(1)

hodnota	0	1	2	3	4	5	6	
(naměřená) četnost n_i	57	24	10	6	2	0	1	... $\Sigma = 100$

H_0 ... náh. hodnoty pochází z geometrického r. s $q = 0{,}5$

H_1 .. ?(....)

$\alpha = 5\%$

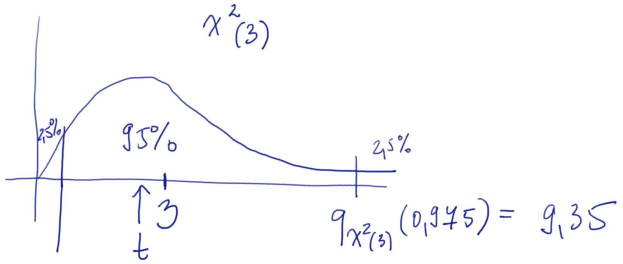
$$T = \sum_{i=1}^{k} \frac{(n_i - np_i)^2}{np_i} \sim \chi^2(k-1)$$

$k = 4$

$$P[X = x] = q^x (1-q) = \left(\frac{1}{2}\right)^x \cdot \frac{1}{2} = \frac{1}{2^{x+1}}$$

	0	1	2	3 a více	
n_i	57	24	10	9	
p_i	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{8}$	
np_i	50	25	12,5	12,5	
$n_i - np_i$	7	-1	-2,5	-3,5	
$(n_i - np_i)^2$	49	1	6,25	12,25	
$\frac{(n_i - np_i)^2}{np_i}$	0,98	0,04	0,5	0,98	... $\Sigma = 2{,}5 = t \sim \chi^2(3)$

H_0 nezamítáme



$q_{\chi^2(3)}(0{,}975) = 9{,}35$

$q_{\chi^2(3)}(0{,}025) = 0{,}216$

(2)

1	2	3	4	5	6
7	5	5	12	6	15

$n = 50$

H_0 ... náh. hod. pochází z rovnoměrného r. $\Rightarrow$ kostka je falešná

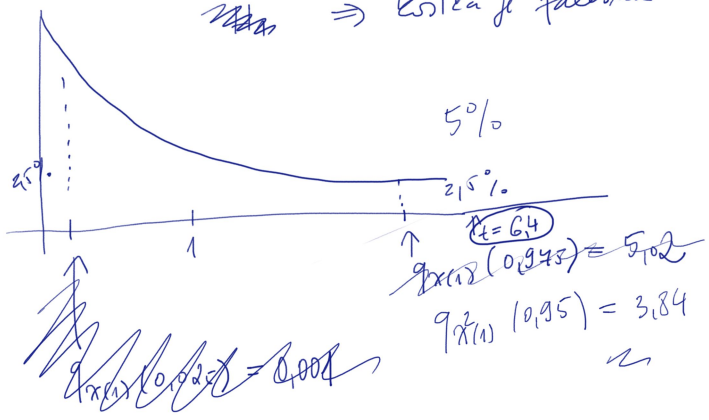
H_1 ... ... ne... ... ...

$$T = \sum \frac{(n_i - np_i)^2}{np_i}$$

	1 až 5	6
n_i	35	15
p_i	$5/6$	$1/6$
np_i	$\frac{5}{6} \cdot 50 = \frac{250}{6}$	$\frac{1}{6} \cdot 50 = \frac{50}{6}$
$n_i - np_i$	$-\frac{20}{6}$	$\frac{20}{6}$
$(n_i - np_i)^2$	$\frac{400}{36}$	$\frac{400}{36}$
$\frac{(n_i - np_i)^2}{np_i}$	$\frac{400}{375}$	$\frac{400}{75}$

$$+ \quad = \quad t = 6{,}4$$

H_0 zamítáme $\Rightarrow$ kostka je falešná



$\chi^2_{(1)}(0{,}975) = 5{,}02$

$\chi^2_{(1)}(0{,}95) = 3{,}84$

$\chi^2_{(1)}(0{,}025) = 0{,}001$

$\alpha = 1\%$ ... $\chi^2_{(1)}(0{,}005) = 0{,}000039$ } H_0 nezamítáme

$\chi^2_{(1)}(0{,}99) = 6{,}64$

(3)

	0	1	2	3	4	
1. sl. m_i	12	8	4	16	20	$m = 60$
2. sl. n_i	8	4	1	9	15	$n = 40$

H_0 ... obě měření pocházejí ze stejného rozdělení

H_1 .. ¬(...)

