① X

hodnota x_i	0	1	2
teor. prav. p_i	a	b	$a+b$
naměřená četnost n_i	13	16	21
p_i	a	$\frac{1}{2}-a$	$\frac{1}{2}$

$a, b \in \langle 0,1 \rangle$

musí platit $a+b+(a+b)=1$

$2a+2b=1$

$2b = 1-2a$

$\boxed{b = \frac{1}{2}-a}$

$a+b = a+\frac{1}{2}-a = \frac{1}{2}$

1. Metoda momentů (MM)

$EX = 0\cdot a + 1\cdot(\frac{1}{2}-a) + 2\cdot\frac{1}{2}$

$= \frac{1}{2} - a + 1$

$EX = \frac{3}{2} - a$

$\bar{x} = \frac{13\cdot 0 + 16\cdot 1 + 21\cdot 2}{13+16+21} = \frac{58}{50}$

$\frac{3}{2} - \hat{a} = \frac{58}{50}$

$\boxed{\hat{a} = \frac{3}{2} - \frac{58}{50} = \frac{14}{50}}$

13 16 21

0,0,0,... | 1,1,... | 2,2,...

2. Metoda max. věrohodnosti (MMV)

věrohodnost (LIKELIHOOD) $L(a) = a^{13}\cdot(\frac{1}{2}-a)^{16}\cdot(\frac{1}{2})^{21}$ .... $= \underbrace{p(0)\cdot p(0)\cdot p(0)\cdot\ldots}_{13}\cdot\underbrace{p(1)\cdot p(1)\cdot\ldots}_{16}\cdot\underbrace{p(2)\ldots}_{21}$

$L(\theta) = \prod p(x_i;\theta)$

$\ell(a) = \ln L(a) = 13\ln a + 16\ln(\frac{1}{2}-a) + 21\ln\frac{1}{2}$

$\ell'(a) = \frac{13}{a} + \frac{-16}{\frac{1}{2}-a} + 0$

$\ell'(a) = \frac{13}{a} - \frac{16}{\frac{1}{2}-a} = 0$

$\frac{13}{2} - 13a - 16a = 0$

$\frac{13}{2} = 29a$

$\boxed{\hat{a} = \frac{13}{58}}$

② X ... geometrické r. $p_X(x) = q^x(1-q)$ pro $q \in \langle 0,1 \rangle$

$EX = \frac{q}{1-q}$ $DX = \frac{q}{(1-q)^2}$

x_i	0	1	2	3
n_i	29	16	4	1

1. MM:

$EX = \frac{q}{1-q}$

$\bar{x} = \frac{29\cdot 0 + 16\cdot 1 + 4\cdot 2 + 1\cdot 3}{29+16+4+1} = \frac{24}{50}$

$\frac{\hat{q}}{1-\hat{q}} = \frac{24}{50}$

$50\hat{q} = 24 - 24\hat{q}$

$\boxed{\hat{q} = \frac{24}{74}}$

2. MMV:

$L(q) = \underbrace{p(0)\cdot p(0)\cdot\ldots}_{29}\cdot\underbrace{p(1)\cdot p(1)\cdot\ldots}_{16}$

$= p^{29}(0)\cdot p^{16}(1)\cdot p^4(2)\cdot p(3)$

$L(q) = [q^0(1-q)]^{29}\cdot[q^1(1-q)]^{16}\cdot[q^2(1-q)]^4\cdot[q^3(1-q)]^1$

$= q^{24}\cdot(1-q)^{50}$

$\ell(q) = 24\ln q + 50\ln(1-q)$

$\ell'(q) = \frac{24}{q} - \frac{50}{1-q} = 0$

$24 - 24q - 50q = 0$

$24 = 74q$

$\boxed{\hat{q} = \frac{24}{74}}$

(3)

správná šestka S

falešná šestka F $\quad p(6) = 1/2 \quad p(1) = \dots = p(5) = 1/10$

$X = Mix_{(1-w,w)}(S, F)$

x_i	1	2	3	4	5	6
n_i	11	10	14	10	18	37

$\Sigma = 100$

teor.

x	1..5	6
$P[S=x]$	$\frac{1}{6}$	$\frac{1}{6}$
$P[F=x]$	$\frac{1}{10}$	$\frac{1}{2}$
$P[X=x]$	$(1-w)\frac{1}{6} + w\frac{1}{10} = \frac{5-2w}{30}$	$(1-w)\frac{1}{6} + w\frac{1}{2} = \frac{1+2w}{6}$

1. MM:

$EX = 1 \cdot \frac{5-2w}{30} + 2 \cdot \dots + 3 \cdot \dots$

$= (1+2+3+4+5)\frac{5-2w}{30} + 6 \cdot \frac{1+2w}{6}$

$= \frac{5-2w}{2} + 1 + 2w = \frac{5}{2} - w + 1 + 2w$

$= \frac{7}{2} + w$

$\bar{x} = \frac{11 \cdot 1 + 10 \cdot 2 + 14 \cdot 3 + 10 \cdot 4 + 18 \cdot 5 + 37 \cdot 6}{100}$

