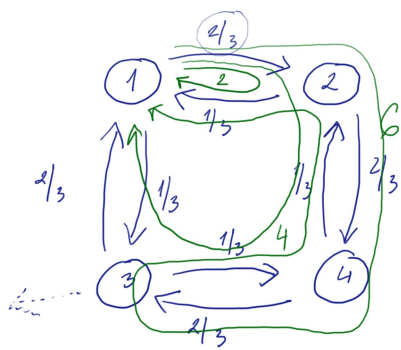


1.

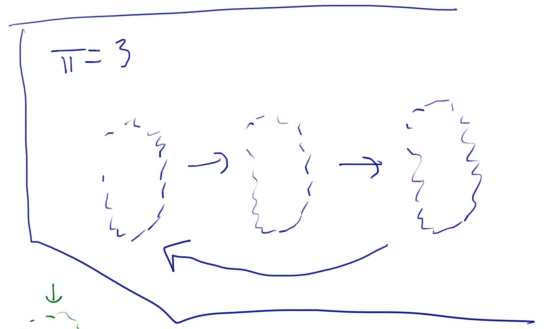


a) matrice přechodu

P:

	1	2	3	4
1	0	2/3	1/3	0
2	1/3	0	0	2/3
3	2/3	0	0	1/3
4	0	1/3	2/3	0

$$p_{ij} = p(i \rightarrow j)$$



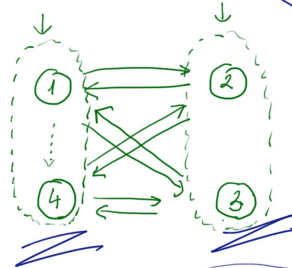
$$\vec{p}_1 = (1, 0, 0, 0)$$

$$\vec{p}_2 = \vec{p}_1 \cdot P$$

$$\vec{p}_2^T = P^T \cdot \vec{p}_1^T$$

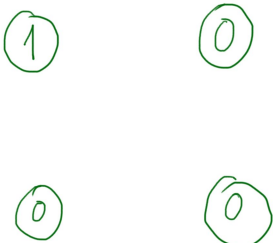
b) všechny stavy jsou trvale
perioda stavu $\pi = 2$ společný dělitel = 2
 $\pi = 2$

Funkce není ergodická

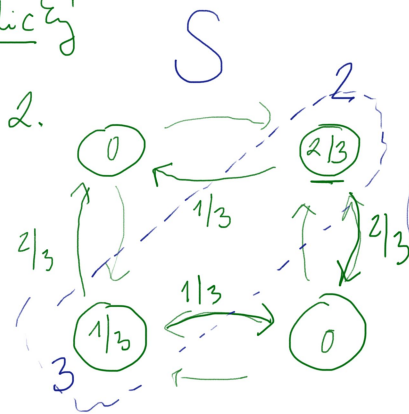


$$\frac{8}{24} + \frac{5}{24}$$

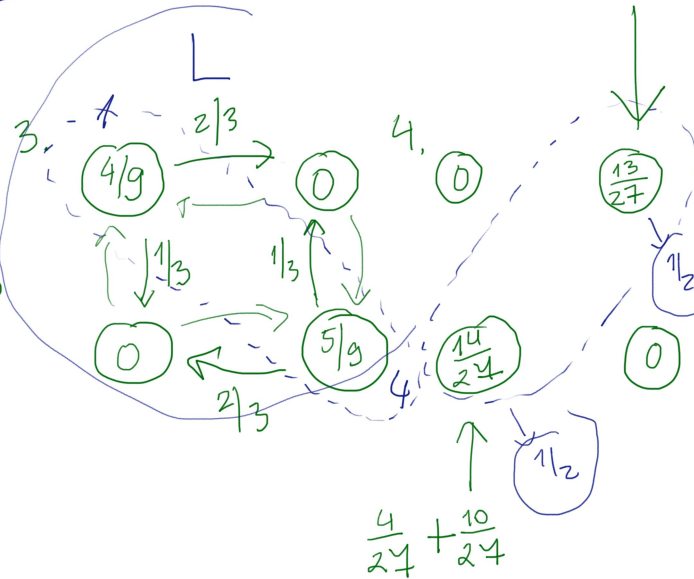
1.



