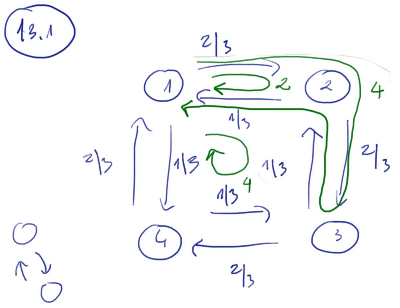
13.1



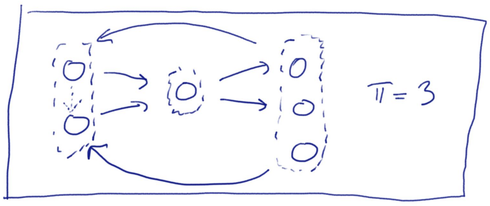
a) matice přechodu $\quad p_{ij} = p(i \to j)$

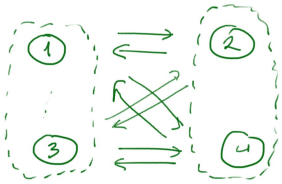
(Do)

$P=$		1	2	3	4	
	1	0	2/3	0	1/3	.... Σ = 1
(z)	2	1/3	0	2/3	0	
	3	0	1/3	0	2/3	
	4	2/3	0	1/3	0	

všechny stavy jsou zde trvalé, tvoří 1 komponentu

řetězec je ergodický ⟺ stavy trvalé & perioda $T = 1$

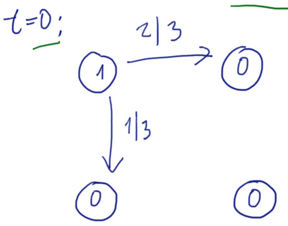


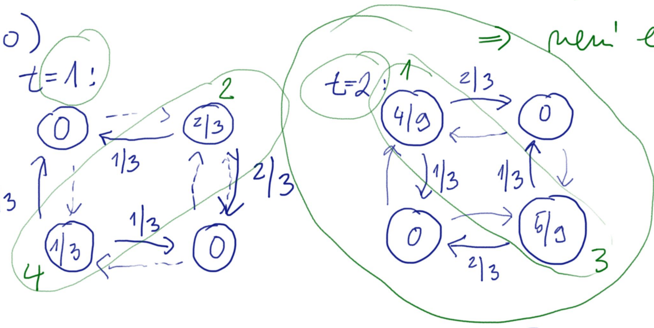


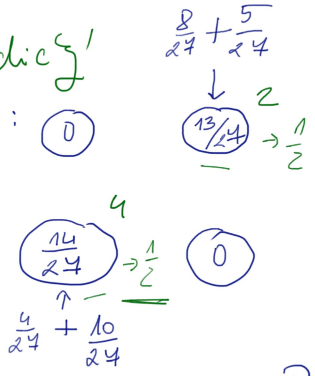
$\Rightarrow T = 2$

$\Rightarrow$ není ergodický!

b) $p_1 = (1 \;\; 0 \;\; 0 \;\; 0)$







$p_2 = p_1 \cdot P$

$(0 \;\; \frac{2}{3} \;\; 0 \;\; \frac{1}{3})$

$p_3 = p_2 \cdot P$

$\left(\frac{4}{9} \;\; 0 \;\; \frac{5}{9} \;\; 0\right) = \left(0 \;\; \frac{2}{3} \;\; 0 \;\; \frac{1}{3}\right) \cdot P$

$p_4 = p_3 \cdot P$

$p_4 = p_2 \cdot P \cdot P$

$p_4 = p_1 \underbrace{(P \cdot P \cdot P)}_{P^3}$

Stacionární rozdělení

(Do)

$P=$		1	2	3	4
	1	0	2/3	0	1/3
(z)	2	1/3	0	2/3	0
	3	0	1/3	0	2/3
	4	2/3	0	1/3	0

$(p_s \cdot P)^T = p_s^T$

$P^T \cdot p_s^T = p_s^T$

$P^T \cdot p_s^T - p_s^T = o^T$

$(P^T - 1 \cdot E) \cdot p_s^T = o^T$

$$P^T - E = \begin{pmatrix} -1 & 1/3 & 0 & 2/3 \\ 2/3 & -1 & 1/3 & 0 \\ 0 & 2/3 & -1 & 1/3 \\ 1/3 & 0 & 2/3 & -1 \end{pmatrix} \sim \begin{pmatrix} -1 & 1/3 & 0 & 2/3 \\ 0 & -7/9 & 1/3 & 4/9 \\ 0 & 2/3 & -1 & 1/3 \\ 0 & 1/9 & 2/3 & -7/9 \end{pmatrix} \sim \begin{pmatrix} -1 & 1/3 & 0 & 2/3 \\ 0 & -5/9 & 0 & 5/9 \\ 0 & 2/3 & -1 & 1/3 \\ 0 & 5/9 & 0 & -5/9 \end{pmatrix}$$

