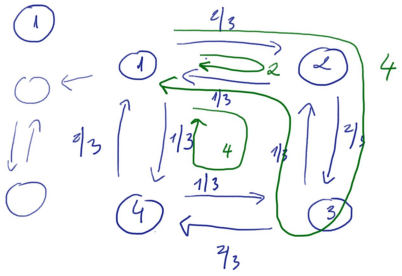


a) matice přechodu $p_{ij} = p(i \to j)$

(DO)

	1	2	3	4	
1	0	2/3	0	1/3	= 1
2	1/3	0	2/3	0	
3	0	1/3	0	2/3	
4	2/3	0	1/3	0	

(Z)

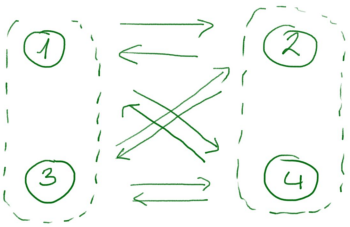
nsd = 2

$p_0 = (1\ 0\ 0\ 0)$

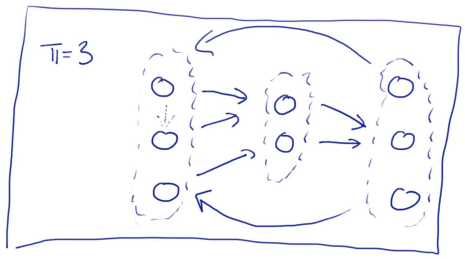
$p_1 = p_0 \cdot P = (1\ 0\ 0\ 0)\begin{pmatrix} \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot \end{pmatrix} = (0\ \tfrac{2}{3}\ 0\ \tfrac{1}{3})$

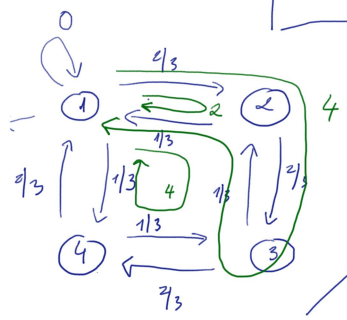
nů. stavy jsou zde trvalé
nů. stavy tvůj 1 komponentu
nů. stavy mají periodu $\Pi = 1$

} ~~ergodické~~



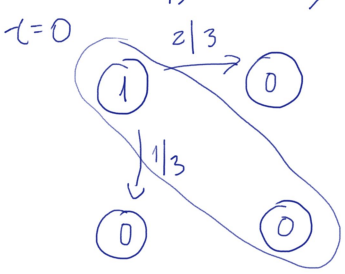
$\Rightarrow \Pi = 2$





Π

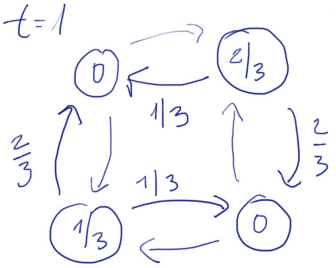
$p_2 = p_0 \cdot P^2$



t=0

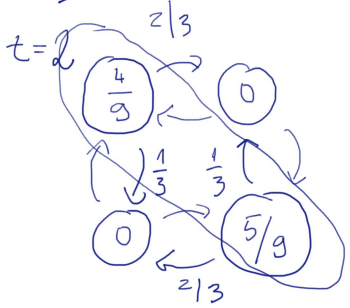
$p_0 = (1\ 0\ 0\ 0)$

t=1



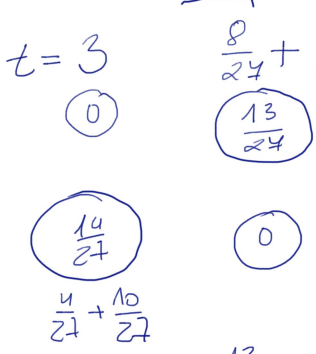
$p_1 = (0\ \tfrac{1}{3}\ 0\ \tfrac{1}{3})$