$$\underline{p_1 = (1 \ 0 \ 0 \ 0)}$$



$$p_2 = p_1 \cdot P$$



$$\frac{4}{24} + \frac{10}{24}$$

$$p_3 = p_2 \cdot P$$

$$p_4 = p_3 \cdot P$$

$$p_3 = p_1 \cdot (P \cdot P)$$

$$p_4 = p_1 \cdot P^3$$

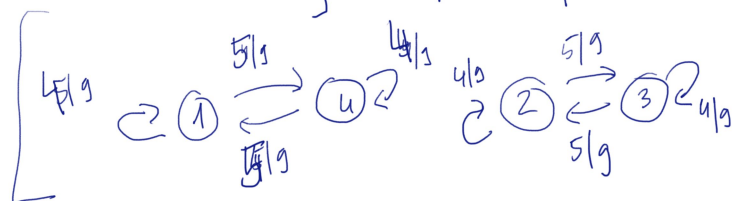
$$p_3 = p_1 \cdot P^2$$

c) $\pi = 2$

$$p_1 \neq P^2$$

$$P^2 = P \cdot P = \begin{matrix} & \begin{matrix} 1 & 2 & 3 & 4 \end{matrix} \\ \begin{matrix} 1 \\ 2 \\ 3 \\ 4 \end{matrix} & \begin{bmatrix} 4/9 & 0 & 0 & 5/9 \\ 0 & 4/9 & 5/9 & 0 \\ 0 & 5/9 & 4/9 & 0 \\ 5/9 & 0 & 0 & 4/9 \end{bmatrix} \end{matrix}$$

	1	4	2	3
1	4/9	5/9	0	0
4	5/9	4/9	0	0
2	0	0	4/9	5/9
3	0	0	5/9	4/9



stacionární rozdělení $p_s = (\dots)$

$$P: \begin{array}{c|cccc} & 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 1/3 & 1/3 & 0 \\ 2 & 1/3 & 0 & 0 & 1/3 \\ 3 & 2/3 & 0 & 0 & 1/3 \\ 4 & 0 & 1/3 & 1/3 & 0 \end{array}$$

$$(p_s \cdot P)^T = p_s^T$$

$$P^T \cdot p_s^T = p_s^T$$

$$P^T \cdot p_s^T - p_s^T = 0^T$$

$$(P^T - I \cdot E) \cdot p_s^T = 0^T$$

$$P^T - E = \begin{pmatrix} -1 & 1/3 & 2/3 & 0 \\ 1/3 & -1 & 0 & 1/3 \\ 1/3 & 0 & -1 & 2/3 \\ 0 & 2/3 & 1/3 & -1 \end{pmatrix}$$

$$\sim \begin{pmatrix} -1 & 1/3 & 2/3 & 0 \\ 0 & -4/9 & 4/9 & 1/3 \\ 0 & 1/9 & -4/9 & 2/3 \\ 0 & 2/3 & 1/3 & -1 \end{pmatrix} \sim \begin{pmatrix} -1 & 1/3 & 2/3 & 0 \\ 0 & -5/9 & 5/9 & 0 \\ 0 & -5/9 & -5/9 & 0 \\ 0 & 2/3 & 1/3 & -1 \end{pmatrix}$$

$$b = c = 1$$

$$a = 1$$

$$d = 1$$

$$\alpha (1 \ 1 \ 1 \ 1); \alpha \in \mathbb{R}$$

$$\frac{1}{4}$$

$$a + b + c + d = 1! \quad p_s = \left(\frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4}\right)$$

