X ... skok do dálky

výběrový aritm. průměr $\bar{X} = \frac{1}{n} \sum_{i=1}^{n} X_i$

$\bar{X} \to N(EX, \frac{1}{n} DX)$

$E\bar{X} = EX$

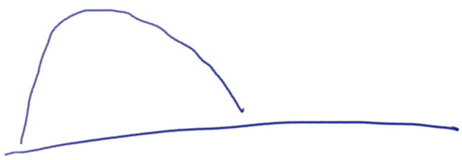
$D\bar{X} = \frac{1}{n} DX$

$\bar{x} = \frac{1}{n} \sum x_i = \frac{55}{10} = 5{,}5\, m$

"useknutý" výb. arit. prů. $\bar{X}_t$ ↑ TRIMMED

10% největších
a nejmenších hodnot
dáme pryč

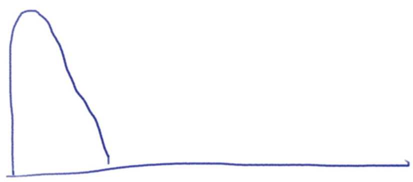
$\bar{x}_t = \frac{5{,}95 + 5{,}9 + \dots + 5{,}42}{8} = \frac{46{,}4}{8} = 5{,}8\, m$



výběrový medián

$\tilde{x}$ někde mezi 5,85 a 5,84

$\tilde{x} = 5{,}845$



výběrový rozptyl $S_X^2 = \frac{1}{n-1} \sum_{i=1}^{n} (X_i - \bar{X})^2$

$s_x^2 = \frac{1}{n-1} \sum (x_i - \bar{x})^2 = \frac{10{,}265}{9} = \underline{\underline{1{,}1406\, m^2}}$

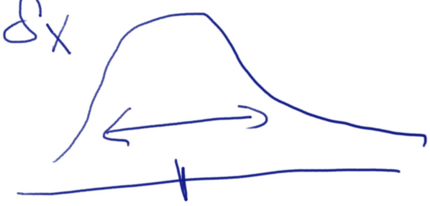
$ES_X^2 = DX$

výběrová směrodatná odchylka

$s_x = \sqrt{s_x^2} = \sqrt{1{,}1406} = \underline{1{,}068\, m}$

$ES_X \neq \sqrt{DX}$ $\quad x = 5{,}5 \pm 1{,}068$

S_X^2



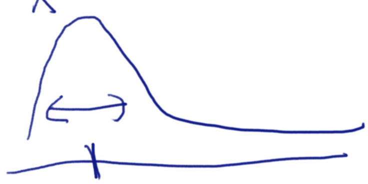
vychýlený výběrový rozptyl

$\Sigma_X^2 = \frac{1}{n} \sum (X_i - \bar{X})^2$

$\sigma_x^2 = \frac{10{,}265}{10} = \underline{1{,}0265\, m^2}$

$E\Sigma_X^2 \neq DX$ $\quad \boxed{D\Sigma_X^2 \leq DS_X^2}$

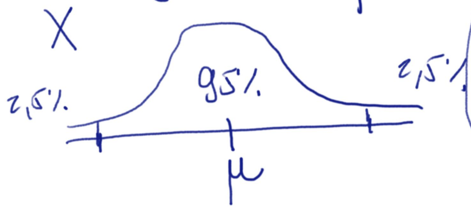
Σ_X^2



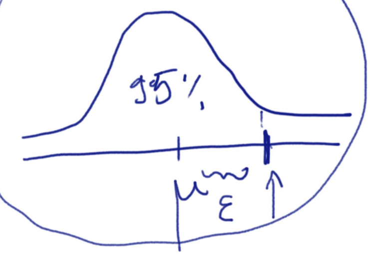
2.1 int. odh. stř. h. μ se spol. $1-\alpha = 0{,}95$ ($\alpha = 0{,}05$)

$X \dots N(\mu, \sigma^2)$

symetrický



horní



dolní

$\bar{x} = 6{,}9$

$s_x^2 = 2{,}445$

$s_x = 1{,}5636$

$D\bar{X} = \frac{1}{n} DX =$

$$P[\bar{X} \le \mu + \varepsilon] \ge 1-\alpha$$

$$P\left[\frac{\bar{X}-\mu}{\sqrt{D\bar{X}}} \le \frac{\varepsilon}{\sqrt{D\bar{X}}}\right] \ge 1-\alpha$$

norm $\bar{X}$

$$P\left[\frac{\bar{X}-\mu}{\sqrt{\frac{DX}{n}}} \le \frac{\varepsilon}{\sqrt{\frac{DX}{n}}}\right] \ge 1-\alpha$$

$$\underbrace{\frac{\bar{X}-\mu}{\sqrt{\frac{S_x^2}{n}}}}_{Z} \dots t(n-1)$$

$$P[Z \le q_{t(n-1)}(0{,}95)] = 0{,}95$$

$$\boxed{\text{známe } DX: \longrightarrow N(0,1) \qquad \Phi\left(\frac{\varepsilon}{\sqrt{\frac{DX}{n}}}\right) \ge 1-\alpha \qquad \varepsilon \ge \sqrt{\frac{DX}{n}}\,\Phi^{-1}(1-\alpha)}$$

