X ... neznáme

odhad EX

výběrový aritmetický průměr $\overline{X}$

$\underline{\underline{\bar{x} = 6{,}9}}$

$E\overline{X} = EX$

$D\overline{X} = \frac{1}{n} DX$

↑ POČET MĚŘENÍ

„uříznutý" aritm. průměr $\overline{X}_t$

↑ trimmed

10% největších a nejmenších hodnot zahodíme

$$\bar{x}_t = \frac{5{,}2 + 5{,}7 + \dots + 8{,}7}{7} = \underline{\underline{6{,}9143}}$$

výběrový medián

~~5,845~~

$\tilde{x} = 6{,}8$

výběrový rozptyl $\underline{\underline{S_X^2}} = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \overline{X})^2$

$ES_X^2 = DX$

$$s_X^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

6,9

$s_X^2 = \underline{\underline{2{,}445}}$

výběrová směrodatná odchylka

$$s_X = \sqrt{s_X^2} = \sqrt{2{,}445} = 1{,}5636$$

$ES_X \neq E\sqrt{DX}$

vychýlený výběrový rozptyl

$$\Sigma_X^2 = \frac{1}{n} \sum (X_i - \overline{X})^2$$

$E\Sigma_X^2 \neq DX$

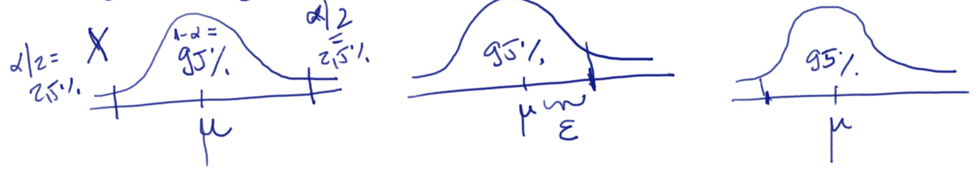
$D\Sigma_X^2 < DS_X^2$

$\bar{x} = 6{,}9$
$s_x^2 = 2{,}445$
$s_x = 1{,}5636$
$n = 9$

symetrický

horní

dolní



intervalový odhad střední hodnoty μ se spolehlivostí $1-\alpha = 95\%$

$$P[\bar{X} \le \mu + \varepsilon] \ge 1-\alpha$$

$$P\left[\underbrace{\frac{\bar{X}-\mu}{\sqrt{D\bar{X}}}}_{\text{norm}\,\bar{X}} \le \frac{\varepsilon}{\sqrt{D\bar{X}}}\right] \ge 1-\alpha$$

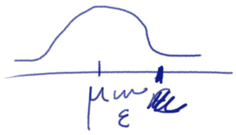
$D\bar{X} = \frac{1}{n} DX$

$$P\left[\frac{\bar{X}-\mu}{\sqrt{\frac{DX}{n}}} \le \frac{\varepsilon}{\sqrt{\frac{DX}{n}}}\right] \ge 1-\alpha$$

pokud známe DX

$$\Phi\left(\frac{\varepsilon}{\sqrt{\frac{DX}{n}}}\right) \ge 1-\alpha$$

$$\varepsilon \ge \sqrt{\frac{DX}{n}} \cdot \Phi^{-1}(1-\alpha)$$

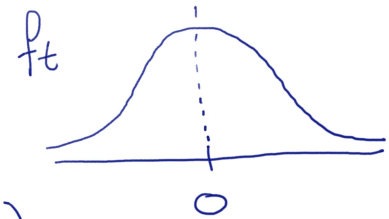


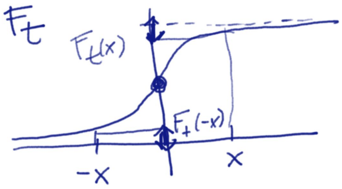
$$\left(-\infty, \bar{x} + \sqrt{\frac{DX}{n}} \cdot \Phi^{-1}(1-\alpha)\right\rangle$$

pokud DX neznáme, tak ho nahradíme odhadem s_x^2

$$P\left[\frac{\overset{N(\mu, DX)}{\bar{X}-\mu}}{\sqrt{\frac{\underset{\chi^2(n-1)}{S_x^2}}{n}}} \le \frac{\varepsilon}{\sqrt{\frac{S_x^2}{n}}}\right] \ge 1-\alpha$$

