2DÚ: rychlost řešení [úlohy/hod.] ... $\underline{X}$ ... rovnoměrné R na $\langle 0,1\rangle$

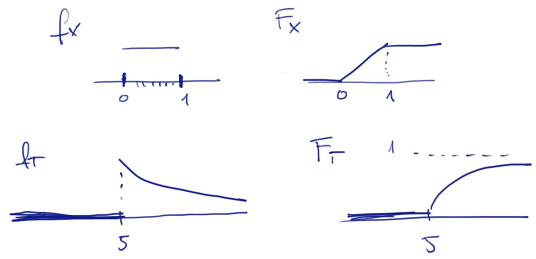
T .. čas pro vyřešení 5 úloh

$ET = ?$

$$F_X(x) = \begin{cases} 0 & (-\infty,0) \\ x & \langle 0,1) \\ 1 & \langle 1,\infty) \end{cases}$$

$T \in \langle 5,\infty)$

$$X = \frac{5}{T} \iff \boxed{T = \frac{5}{X}}$$



$$F_T(x) = P[T \le x] = P\left[\frac{5}{X} \le x\right] = P\left[\frac{X}{5} \ge \frac{1}{x}\right] = P\left[X \ge \frac{5}{x}\right]$$

$$= 1 - P\left[X < \frac{5}{x}\right] = 1 - P\left[X \le \frac{5}{x}\right] = 1 - F_X\left(\frac{5}{x}\right)$$

$x \in \langle 0,1) \Rightarrow \frac{5}{x} \in (5,\infty)$

$$F_T(x) = \begin{cases} 0 & x \in (-\infty,5) \\ 1 - \frac{5}{x} & x \in \langle 5,\infty) \end{cases}$$

$$f_T(x) = \begin{cases} 0 & (-\infty,5) \\ \frac{5}{x^2} & \langle 5,\infty) \end{cases}$$

$$ET = \int_{-\infty}^{\infty} x \cdot f_T(x)\,dx = \int_5^{\infty} x \cdot \frac{5}{x^2}\,dx = \int_5^{\infty} \frac{5}{x}\,dx$$

$$= 5\left[\ln|x|\right]_5^{\infty} = 5(\underbrace{\ln\infty}_{\infty} - \ln 5)$$

$$= \underline{\infty}$$

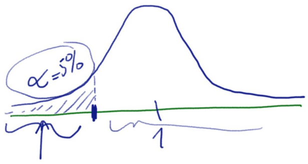
(2)

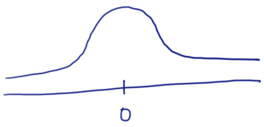
$n = 16$
$\bar{x} = 0{,}98$ kg
$s_x = 0{,}04$ kg

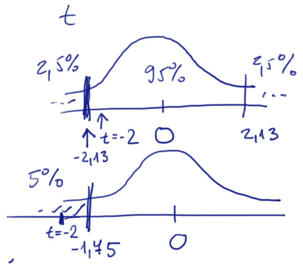
X

$T = \dfrac{\bar{X} - c}{\sqrt{\frac{S_X^2}{n}}} \sim t(n-1)$

$\alpha = 5\%$







a) $H_0: \mu = 1\,\text{kg}$ .... $T = 0$; $H_1: \mu \neq 1\,\text{kg}$ ... $T \neq 0$

b) $H_0: \mu \geq 1\,\text{kg}$ ... $T \geq 0$; $H_1: \mu < 1\,\text{kg}$ ... $T < 0$

a) $t = \dfrac{\bar{x} - c}{s_x}\sqrt{n} = \dfrac{0{,}98 - 1}{0{,}04}\sqrt{16} = \dfrac{-0{,}02}{0{,}04}\cdot 4 = -2$ $\Rightarrow$ H0 nezamítáme

$q_{t(15)}(0{,}025) = -q_{t(15)}(0{,}975) = -2{,}13$

b) $t = -2$ $\Rightarrow$ H0 zamítáme

$q_{t(15)}(0{,}05) = -q_{t(15)}(0{,}95) = -1{,}75$ $\Rightarrow$ $\mu < 1\,\text{kg}$

(3)

$X \sim N(0, 1)$, σ^2

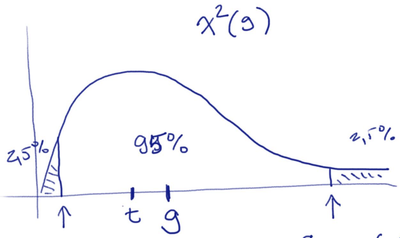
$\alpha = 5\%$

$n = 10$
$\bar{x} = 0{,}3483$
$s_X^2 = 0{,}9082$

$H_0: \sigma^2 = 1$
$H_1: \sigma^2 \neq 1$

$T = (n-1)\dfrac{S_X^2}{c} \sim \chi^2(n-1)$

$t = 9 \cdot \dfrac{0{,}9082}{1} \doteq 8{,}174$



$q_{\chi^2(9)}(0{,}025) = 2{,}7$

$q_{\chi^2(9)}(0{,}975) = 19{,}02$

H0 nezamítáme
$\sigma^2 \neq 1$ jsme nedokázali

$$X \qquad\qquad Y \qquad\qquad \alpha = 5\%$$

$m = 13 \qquad n = 9$

$\bar{x} = 1{,}02 \text{ kg} \qquad \bar{y} = 0{,}83 \text{ kg}$

$s_X^2 = 0{,}04 \text{ kg}^2 \qquad s_Y^2 = 0{,}09 \text{ kg}^2$

$s_X = 0{,}2 \text{ kg} \qquad s_Y = 0{,}3 \text{ kg}$

porovnání rozptylů:

