(1)

i	0	1	2	3	4	5	6
n_i	57	24	10	6	2	0	1
np_i	50	25	12,5	6,25	3,125	1,...	...

$n = 100$

H_0 ... naměřené hodnoty pocházejí z geometrického r. s $q = 1/2$

H_1 ... " " nepocházejí

$\alpha = 5\%$

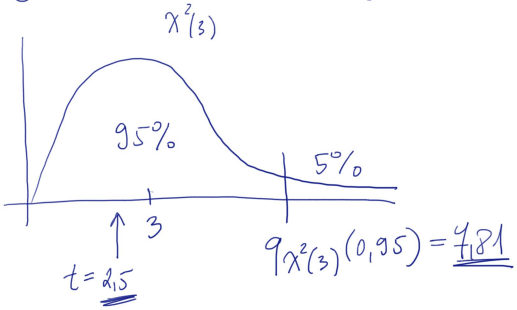
POČET TŘÍD

$$T = \sum \frac{(n_i - np_i)^2}{np_i} \sim \chi^2(k-1)$$

$$P[X = x] = q^x (1-q)$$

	0	1	2	3 a víc
n_i	57	24	10	9
p_i	1/2	1/4	1/8	1/8
np_i	50	25	12,5	12,5
$n_i - np_i$	7	-1	-2,5	-3,5
$(n_i - np_i)^2$	49	1	6,25	12,25
$\frac{(n_i - np_i)^2}{np_i}$	$\frac{49}{50}$	$+\frac{1}{25}$	$+\frac{6,25}{12,5}$	$+\frac{12,25}{12,5}$

$n = 100$ $\alpha = 5\%$



$= t = 2,5 \sim \chi^2(3)$ H_0 nezamítáme

(2)

	1	2	3	4	5	6
	7	5	5	12	6	25
	10	10	10			

... $n = 60$

H_0 ... nam. h. pochází z rovnoměrného r.) $\Rightarrow$ kostka je falešná

H_1 ... ¬(...

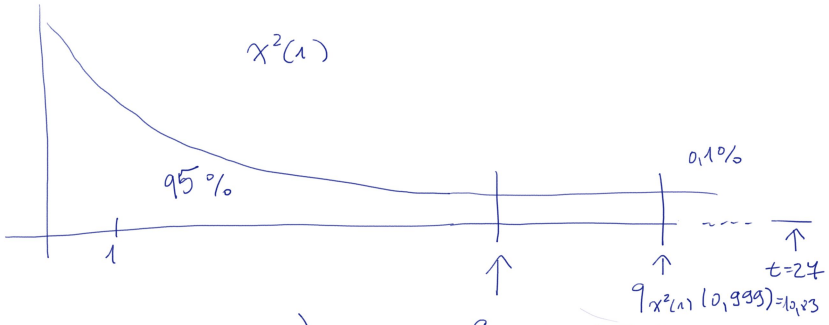
$\alpha = 5\%$

$$T = \sum \frac{(n_i - np_i)^2}{np_i} \sim \chi^2(n-1)$$

	1 až 5	6
n_i	35	25
p_i	5/6	1/6
$n \cdot p_i$	50	10
$n_i - np_i$	-15	15
$(n_i - np_i)^2$	225	225
$\frac{(n_i - np_i)^2}{np_i}$	$\frac{225}{50}$	$+\frac{225}{10}$

$n = 60$

$= t = 24 \sim \chi^2(1)$



H_0 zamítáme (i pro $\alpha = 0,1\%$)

$\Rightarrow$ kostka je falešná

③

	0	1	2	3	4	
m_i	12	8	4	16	20	$m = 60$
n_i	8	7	1	9	15	$n = 40$

H_0 ... obě měření pocházejí ze stejného r. .... $T = 0$

H_1 ... ?(...)

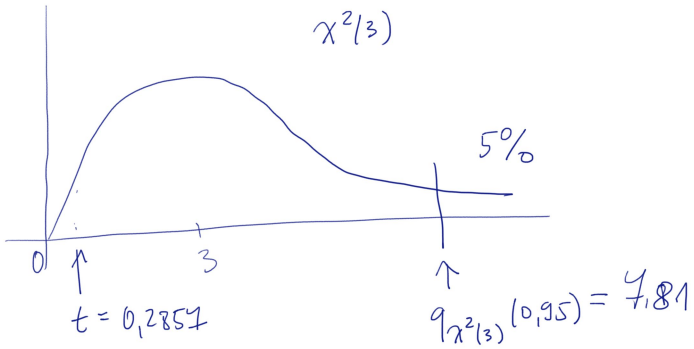
$\alpha = 5\%$

$$T = \sum_{i=1}^{k} \frac{(m_i - m \cdot p_i)^2}{m \cdot p_i} + \sum_{i=1}^{k} \frac{(n_i - n \cdot p_i)^2}{n \cdot p_i} \qquad k-1 + k-1$$

$$p_i = \frac{m_i + n_i}{m + n} \quad \ldots \quad t \sim \chi^2(k-1)$$

	0	1,2	3	4	
m_i	12	12	16	20	$m = 60$
n_i	8	8	9	15	$n = 40$
$m_i + n_i$	20	20	25	35	
p_i	0,2	0,2	0,25	0,35	
mp_i	12	12	15	21	
np_i	8	8	10	14	
$m_i - mp_i$	0	0	1	−1	
$(m_i - mp_i)^2$	0	0	1	1	
$\frac{(m_i - mp_i)^2}{mp_i}$	0	0	$\frac{1}{15}$	$\frac{1}{21}$	
$n_i - np_i$	0	0	−1	1	
$(n_i - np_i)^2$	0	0	1	1	
$\frac{(n_i - np_i)^2}{np_i}$	0	0	$\frac{1}{10}$	$\frac{1}{14}$	

$$t = \frac{1}{15} + \frac{1}{21} + \frac{1}{10} + \frac{1}{14} = 0{,}2857 \sim \chi^2(3)$$