t=2



p_2

t=3



$p_3 = (0\ \tfrac{13}{27}\ 0\ \tfrac{14}{27})$

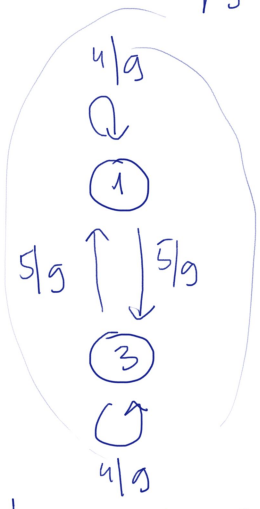
$p_1 = p_0 \cdot P$

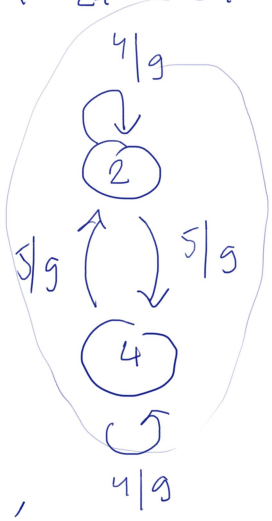
$P^2 = P \cdot P =$

	1	2	3	4
1	4/9	0	5/9	0
2	0	4/9	0	5/9
3	5/9	0	4/9	0
4	0	5/9	0	4/9

~

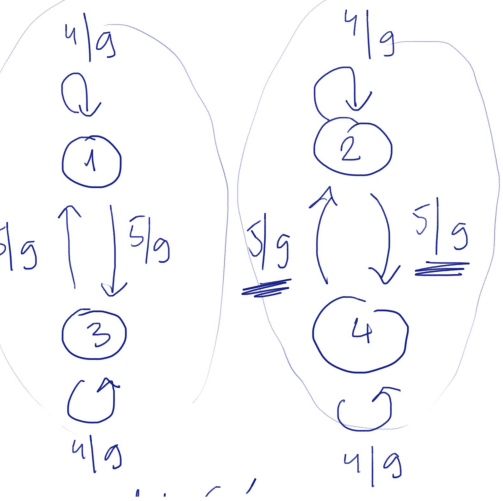
	1	3	2	4
1	4/9	5/9	0	0
3	5/9	4/9	0	0
2	0	0	4/9	5/9
4	0	0	5/9	4/9





Stavy trvalé
$\Pi = 1$
2 komponenty

} není ergodický



$$\hat{P} = \begin{array}{c|cc} & 1 & 3 \\ \hline 1 & 4/9 & 5/9 \\ 3 & 5/9 & 4/9 \end{array}$$

$$\hat{P}^T - E = \begin{bmatrix} \cancel{4/9} - \frac{5}{9} & 5/9 \\ 5/9 & \cancel{\frac{4}{9}} - \frac{5}{9} \end{bmatrix}$$

$$(p_S \cdot P)^T = p_S^T$$

$$P^T \cdot p_S^T = p_S^T$$

$$P^T p_S^T - E p_S^T = 0^T$$

$$(P^T - E)\, p_S^T = 0^T$$

$$\underbrace{\alpha (1 \quad 1)}_{2} : \alpha \in \mathbb{R}$$

$$p_S = (1/2 \quad 1/2)$$

$$\begin{matrix} 1 & & 3 & & 2 & & 4 \end{matrix}$$

$$\left(\tfrac{1}{2} \; 0 \; \tfrac{1}{2} \; 0\right) \cdot P = \left(0 \; \tfrac{1}{2} \; 0 \; \tfrac{1}{2}\right)$$

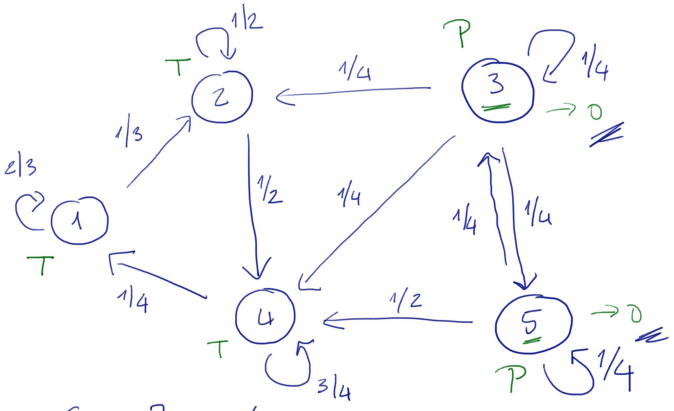
$$\left(\tfrac{1}{4} \; \tfrac{1}{4} \; \tfrac{1}{4} \; \tfrac{1}{4}\right) \cdot P = \left(\tfrac{1}{4} \; \tfrac{1}{4} \; \tfrac{1}{4} \; \tfrac{1}{4}\right)$$

p_S

3

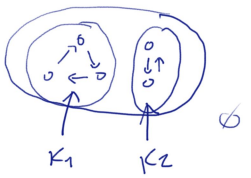
$P =$

	1	2	3	4	5
1	2/3	1/3	0	0	0
2	0	1/2	0	1/2	0
3	0	1/4	1/4	1/4	1/4
4	1/4	0	0	3/4	0
5	0	0	1/4	1/2	1/4



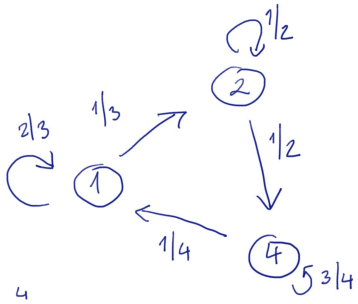
uzavřené množiny stavů $\{1,2,4\}$, $\emptyset$

komp.



