$X \in \{0,1,2\}$

hodnoty	0	1	2
teor. prav.	a	b	$a+b$
naměřené četnosti	13	16	21

$a, b \in \mathbb{R}$

$a+b+(a+b)=1$

$2a+2b=1$

$b = \frac{1}{2}-a$

$a+b = a+\frac{1}{2}-a = \frac{1}{2}$

1. Metoda momentů (MM)

$EX = \frac{0\cdot a + 1\cdot(\frac{1}{2}-a) + 2\cdot\frac{1}{2}}{}$

$= \frac{3}{2}-a$

$\bar{x} = \frac{13\cdot 0 + 16\cdot 1 + 21\cdot 2}{13+16+21} = \frac{58}{50}$

$\frac{3}{2} - \hat{a} = \frac{58}{50}$

$\hat{a} = \frac{3}{2} - \frac{58}{50} = \frac{17}{50}$

2. Metoda max. věrohodnosti (MMV)

LIKELIHOOD $L(a) = \prod p(x_i; a)$

$= \underbrace{p(0)\cdot p(0)\cdot\ldots}_{13} \underbrace{p(1)p(1)\ldots}_{16} \underbrace{p(2)p(2)\ldots}_{21}$

$= a^{13}\cdot\left(\frac{1}{2}-a\right)^{16}\cdot\left(\frac{1}{2}\right)^{21}$

LOG-LIKELIHOOD $l(a) = \ln L(a) = 13\ln a + 16\ln\left(\frac{1}{2}-a\right) + 21\ln\frac{1}{2}$

$l'(a) = \frac{13}{a} + \frac{-16}{\frac{1}{2}-a} = 0$

$\frac{13}{2} - 13\hat{a} - 16\hat{a} = 0$

$-29a$

$\hat{a} = \frac{13}{58}$

(2) X .. geometrické r.

$p_X(x) = q^x(1-q)$; $q \in (0,1)$

0	1	2	3
29	16	4	1

1. MM:

$EX = \frac{q}{1-q}$

$\bar{x} = \frac{29\cdot 0 + 16\cdot 1 + 4\cdot 2 + 1\cdot 3}{50} = \frac{24}{50}$

$\frac{\hat{q}}{1-\hat{q}} = \frac{24}{50}$

$50\hat{q} = 24 - 24\hat{q}$

$\hat{q} = \frac{24}{74}$

2 MMV: $L(q) = \underbrace{p(0)\,p(0)\ldots}_{29}\underbrace{p(1)\ldots}_{16}$

$= p(0)^{29}\,p(1)^{16}\,p(2)^{4}\,p(3)^{1}$

$= [q^0(1-q)]^{29}\,[q^1(1-q)]^{16}\,[q^2(1-q)]^{4}\,[q^3(1-q)]^{1}$

$= q^{16+8+3}\cdot(1-q)^{29+16+4+1}$

$= q^{27}\cdot(1-q)^{50}$

$l(q) = 27\ln q + 50\ln(1-q)$

$l'(q) = \frac{27}{q} + \frac{-50}{1-q} = 0$

$27 - 27\hat{q} - 50\hat{q} = 0$

$\hat{q} = \frac{27}{77}$

S ... správná kostka

F ... falešná

	1 ... 5	6
$P[S=x]$	$1/6$	$1/6$
$P[F=x]$	$1/10$	$1/2$
$P[X=x]$	$(1-w)\frac{1}{6}+w\frac{1}{10} = \frac{5-2w}{30}$	$(1-w)\frac{1}{6}+w\frac{1}{2} = \frac{1+2w}{6}$

1	2	3	4	5	6
11	10	14	10	18	37

$X = Mix_{(1-w,w)}(S,F)$

$EX = (1-w)ES + wEF$

1. mm:

$EX = \bar{x}$

$EX = (1+2+3+4+5)\frac{5-2w}{30} + 6\cdot\frac{1+2w}{6}$

$= \frac{5}{2} - w + 1 + 2w = w + \frac{7}{2}$

$\bar{x} = \frac{11+102+\dots}{11+10+\dots} = \frac{425}{100} = \frac{17}{4}$

$\hat{w} + \frac{7}{2} = \frac{17}{4}$

$\hat{w} = \frac{17}{4} - \frac{14}{4} = \frac{3}{4}$

2. mm: $L(w) = \underbrace{p(1)\cdot p(1)\cdot\dots}_{11} \cdot \underbrace{p(2)\cdot p(2)\cdot\dots}_{10}$

