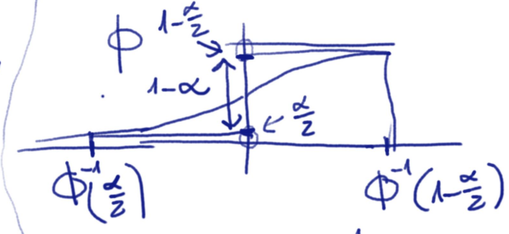
$s_x = 1{,}5636$

$$\frac{n-1}{\sigma^2} s_x^2 \quad \dots \quad \chi^2(n-1)$$



$$s_x^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

$$s_x^2 = \frac{\sigma^2}{n-1} \sum \left(\underbrace{\frac{x_i - \bar{x}}{\sigma}}_{\text{nom } x_i} \right)^2$$



$$P\left[\Phi^{-1}\left(\tfrac{\alpha}{2}\right) \le X \le \Phi^{-1}\left(1-\tfrac{\alpha}{2}\right)\right] = 1-\alpha$$

$$\frac{n-1}{\sigma^2} s_x^2 = \underbrace{\sum \dots}_{\chi^2(n-1)}$$

$$P\left[q_{\chi^2(n-1)}(\alpha/2) \le \frac{n-1}{\sigma^2} s_x^2 \le q_{\chi^2(n-1)}\left(1-\frac{\alpha}{2}\right)\right] \ge 1-\alpha$$

$$q_{\chi^2(8)}(0{,}025) \qquad \frac{n-1}{\sigma^2} s_x^2 \qquad q_{\chi^2(8)}(0{,}975)$$

$$2{,}18 \le \frac{8}{\sigma^2} \cdot 2{,}445 \le 17{,}53$$

$$\frac{1}{2{,}18} \ge \frac{\sigma^2}{8 \cdot 2{,}445} \ge \frac{1}{17{,}53}$$

$$\frac{8 \cdot 2{,}445}{2{,}18} \ge \sigma^2 \ge \frac{8 \cdot 2{,}445}{17{,}53} = \frac{(n-1)\, s_x^2}{q_{\chi^2(8)}(0{,}975)}$$

$$8{,}9725 \ge \sigma^2 \ge 1{,}1158$$

symetric?

$$\sigma^2 \in \langle 1{,}1158 ; 8{,}9725 \rangle$$