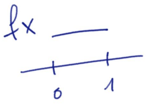
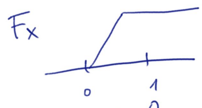
2.DÚ: rychlost řešení úloh ... $X$ ... rovnoměrné r. na $\langle 0,1\rangle$

$[\text{úlohy}/h]$

5 úloh

stř. hodnota času $T$





$$F_X(x) = \begin{cases} 0 & (-\infty,0) \\ x & \langle 0,1) \\ 1 & \langle 1,\infty) \end{cases}$$

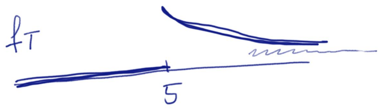
$$\boxed{T = \frac{5}{X}} \Leftrightarrow X = \frac{5}{T}$$

$x \in \langle 5,\infty)$:

$$F_T(x) = P[T \le x] = P\left[\frac{5}{X} \le x\right] = P\left[\frac{X}{5} \ge \frac{1}{x}\right] = P\left[X \ge \frac{5}{x}\right]$$

$$= 1 - P\left[X < \frac{5}{x}\right] = 1 - P\left[X \le \frac{5}{x}\right] = 1 - F_X\left(\frac{5}{x}\right)$$

$$F_T(x) = \begin{cases} 0 & (-\infty,5) \\ 1 - \frac{5}{x} & \langle 5,\infty) \end{cases}$$



$$f_T(x) = \begin{cases} 0 & (-\infty,5) \\ +\frac{5}{x^2} & \langle 5,\infty) \end{cases}$$

$$ET = \int_{-\infty}^{\infty} x \cdot f_T(x)\,dx = \int_5^{\infty} x \frac{5}{x^2}\,dx = \int_5^{\infty} \frac{5}{x}\,dx = 5\left[\ln|x|\right]_5^{\infty}$$

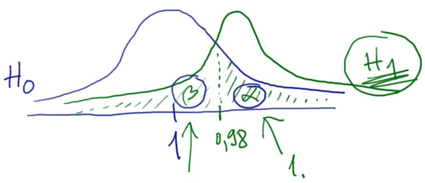
$$= 5(\underbrace{\ln\infty}_{\infty} - \ln 5) = \infty$$

(2) $n = 16$

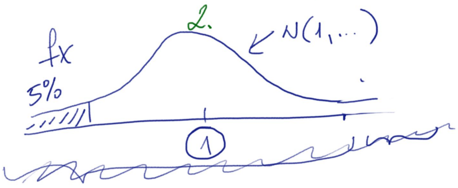
$\bar{X} = 0{,}98\ kg$

$s_x = 0{,}04\ kg$

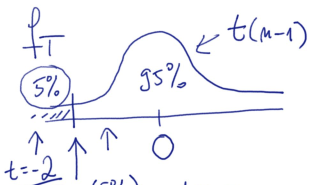
X… hmotnost balíčku … $N(\mu, \sigma^2)$




$$\boxed{T = \frac{\bar{X} - c}{\frac{S_X}{\sqrt{n}}} \sim t(n-1)}$$




| | |
|---|---|
| $H_0: \mu \geq 1$ | $\mu = 1$ |
| $H_1: \mu < 1$ | $\mu \neq 1$ |




$q_{t(n-1)}(5\%) = -1{,}75$

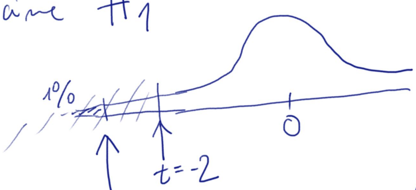
$$t = \frac{\bar{x} - c}{\frac{s_x}{\sqrt{n}}}$$

$$t = \frac{0{,}98 - 1}{\frac{0{,}04}{\sqrt{16}}} = \frac{-0{,}02}{\frac{0{,}04}{4}} = -\frac{0{,}02}{0{,}01} = -2$$

$q_{t(15)}(0{,}05) = -q_{t(15)}(0{,}95) = -1{,}75$

zamítáme H0

$\Rightarrow$ ~~př~~ přijímáme H1




$q_{t(15)}(0{,}01) = -q_{t(15)}(0{,}99) = -2{,}6 \Rightarrow$ nezamítáme H0


(4)

| počet bal. | X | Y |
|---|---|---|
| | $n = 13$ | $n = 9$ |
| | $\bar{x} = 1{,}02\ kg$ | $\bar{y} = 0{,}83\ kg$ |
| | $s_x^2 = 0{,}04\ kg^2$ | $s_y^2 = 0{,}09\ kg$ |
| | $s_x = 0{,}2\ kg$ | $s_y = 0{,}3\ kg$ |

④

| | X | Y |
|---|---|---|
| počet bal. | $n=13$ | $n=9$ |
| | $\bar{x}=1{,}02\,kg$ | $\bar{y}=0{,}83\,kg$ |
| | $s_x^2=0{,}04\,kg^2$ | $s_y^2=0{,}09\,kg$ |
| | $s_x=0{,}2\,kg$ | $s_y=0{,}3\,kg$ |

test rovnosti rozptylů:

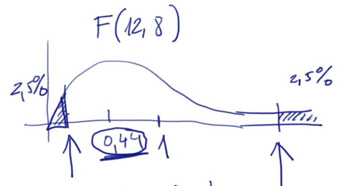
$H_0: \sigma_X^2=\sigma_Y^2$

$H_1: \sigma_X^2\neq\sigma_Y^2$

$$T=\frac{S_X^2}{S_Y^2}\sim F(m-1,n-1)=F(12,8)$$

$$t=\frac{0{,}04}{0{,}09}\doteq 0{,}44\ldots$$

H₀ nezamítáme




$q_{F(12,8)}(0{,}025)=\dfrac{1}{q_{F(8,12)}(0{,}975)}=\dfrac{1}{3{,}51}=0{,}2849$