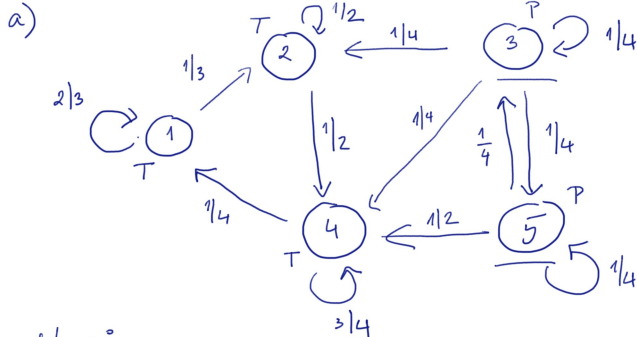
$$\left(\frac{1}{2} \ 0 \ 0 \ \frac{1}{2}\right) P = \left(0 \ \frac{1}{2} \ \frac{1}{2} \ 0\right)$$

d) rozdělení po 1000 krocích

$$p_1 = (1 \ 0 \ 0 \ 0)$$

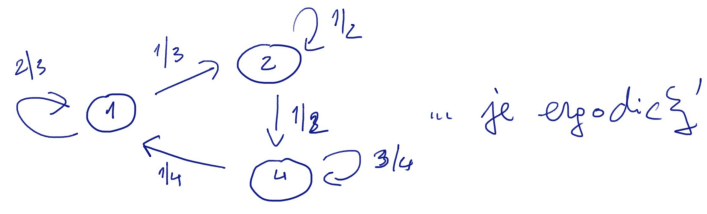
$$P_{1001} \doteq \left(\frac{1}{2} \ 0 \ 0 \ \frac{1}{2}\right)$$

3

$$P = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 2/3 & 1/3 & 0 & 0 & 0 \\ 2 & 0 & 1/2 & 0 & 1/2 & 0 \\ 3 & 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 4 & 1/4 & 0 & 0 & 3/4 & 0 \\ 5 & 0 & 0 & 1/4 & 1/2 & 1/4 \end{array}$$


b) možemo' možjina stani
 \emptyset
 $\{1, 2, 4\}$

c) nisu ergodici'



$$P = \begin{array}{c|ccc} & 1 & 2 & 4 \\ \hline 1 & 2/3 & 1/3 & 0 \\ 2 & 0 & 1/2 & 1/2 \\ 4 & 1/4 & 0 & 3/4 \end{array}$$

$$(P-E)^T = \begin{bmatrix} 1 & 2 & 4 \\ -1/3 & 1/3 & 0 \\ 0 & -1/2 & 1/2 \\ 1/4 & 0 & -3/4 \end{bmatrix} = \begin{bmatrix} -1/3 & 0 & 1/4 \\ 1/3 & -1/2 & 0 \\ 0 & 1/2 & -1/4 \end{bmatrix} \sim \begin{bmatrix} -1/3 & 0 & 1/4 \\ 0 & -1/2 & 1/4 \\ 0 & 1/2 & -1/4 \end{bmatrix}$$

$a \quad b \quad c \quad -\frac{1}{3}a + \frac{2}{4} = 0$
 $a = \frac{-\frac{1}{2}}{-\frac{1}{3}} = \frac{3}{2}$

$$p_s = \left(\frac{3}{9} \quad \frac{2}{9} \quad 0 \quad \frac{4}{9} \quad 0 \right)$$

$$\underbrace{\left(\frac{3}{2} \quad 1 \quad 2 \right)}_{\frac{3}{2} + \frac{2}{2} + \frac{4}{2} = \frac{9}{2}}; \alpha \in \mathbb{R}$$

$$x = \left(\frac{3}{9} \quad \frac{2}{9} \quad \frac{4}{9} \right)$$

d) u case $t+2$ u stan 2
 u jakom stan u case t ?

$$L(i) = \frac{1}{5} \cdot P(X_{t+1} = i, X_{t+2} = 2)$$

$$L(2) = \frac{1}{5} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{5} \cdot \frac{1}{4}$$

$$L(1) = \frac{1}{5} \cdot \frac{1}{3} \cdot \frac{1}{2} + \frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{5} \cdot \left(\frac{4}{18} \right)$$

$$L(3) = \frac{1}{5} \left(\frac{1}{4} \cdot \frac{1}{2} + \frac{1}{3} \cdot \frac{1}{4} \right) = \frac{1}{5} \cdot \frac{3}{16}$$

$$L(4) = \frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{5} \cdot \frac{1}{12}$$

$$L(5) = \frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{5} \cdot \frac{1}{16}$$