H_0 nezamítáme

(4)

i \ j	1 modré	2 šedé	3 hnědé
1 tmavé	10	$n_{12}=10$	40
2 světlé	20	10	10

H_0 … jevy jsou nezávislé … $T=0$

H_1 … nejsou … $T>0$

$\alpha = 5\%$

$$T=\sum_{i=1}^{k}\sum_{j=1}^{m}\frac{(n_{ij}-n\cdot p_i\cdot q_j)^2}{n\cdot p_i\cdot q_j}$$

$$p_i=\frac{1}{n}\sum_{j=1}^{m} n_{ij} \qquad q_j=\frac{1}{n}\sum_{i=1}^{k} n_{ij}$$

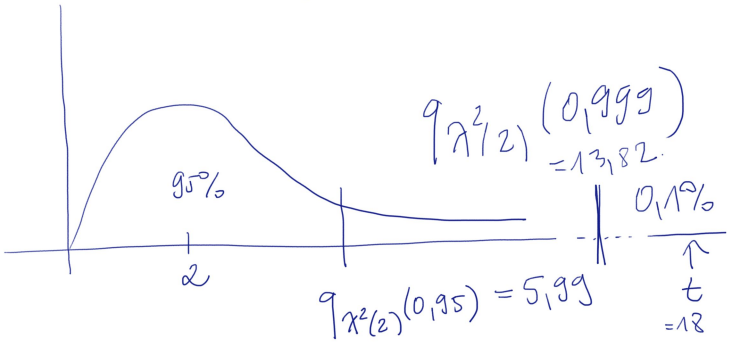
i \ j	1	2	3	
1	10	10	40	$60=\sum_{j=1}^{3} n_{1j}$
2	20	10	10	40
	30	20	50	$n=100$

i \ j	1	2	3	p_i
1	0,18	0,12	0,30	0,6
2	0,12	0,08	0,20	0,4
	0,3	0,2	0,5	1
q_j				

i \ j	1	2	3	
1	18	12	30	60
2	12	8	20	40
	30	20	50	100

$$t=\frac{(10-18)^2}{18}+\frac{(10-12)^2}{12}+\frac{(40-30)^2}{30}+\frac{(20-12)^2}{12}+\frac{(10-8)^2}{8}+\frac{(10-20)^2}{20}=18{,}056$$

$$\chi^2\left((k-1)(m-1)\right)=\chi^2(2) \qquad (k-1=2-1,\ m-1=3-1)$$



H_0 zamítáme ⇒ jevy jsou závislé

$\alpha = 0{,}01\%$ ⇒ H_0 zamítáme

(5)

H_0 ... korelace je nulová $\Leftrightarrow T = 0$

H_1 ... její existuje korelace $\Leftrightarrow T \neq 0$

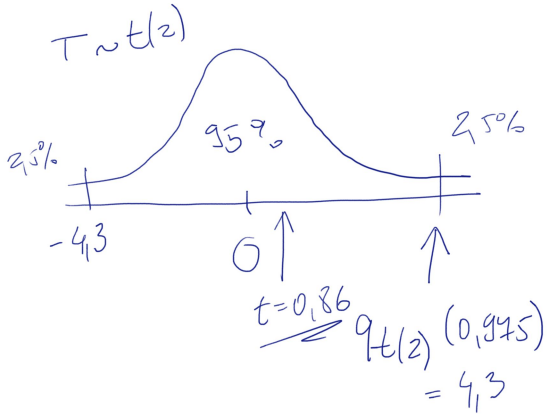
$\alpha = 5\%$

$$T = \frac{R_{XY}\sqrt{n-2}}{\sqrt{1-R_{XY}^2}} \sim t(n-2) \qquad R_{XY} = \frac{\mathcal{E}(x_i-\bar{x})(y_i-\bar{y})}{\sqrt{\mathcal{E}(x_i-\bar{x})^2 \cdot \mathcal{E}(y_i-\bar{y})^2}}$$

i	1	2	3	4	
výška x_i	205	155	185	155	$\bar{x} = 175$
váha y_i	95	55	65	85	$\bar{y} = 75$
$x_i - \bar{x}$	30	-20	10	-20	
$y_i - \bar{y}$	20	-20	-10	10	
$(x_i-\bar{x})(y_i-\bar{y})$	600	400	-100	-200	$\mathcal{E} = 700$
$(x_i-\bar{x})^2$	900	400	100	400	$\mathcal{E} = 1800$
$(y_i-\bar{y})^2$	400	400	100	100	$\mathcal{E} = 1000$

$$r_{XY} = \frac{700}{\sqrt{1800 \cdot 1000}} \doteq 0{,}5217$$

$$t = \frac{0{,}5217\sqrt{2}}{\sqrt{1-0{,}5217^2}} \doteq 0{,}8648$$



H_0 nezamítáme