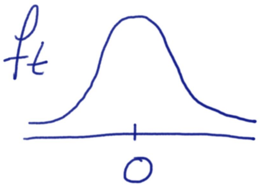
neznáme DX $\Rightarrow$ odhadneme pomocí ~~s_x~~ S_x^2

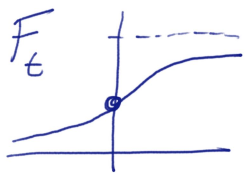
Norm.

$$P\left[\frac{\bar{X}-\mu}{\sqrt{\frac{S_x^2}{n}}} \le \frac{\varepsilon}{\sqrt{\frac{S_x^2}{n}}}\right] \ge 1-\alpha$$

$\chi^2(n-1)$

Studentovo r. ... $t(n-1)$



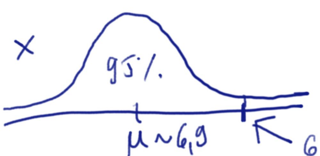


$$F_{t(n-1)}\left(\frac{\varepsilon}{\sqrt{\frac{S_x^2}{n}}}\right) \ge 1-\alpha$$

$$\varepsilon \ge \frac{S_x}{\sqrt{n}}\, q_{t(n-1)}(1-\alpha) = \frac{1{,}5636}{\sqrt{9}}\, q_{t(8)}(0{,}95)$$

$S_x \dots s_x$

$$= \frac{1{,}5636 \cdot 1{,}86}{3} \doteq 0{,}97$$



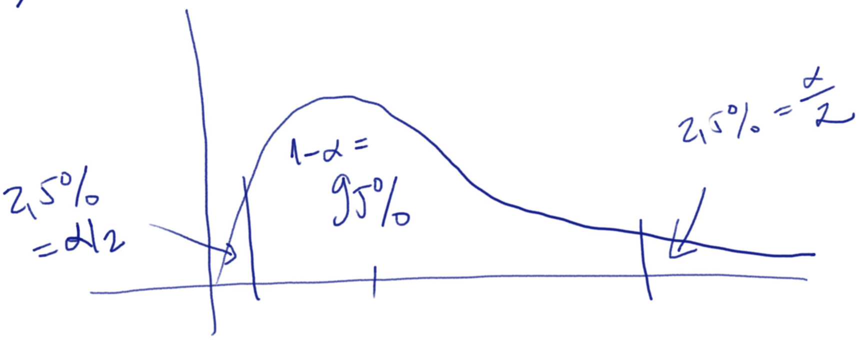
$\varepsilon \doteq 0{,}97$

$6{,}9 + 0{,}97 = 7{,}87$

$\mu \in (-\infty, 7{,}87\rangle$

$$\frac{n-1}{\sigma^2} S_X^2 \quad \ldots \quad \chi^2(n-1)$$

ROZPTYL

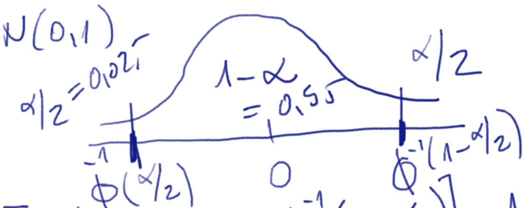


$$S_X^2 = \frac{1}{n-1}\sum (X_i-\bar{X})^2$$

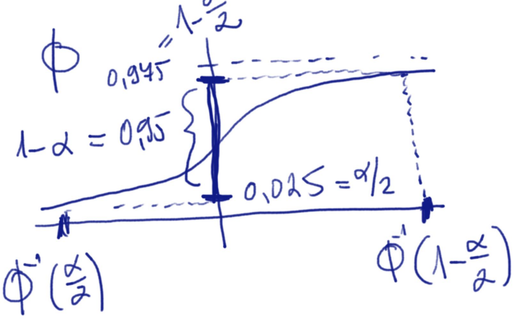
$$S_X^2 = \frac{\sigma^2}{n-1}\underbrace{\sum\Big(\underbrace{\frac{X_i-\bar{X}}{\sigma}}_{N(0,1)}\Big)^2}_{\chi^2(n-1)}$$

$$P\left[q_{\chi^2(n-1)}\left(\frac{\alpha}{2}\right) \le \frac{n-1}{\sigma^2} S_X^2 \le q_{\chi^2(n-1)}\left(1-\frac{\alpha}{2}\right)\right] = 1-\alpha$$

$Y \ldots N(0,1)$



$$P\left[\Phi^{-1}\left(\frac{\alpha}{2}\right) \le Y \le \Phi^{-1}\left(1-\frac{\alpha}{2}\right)\right] = 1-\alpha$$



$$q_{\chi^2(8)}(0{,}025) \le \frac{8}{\sigma^2}\cdot 2{,}445 \le q_{\chi^2(8)}(0{,}975)$$

$$\frac{1}{2{,}18} \ge \frac{\sigma^2}{8\cdot 2{,}445} \ge \frac{1}{17{,}53}$$

$$\frac{8\cdot 2{,}445}{2{,}18} \ge \sigma^2 \ge \frac{8\cdot 2{,}445}{17{,}53}$$

$$8{,}9725 \ge \sigma^2 \ge 1{,}1158$$

$$\sigma^2 \in \langle 1{,}1158 \,;\, 8{,}9725 \rangle$$