$\alpha = 5\%$

$$T = \sum_{i=1}^{k} \frac{(m_i - mp_i)^2}{mp_i} + \sum_{i=1}^{k} \frac{(n_i - np_i)^2}{np_i}$$

$p_i = \frac{m_i + n_i}{m + n}$ ... testujeme na $\chi^2(k-1)$

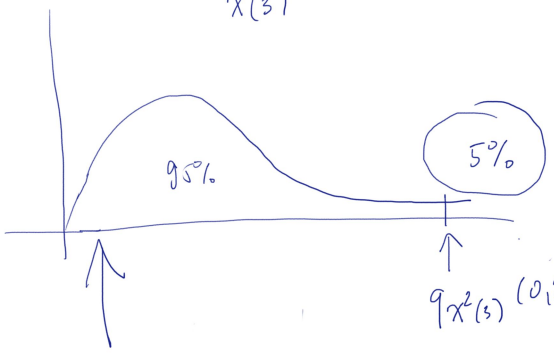
	0	1,2	3	4	Σ
m_i	12	12	16	20	$m = 60$
n_i	8	8	9	15	$n = 40$
$m_i + n_i$	20	20	25	35	
p_i	0,2	0,2	0,25	0,35	
$m \cdot p_i$	12	12	15	21	
$n \cdot p_i$	8	8	10	14	
$m_i - mp_i$	0	0	1	-1	
$n_i - np_i$	0	0	-1	1	
$(m_i - mp_i)^2$	0	0	1	1	
$(n_i - np_i)^2$	0	0	1	1	
$\frac{(m_i - mp_i)^2}{mp_i}$	0	0	$\frac{1}{15}$	$\frac{1}{21}$	
$\frac{(n_i - np_i)^2}{np_i}$	0	0	$\frac{1}{10}$	$\frac{1}{14}$	

$t = \frac{1}{15} + \frac{1}{21} + \frac{1}{10} + \frac{1}{14}$

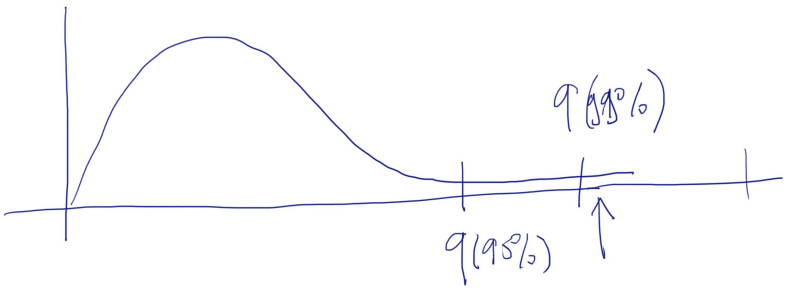
$= 0{,}2857$

$\alpha = 5\%$

$\chi^2(3)$



$q_{\chi^2(3)}(0{,}95) = 7{,}81$ ... H_0 nezamítáme



(4)

oči / vlasy	modré	šedé	hnědé
tmavé	10	10	40
světlé	20	10	10

H_0 ... jevy jsou nezávislé ... $T = 0$

H_1 ... nejsou

$$T = \sum_{i=1}^{k}\sum_{j=1}^{m} \frac{(n_{ij} - n p_i q_j)^2}{n p_i q_j}$$

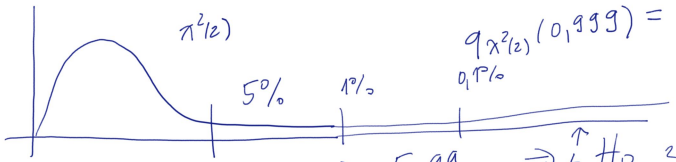
$\alpha = 5\%$

$$p_i = \frac{1}{n}\sum_{j=1}^{m} n_{ij} \qquad q_j = \frac{1}{n}\sum_{i=1}^{n} n_{ij}$$