$q_{F(12,8)}(0{,}975)=4{,}2$

test stř. hodnot:

$H_0: \mu_X=\mu_Y$ $\quad H_1: \mu_X\neq\mu_Y$

$H_0: \mu_X\geq\mu_Y$ $\quad H_1: \mu_X<\mu_Y$

$H_0: \boxed{\mu_X\leq\mu_Y}$ $\quad H_1: \mu_X>\mu_Y$

$$T=\frac{\bar{X}-\bar{Y}}{\ldots}\leq 0$$

$$\frac{\bar{X}-c}{\frac{S}{\sqrt{n}}}\sim t(n-1)$$

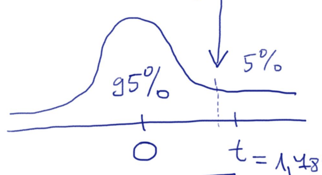
$$T=\frac{\bar{X}-\bar{Y}}{S\sqrt{\frac{1}{m}+\frac{1}{n}}}\sim t\big(\overset{12}{(m-1)}+\overset{8}{(n-1)}\big)$$

$$S^2=\frac{\overset{12}{(m-1)}S_X^2+\overset{8}{(n-1)}S_Y^2}{m-1+n-1}$$

$$s^2=\frac{12\cdot 0{,}04+8\cdot 0{,}09}{12+8}=0{,}06$$

$s=\sqrt{0{,}06}$

$q_{t(20)}(0{,}95)=1{,}72$




$$t=\frac{\bar{x}-\bar{y}}{\sqrt{0{,}06}\sqrt{\frac{1}{13}+\frac{1}{9}}}=\frac{\overbrace{1{,}02-0{,}83}^{+0{,}19}}{\sqrt{0{,}06}\sqrt{\frac{1}{13}+\frac{1}{9}}}\doteq\frac{+0{,}19}{0{,}1062}\doteq 1{,}4891$$

H₀ zamítáme

⇒ přijímáme H₁

$\mu_X>\mu_Y$

⑤ párový test

| | $x_i$ | | $y_i$ | $\delta_i$ | $\delta_i-\bar{\delta}$ | $(\delta_i-\bar{\delta})^2$ |
|---|---|---|---|---|---|---|
| 1 | 26,5 | > | 24,0 | 2,5 | 1,5 | 2,25 |
| 2 | 25,0 | > | 23,5 | 1,5 | 0,5 | 0,25 |
| 3 | 24,3 | < | 24,4 | −0,1 | −1,1 | 1,21 |
| 4 | 26,3 | > | 25,0 | 1,3 | 0,3 | 0,09 |
| 5 | 22,0 | < | 22,2 | −0,2 | −1,2 | 1,44 |
| | | | | $\Sigma = 5{,}0$ | | $\Sigma = 5{,}24$ |

$\boxed{\bar{\delta} = \frac{1}{n}\Sigma = 1{,}0}$

$s_\delta^2 = \frac{1}{n-1}\Sigma \ldots = 1{,}31$

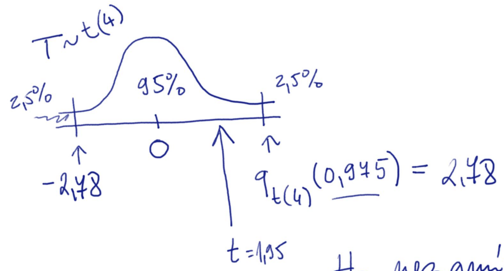
$H_0: X = Y \qquad \mu_x = \mu_y$

$H_1: X \neq Y \qquad \mu_x \neq \mu_y$

$T = \frac{\bar{\Delta}}{s_\Delta}\sqrt{n} \sim t(n-1)$

$\Delta = X - Y$

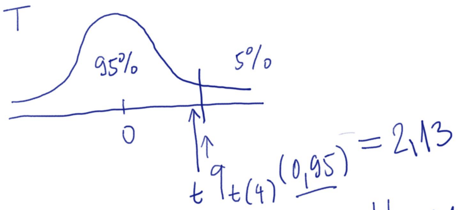
$t = \frac{1{,}0}{\sqrt{1{,}31}}\sqrt{5} \doteq 1{,}9536$




$H_0$ nezamítáme

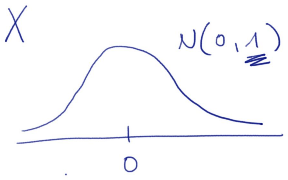
$H_0: \mu_x \leq \mu_y \Rightarrow T \leq 0$

$H_1: \mu_x > \mu_y$




$H_0$ nezamítáme

3




$n = 10$

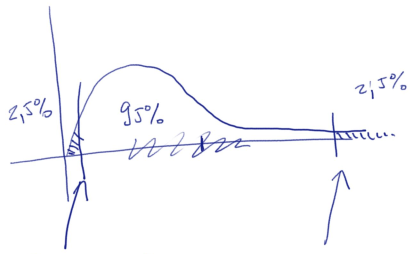
$\bar{x} = 0{,}3483$

$s_x^2 = 0{,}9082$

$H_0: \sigma^2 = 1 = c$

$H_1: \sigma^2 \neq 1$

$$T = (n-1)\frac{s_x^2}{c} \sim \chi^2(n-1)$$




$$t = 9 \cdot \frac{0{,}9082}{1} \doteq 8{,}174$$

$q_{\chi^2(9)}(0{,}025) = 2{,}7$

$q_{\chi^2(9)}(0{,}975) = 19{,}02$

$H_0$ nezamítáme