$= \frac{425}{100}$

$\frac{7}{2} + \hat{w} = \frac{425}{100} = \frac{14}{4}$

$\hat{w} = \frac{14}{4} - \frac{14}{4} = \frac{3}{4} = 75\%$

2. MMV:

$L(w) = \left(\frac{5-2w}{30}\right)^{11+10+14+10+18} \cdot \left(\frac{1+2w}{6}\right)^{37}$

$= \left(\frac{5-2w}{30}\right)^{63} \cdot \left(\frac{1+2w}{6}\right)^{37}$

$\ell(w) = 63 \ln(5-2w) - 63 \ln(30) + 37 \ln(1+2w) - 37 \ln(6)$

$\ell'(w) = \frac{-2 \cdot 63}{5-2w} + \frac{2 \cdot 37}{1+2w} = 0$

$\frac{244 - 400w}{(5-2w)(1+2w)} = 0$

$244 - 400w = 0$

$\hat{w} = \frac{244}{400} = 0.61$

(5) X ... rovnoměrné r. na $\langle -h, h\rangle$; $h > 0$

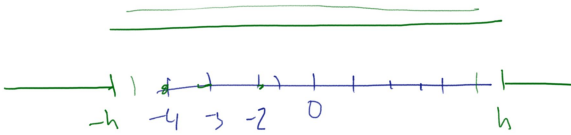
nam. h. $\quad -4 \quad -3 \quad -2 \quad -1{,}5 \quad 0{,}5 \quad 1 \quad 2{,}5 \quad 3$

$\Rightarrow \underline{h \geq 4}$

$$f_X(x) = \begin{cases} \frac{1}{2h} & \text{pro } x \in \langle -h, h\rangle \\ 0 & \text{jinde} \end{cases}$$

$$EX = \int_{-\infty}^{\infty} x f_X(x)\,dx = 0 = \bar{x}$$

$$EX^2 = \int_{-h}^{h} x^2 \frac{1}{2h}\,dx = \frac{1}{2h}\left[\frac{x^3}{3}\right]_{-h}^{h} = \frac{2h^3}{2h\cdot 3} = \frac{h^2}{3} = m_2$$



$$m_2 = \frac{1}{n}\sum x_i^2 = \frac{16 + 9 + 4 + 2{,}25 + 0{,}25 + 1 + \frac{25}{4} + 9}{8} = \frac{47{,}75}{8} \doteq 6$$

$$\frac{\hat{h}^2}{3} = 6 \quad \rightarrow \quad \hat{h} = \pm\sqrt{6\cdot 3} = \sqrt{18} = 3\sqrt{2} \doteq 4{,}4$$

2. MMV:

$$\Lambda(h) = \prod f_X(x_i; h) = f_X(-4)\cdot f_X(-3)\cdot f_X(-2)\cdot \ldots$$

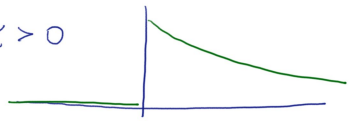
$$= \frac{1}{2h}\cdot\frac{1}{2h}\cdot\frac{1}{2h}\cdot\ldots = \left(\frac{1}{2h}\right)^8$$

$$\Rightarrow h = 4$$

(6) X ... exponenciální r.

$0{,}87 \quad 1{,}28 \quad 0{,}9 \quad 0{,}37 \quad 0{,}25$

$$f_X(x;\tau) = \frac{1}{\tau} e^{-\frac{x}{\tau}} \quad ; \tau > 0$$



$EX = \tau$

$DX = \tau^2$

1. MM:

$$\bar{x} = \frac{3.67}{5} = 0{,}734$$

$$\boxed{0{,}734 = \hat{\tau}}$$

2. MMV: $L(\tau) = \prod f(x_i;\tau)$

$$L(\tau) = f(0{,}87)\cdot f(1.28)\cdot f(0.9)\cdot f(0.37)\cdot f(0.25)$$

$$= \frac{1}{\tau} e^{-\frac{0.87}{\tau}}\cdot\frac{1}{\tau} e^{-\frac{1{,}28}{\tau}}\cdot\ldots\ldots$$

$$= \frac{1}{\tau^5}\cdot e^{-\frac{1}{\tau}\underbrace{(0.87 + 1.28 + \ldots)}_{3.67}} = \frac{1}{\tau^5} e^{-\frac{1}{\tau}\cdot 3.67}$$

$$l(\tau) = \ln\frac{1}{\tau^5} + \left(-\frac{1}{\tau}\right)\cdot 3{,}67$$

$$= -5\ln\tau - \frac{3{,}67}{\tau}$$

$$l'(\tau) = \frac{-5}{\tau} + \frac{3.67}{\tau^2} = 0$$

$$-5\tau^2 + 3.67\tau = 0$$

$$\tau(3.67 - 5\tau) = 0$$

$$\tau \in \left\{0, \frac{3.67}{5}\right\} \quad \rightarrow \quad \hat{\tau} = 0{,}734$$