Studentovo rozdělení ... $t(n-1)$
s $n-1$ stupni volnosti





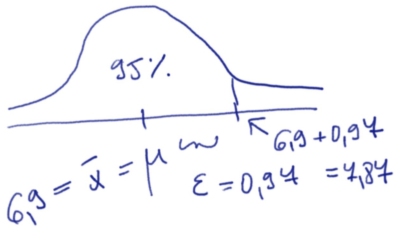
$F_t(-x) = 1 - F_t(x)$

$$F_{t(n-1)}\left(\frac{\varepsilon}{\sqrt{\frac{s_x^2}{n}}}\right) \ge 1-\alpha$$

$$\frac{\varepsilon}{\sqrt{\frac{s_x^2}{n}}} \ge q_{t(n-1)}(1-\alpha)$$

$$\varepsilon \ge \frac{s_x}{\sqrt{n}} q_{t(n-1)}(1-\alpha)$$

$$\varepsilon \ge \frac{1{,}5636}{\sqrt{9}} \underbrace{q_{t(8)}(0{,}95)}_{1{,}86} \doteq 0{,}97$$



horní int. odhad střední h.

$(-\infty; 7{,}87\rangle$

$\bar{x} = 6{,}9$

$s_x^2 = 2{,}445$

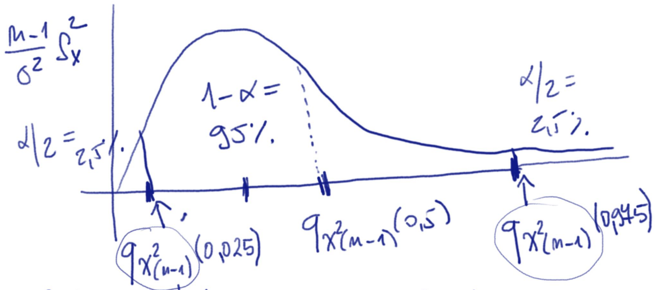
$s_x = 1{,}5636$

$n = 9$

$$\frac{n-1}{\sigma^2} S_X^2 \dots \chi^2(n-1)$$

$$S_X^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2$$

$$S_X^2 = \frac{\sigma^2}{n-1} \sum \underbrace{\left(\frac{x_i - \bar{x}}{\sigma}\right)^2}_{\chi^2(n-1)}$$



symetrický intervalový odhad rozptylu se spolehlivostí $1-\alpha = 95\%$

$$P\left[q_{\chi^2(n-1)}(0{,}025) \le \frac{n-1}{\sigma^2} S_X^2 \le q_{\chi^2(n-1)}(0{,}975)\right] = 1-\alpha = 0{,}95$$

$0{,}975$

$$\underbrace{q_{\chi^2(8)}(0{,}025)}_{2{,}18} \le \frac{n-1}{\sigma^2} S_X^2 \le \underbrace{q_{\chi^2(8)}(0{,}975)}_{17{,}53}$$

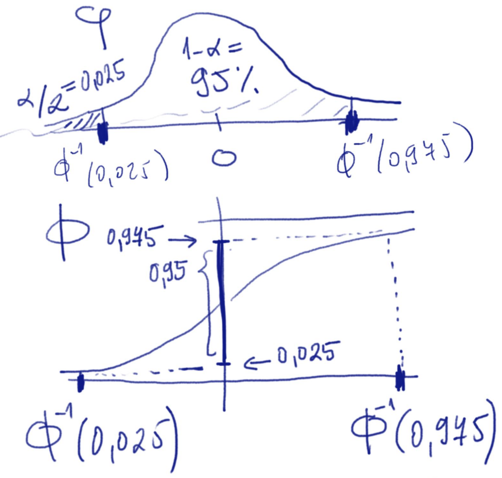
$$2{,}18 \le \frac{8}{\sigma^2} 2{,}445 \le 17{,}53$$

$$\frac{1}{2{,}18} \ge \frac{\sigma^2}{8 \cdot 2{,}445} \ge \frac{1}{17{,}53}$$

$$\frac{8 \cdot 2{,}445}{2{,}18} \ge \sigma^2 \ge \frac{8 \cdot 2{,}445}{17{,}53}$$

$$8{,}97 \ge \sigma^2 \ge 1{,}12$$

$$\sigma^2 \in \langle 1{,}12;\ 8{,}97 \rangle$$



$$P\left[\Phi^{-1}(0{,}025) \le X \le \Phi^{-1}(0{,}975)\right] = 0{,}95$$