$\emptyset$

$p_S = \left(\frac{3}{9}\ \frac{2}{9}\ 0\ \frac{4}{9}\ 0\right)$



(ERGODICKÁ)

$\hat{P} =$

	1	2	4
1	2/3	1/3	0
2	0	1/2	1/2
4	1/4	0	3/4

$(\hat{P} - E)^T =$

~~2/3~~ $-\frac{1}{3}$	0	1/4
1/3	~~1/2~~ $-\frac{1}{2}$	0
0	1/2	~~3/4~~ $-\frac{1}{4}$

$\hat{p}_S = (a\ \ b\ \ c)$

$-1/3$	0	1/4
1/3	$-1/2$	0
0	1/2	$-1/4$

$+$

$-1/3$	0	1/4
0	$-1/2$	1/4
~~0~~	~~1/2~~	~~$-1/4$~~

$b = 1$

$c = 2$

$-\frac{1}{3}a + \frac{1}{2} = 0$

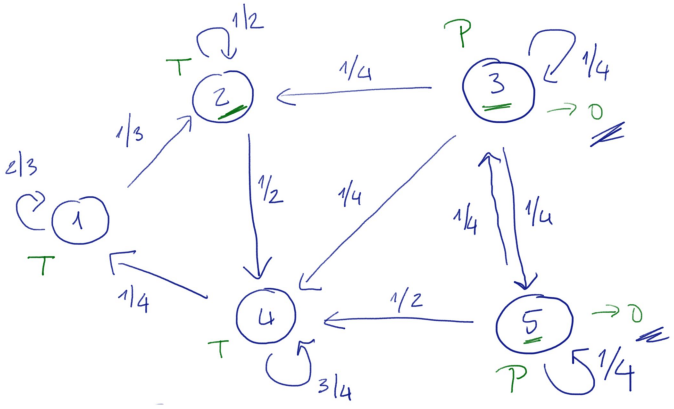
$a = \frac{1/2}{1/3} = \frac{3}{2}$

2/9

$\alpha\left(\frac{3}{2}\ 1\ 2\right) ; \alpha \in R$

$\frac{3}{2} + \frac{2}{2} + \frac{4}{2} = \frac{9}{2}$

$\hat{p}_S = \left(\frac{3}{9}\ \frac{2}{9}\ \frac{4}{9}\right)$



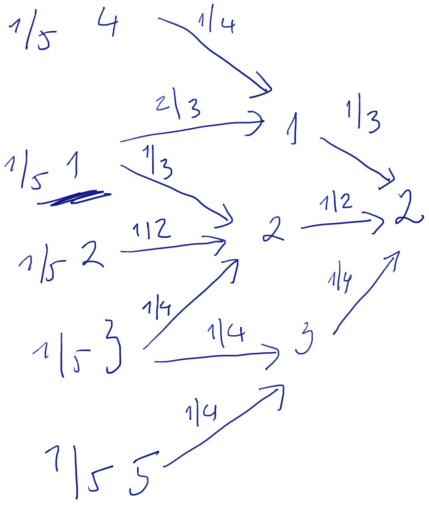
v čase $t=2$ je stav $=2$

$t=0$ ... ?

$$L(i) = p(i \to \cdot \to 2)$$

$$= P[X_0 = i] \cdot p_{i\cdot} \cdot p_{\cdot 2}$$

$t=0$



$$L(4) = p(4 \to 1 \to 2) = \frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{3} = \frac{1}{5} \cdot \frac{1}{12}$$

$$L(1) = p(1 \to 1 \to 2) + p(1 \to 2 \to 2)$$

$$= \frac{1}{5} \cdot \frac{2}{3} \cdot \frac{1}{3} + \frac{1}{5} \cdot \frac{1}{3} \cdot \frac{1}{2} = \frac{1}{5} \cdot \frac{4}{18}$$

$$L(2) = p(2 \to 2 \to 2) = \frac{1}{5} \cdot \frac{1}{2} \cdot \frac{1}{2} = \frac{1}{5} \cdot \frac{1}{4}$$

$$L(3) = p(3 \to 2 \to 2) + p(3 \to 3 \to 2)$$

$$= \frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{2} + \frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{5} \cdot \frac{3}{16}$$

$$L(5) = p(5 \to 3 \to 2) = \frac{1}{5} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{5} \cdot \frac{1}{16}$$