$\alpha(1 \;\; 1 \;\; 1 \;\; 1);\ \alpha \in \mathbb{R}$

$\frac{1}{4} \qquad 4$

$$p_s = \left(\frac{1}{4} \;\; \frac{1}{4} \;\; \frac{1}{4} \;\; \frac{1}{4}\right)$$

$P =$

(z) \ (do)	1	2	3	4
1	0	2/3	0	1/3
2	1/3	0	2/3	0
3	0	1/3	0	2/3
4	2/3	0	1/3	0

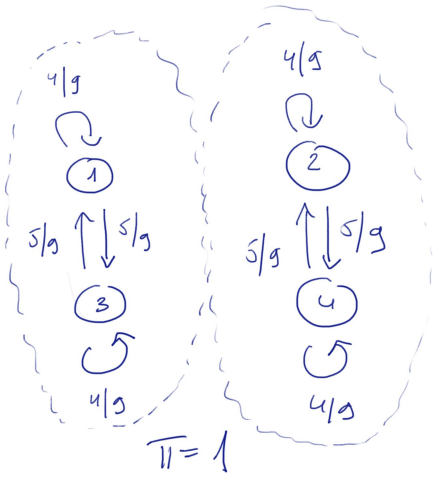
c) slučíme $\Pi = 2$ třídy

$$p_{t+1} = p_t \cdot P^2$$

$P^2 = P \cdot P =$

	1	2	3	4
1	4/9	0	5/9	0
2	0	4/9	0	5/9
3	5/9	0	4/9	0
4	0	5/9	0	4/9

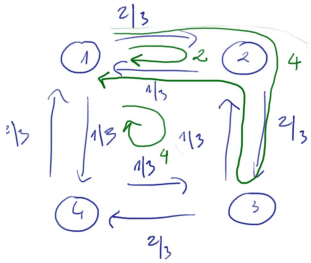
	1	3	2	4
1	4/9	5/9	0	0
3	5/9	4/9	0	0
2	0	0	4/9	5/9
4	0	0	5/9	4/9

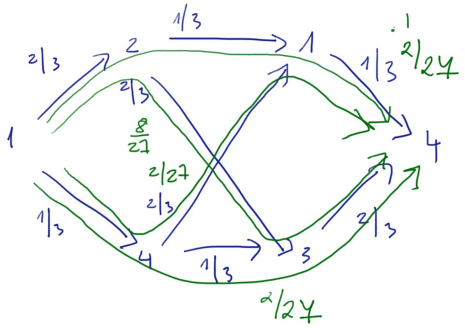


$\Pi = 1$

2 komponenty ⇒ rozložitelný
⇒ není ergodický

b) $P[1 \to \cdot \to \cdot \to 4] \equiv P[X_{t+3} = 4 \mid X_t = 1] = \frac{14}{24}$





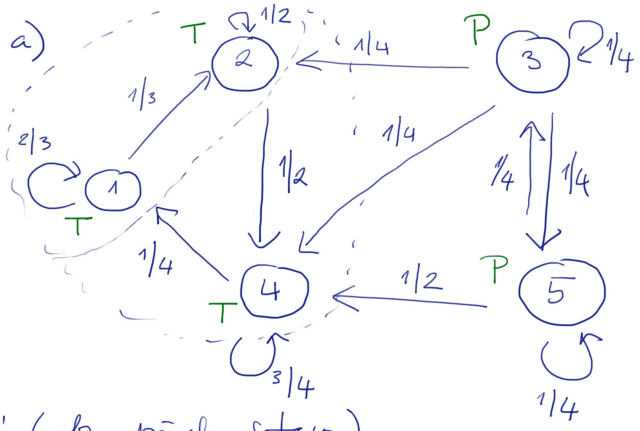
c) rozdělení po 1000 krocích ; $p_1 = (1\ 0\ 0\ 0)$