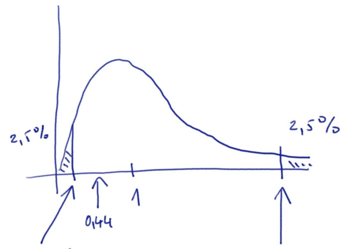
$H_0 : \sigma_X^2 = \sigma_Y^2$

$H_1 : \sigma_X^2 \neq \sigma_Y^2$

$$T = \frac{S_X^2}{S_Y^2} \sim F(\overset{12}{m-1}, \overset{8}{n-1})$$

$$t = \frac{0{,}04}{0{,}09} \doteq 0{,}44\ldots$$

Ho nezamítáme



$q_{F(12,8)}(0{,}025) = \dfrac{1}{q_{F(8,12)}(0{,}975)} = \dfrac{1}{3{,}51} \doteq 0{,}2849$

$q_{F(12,8)}(0{,}975) = 4{,}2$

porovnání stř. h.

$$T = \frac{\bar{X} - \bar{Y}}{S\sqrt{\frac{1}{m} + \frac{1}{n}}} \sim t(\underset{12}{m-1} + \underset{8}{n-1}) \sim t(20)$$

$\alpha = 5\%$

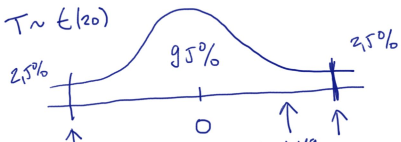
$$S^2 = \frac{(m-1)S_X^2 + (n-1)S_Y^2}{m-1 + n-1}$$

$$s^2 = \frac{12 \cdot 0{,}04 + 8 \cdot 0{,}09}{12+8} = 0{,}06 \qquad s = \sqrt{0{,}06}$$

$$t = \frac{\bar{x} - \bar{y}}{s\sqrt{\frac{1}{m} + \frac{1}{n}}} = \frac{1{,}02 - 0{,}83}{\sqrt{0{,}06}\sqrt{\frac{1}{13} + \frac{1}{9}}} \doteq \frac{0{,}19}{0{,}1062} \doteq 1{,}7891$$

a) $H_0 : \mu_X = \mu_Y \ldots T = 0$; $H_1 : \mu_X \neq \mu_Y$

Ho nezamítáme

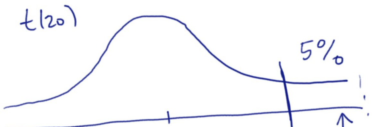


$q_{t(20)}(0{,}025) = -2{,}09 \qquad t = 1{,}79 \qquad q_{t(20)}(0{,}975) = 2{,}09$

b) $H_0 : \mu_X \leq \mu_Y \ldots T \leq 0$; $H_1 \; \mu_X > \mu_Y$

Ho … zamítáme

$\Rightarrow \mu_X > \mu_Y$



$t = 1{,}79$

$q_{t(20)}(0{,}95) = 1{,}72$

5 párový test

$\alpha = 5\%$

i	x_i >	y_i	δ_i	$\delta_i - \bar{\delta}$	$(\delta_i - \bar{\delta})$
1	26,5	24,0	2,5	1,5	2,25
2	25,0	23,5	1,5	0,5	0,25
3	24,3	24,4	-0,1	-1,1	1,21
4	26,3	25,0	1,3	0,3	0,09
5	22,0	22,2	-0,2	-1,2	1,44
		součet	5,0		5,24

$\bar{\delta} = 1,0$

$$s_\Delta^2 = \frac{1}{n-1} \Sigma(\ldots)^2 = \frac{5,24}{4} = 1,31$$

$$s_\Delta = \sqrt{1,31}$$

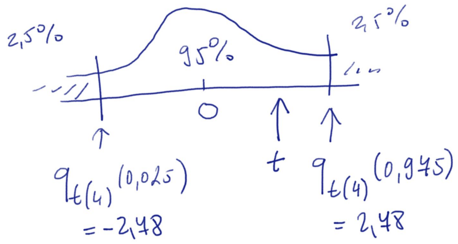
$$T = \frac{\bar{\Delta}}{s_\Delta}\sqrt{n} \sim t(\underbrace{n-1}_{4})$$

$$\underline{\Delta = X - Y}$$

$$t = \frac{1,0}{\sqrt{1,31}}\sqrt{5} \doteq \underline{\underline{1,9536}}$$

a) $H_0: \mu_X = \mu_Y \;\ldots\; T = 0$; $H_1: \underline{\underline{\mu_X \neq \mu_Y}}$

H0 nezamítáme



b) $H_0: \mu_X \leq \mu_Y \;\ldots\; T \leq 0$; $H_1: \underline{\underline{\mu_X > \mu_Y}}$

H0 nezamítáme