$= p^{11}(1)\, p^{10}(2)\, p^{14}(3)\, p^{10}(4)\, p^{18}(5)\, p^{37}(6)$

$= \left[\frac{5-2w}{30}\right]^{11+10+14+10+18} \cdot \left[\frac{1+2w}{6}\right]^{37}$

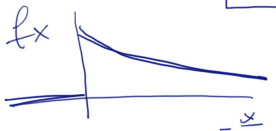
$l(w) = 63\ln(5-2w) - 63\ln 30 + 37\ln(1+2w) - 37\ln 6$

$l'(\hat{w}) = \frac{-2\cdot 63}{5-2w} + \frac{2\cdot 37}{1+2w} = \frac{244-400\hat{w}}{(5-2\hat{w})(1+2\hat{w})} = 0$

$\hat{w} = \frac{244}{400} \doteq 0{,}61$

X ... exponenciální

$0.87 + 1.28 + 0.9 + 0.37 + 0.25 = 3.67$

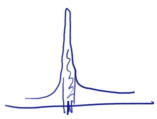


$f_X(x) = \frac{1}{\tau} e^{-\frac{x}{\tau}}$ $\quad \tau > 0$

$EX = \tau$

1. MM

$\bar{x} = \frac{3.67}{5} = 0{,}734 = \hat{\tau}$



2. mm $\quad \Lambda(\tau) = \prod f(x_i;\tau)$

$= f(0.87)\cdot f(1.28)\cdot f(0.9)\cdot f(0.37)\cdot f(0.25)$

$= \frac{1}{\tau} e^{-\frac{0.87}{\tau}} \cdot \frac{1}{\tau} e^{-\frac{1.28}{\tau}} \cdot \dots$

$= \frac{1}{\tau^5} \cdot e^{-\frac{0.87+1.28+0.9+0.37+0.25}{\tau}} = \frac{1}{\tau^5} e^{-\frac{3{,}67}{\tau}}$

$\lambda(\tau) = -5\ln\tau - \frac{3{,}67}{\tau}$

$\lambda'(\tau) = -\frac{5}{\tau} + \frac{3{,}67}{\tau^2} = 0$

$-5\tau^2 + 3{,}67\tau = 0$

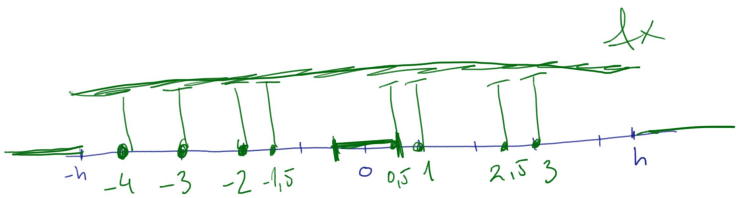
$\tau(3{,}67 - 5\tau) = 0$

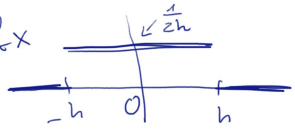
$\tau_1 = 0 \leftarrow \times$

$\tau_2 = \frac{3{,}67}{5}$

$\hat{\tau} = \frac{3{,}67}{5}$

X ... rovnoměrné r. na $\langle -h, h \rangle$





$$f_X(x) = \begin{cases} \frac{1}{2h} & x \in \langle -h, h \rangle \\ 0 & \text{jinak} \end{cases}$$

1. mm $\quad EX = \int_{-h}^{h} x \frac{1}{2h} dx = 0$

$$E(X^2) = \int_{-h}^{h} x^2 \frac{1}{2h} dx = \frac{1}{2h} \frac{1}{3} \left[x^3\right]_{-h}^{h} = \frac{2h^3}{2h \cdot 3} = \frac{h^2}{3}$$

$$m_2 = \frac{1}{n} \sum x_i^2 = \frac{16 + 9 + 4 + 2{,}25 + 0{,}25 + 1 + (2{,}5)^2 + 9}{8} = \frac{47{,}5}{8} \doteq 6$$

$$\frac{\hat{h}^2}{3} = 6 \Rightarrow \hat{h} = \pm\sqrt{18} \doteq 4{,}2$$ ✓

$h \geq 4!$

2. mm $\quad \Lambda(h) = f(-4) \cdot f(-3) \cdot f(-2) \cdots$

$$= \frac{1}{2h} \cdot \frac{1}{2h} \cdot \frac{1}{2h} \cdots = \left(\frac{1}{2h}\right)^8$$

$h = 4$