$p_{1001} \doteq \left(\frac{1}{2}\ 0\ \frac{1}{2}\ 0\right)$

3

$$P = \begin{array}{c|ccccc} & 1 & 2 & 3 & 4 & 5 \\ \hline 1 & 2/3 & 1/3 & 0 & 0 & 0 \\ 2 & 0 & 1/2 & 0 & 1/2 & 0 \\ 3 & 0 & 1/4 & 1/4 & 1/4 & 1/4 \\ 4 & 1/4 & 0 & 0 & 3/4 & 0 \\ 5 & 0 & 0 & 1/4 & 1/2 & 1/4 \end{array}$$

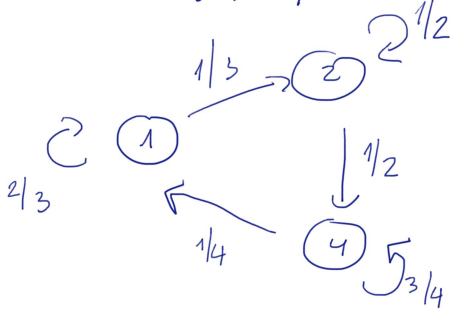
a)



není ergodický (obs. přech. stavy)

b) uzavřené množiny stavů $\{1,2,4\}$; $\emptyset$

c) $p_S(\,.\,,\,0\,,\,0\,)$



$$\hat{P} = \begin{pmatrix} 2/3 & 1/3 & 0 \\ 0 & 1/2 & 1/2 \\ 1/4 & 0 & 3/4 \end{pmatrix}$$

ergodický

$$(\hat{P}-E)^T \hat{p}_S^T = o^T \quad \& \quad \sum \hat{p}_{S_i} = 1$$

$$\begin{array}{ccc} a & b & c \\ -1/3 & 0 & 1/4 \\ 0 & -1/2 & 1/4 \\ 0 & 1/2 & -1/4 \end{array} \Rightarrow -\frac{1}{3}a + \frac{2}{4} = 0 \Rightarrow a = \frac{3}{2}$$

$$\sim \quad \Rightarrow \quad b = 1\,,\ c = 2$$

$$\alpha \left(\frac{3}{2}\ 1\ 2\right);\ \alpha \in \mathbb{R}$$

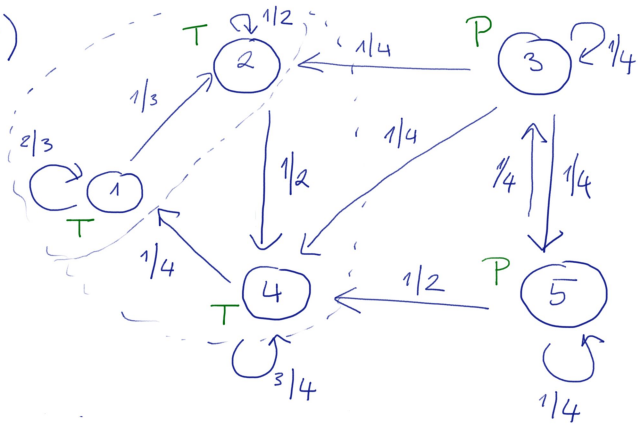
$$\frac{2}{9}$$

$$\frac{3}{2} + \frac{2}{2} + \frac{4}{2} = \frac{9}{2}$$

$$\hat{p}_S = \left(\frac{3}{9}\ \ \frac{2}{9}\ \ \frac{4}{9}\right)$$

$$p_S = \left(\frac{3}{9}\ \ \frac{2}{9}\ \ 0\ \ \frac{4}{9}\ \ 0\right)$$

d)



$t+2 \dots start\,2$

$t \dots 2$

$L(i) = P[i \to . \to 2]$

$L(4) = P[4 \to 1 \to 2] = \frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{3}$

$L(1) = P[1 \to 1 \to 2] + P[1 \to 2 \to 2]$

$L(2) = P[2 \to 2 \to 2]$

$L(3) = P[3 \to 2 \to 2] + P[3 \to 3 \to 2]$

$L(5) = P[5 \to 3 \to 2]$

$= \frac{1}{5} \cdot \frac{1}{12}$ $\frac{1}{5}$ 4

$= \frac{1}{5} \cdot \frac{7}{18}$ $\frac{1}{5}$ 1

$= \frac{1}{5} \cdot \frac{1}{4}$ $\frac{1}{5}$ 2

$= \frac{1}{5} \cdot \frac{3}{16}$

$= \frac{1}{5} \cdot \frac{1}{16}$ $\frac{1}{5}$ 3

$\frac{1}{5}$ 5