i \ j	1	2	3	
1	$\frac{(10 - n\cdot p_1 q_1)^2}{n p_1 q_1} = \frac{(10-100\cdot 0{,}3\cdot 0{,}6)^2}{100\cdot 0{,}3\cdot 0{,}6}$	$\frac{(10-100\cdot 0{,}2\cdot 0{,}6)^2}{100\cdot 0{,}2\cdot 0{,}6}$	$\frac{(40-100\cdot 0{,}5\cdot 0{,}6)^2}{100\cdot 0{,}5\cdot 0{,}6}$	60, $p_1 = 0{,}6$
2	$\frac{(20-12)^2}{12}$	$\frac{(10-8)^2}{8}$	$\frac{(10-20)^2}{20}$	40, $p_2 = 0{,}4$
	30	20	50	$n = 100$
	$q_1 = 0{,}3$	$q_2 = 0{,}2$	$q_3 = 0{,}5$	

$$t = \frac{(10-18)^2}{18} + \frac{(10-12)^2}{12} + \frac{(40-30)^2}{30} + \frac{(20-12)^2}{12} + \frac{(10-8)^2}{8} + \frac{(10-20)^2}{20} \sim \chi^2((k-1)(m-1)) = \chi^2(2)$$

$$= \frac{64}{18} + \frac{4}{12} + \frac{100}{30} + \frac{64}{12} + \frac{4}{8} + \frac{100}{20} = 18{,}056$$



$q_{\chi^2(2)}(0{,}999) = 10{,}83$

$q_{\chi^2(2)}(0{,}95) = 5{,}99$ $\Rightarrow t$ H_0 zamítáme

$\Rightarrow$ jevy nejsou nezávislé

$\alpha = 0{,}1\% \Rightarrow H_0$ zamítáme

(5)

výška	205	155	185	155
váha	95	55	65	85

H_0 ... korelace je nulová

H_1 ... jsou spolu korelované

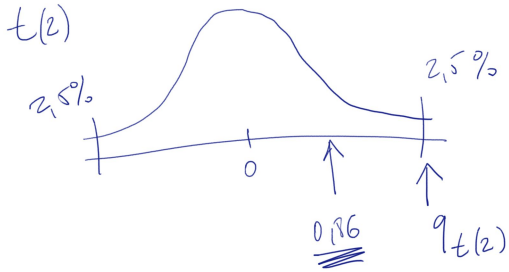
$$T = \frac{R_{xy}\cdot\sqrt{n-2}}{\sqrt{1-R_{xy}^2}} \sim t(n-2)$$

$$R_{xy} = \frac{\Sigma (x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\Sigma(x_i-\bar{x})^2 \cdot \Sigma(y_i-\bar{y})^2}}$$

	1	2	3	4	
x_i	205	155	185	155	$\bar{x} = 175$
y_i	95	55	65	85	$\bar{y} = 75$
$x_i-\bar{x}$	30	-20	10	-20	
$y_i-\bar{y}$	20	-20	-10	10	Σ
$(x_i-\bar{x})(y_i-\bar{y})$	600	400	-100	-200	700
$(x_i-\bar{x})^2$	900	400	100	400	1800
$(y_i-\bar{y})^2$	400	400	100	100	1000

$$r_{xy} = \frac{700}{\sqrt{1800\cdot 1000}} \doteq 0{,}5217$$

$$t = \frac{0{,}5217\sqrt{2}}{\sqrt{1-0{,}5217^2}} \doteq 0{,}8648$$



H_0 nezamítáme