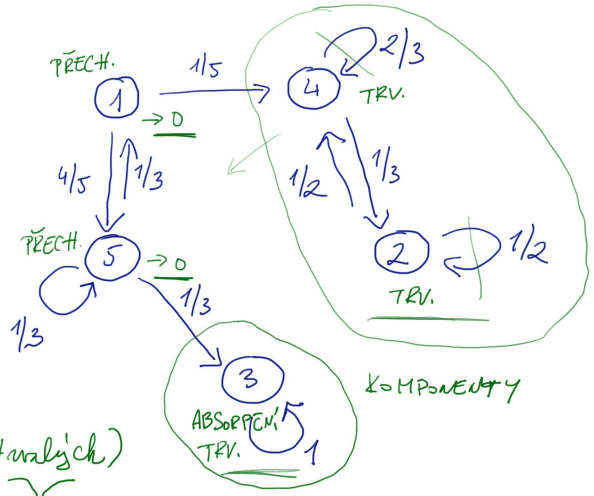
13.4

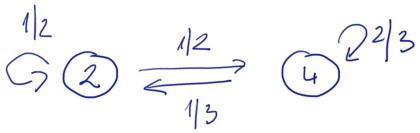
$P =$

	1	2	3	4	5
1	0	0	0	1/5	4/5
2	0	1/2	0	1/2	0
3	0	0	1	0	0
4	0	1/3	0	2/3	0
5	1/3	0	1/3	0	1/3



(tranzitních) množina stavů: $\emptyset, \{3\}, \{4,2\}, \{2,3,4\}$

$\pi_S = (0 \; p_2 \; p_3 \; p_4 \; 0)$



$\pi_{S,3} = (1)$

$P_{24} =$

	2	4
2	1/2	1/2
4	1/3	2/3

$\pi_{S,24} \cdot P_{24} = \pi_{S,24}$

$P_{24}^T - E = \begin{pmatrix} 1/2 - 1 & 1/3 \\ 1/2 & 2/3 - 1 \end{pmatrix} = \begin{pmatrix} -1/2 & 1/3 \\ 1/2 & -1/3 \end{pmatrix}$ (2, 3)

$\alpha(2, 3)$; $\alpha \in \mathbb{R}$

$\pi_{S,3} = (0 \; 0 \; 1 \; 0 \; 0)$

$\pi_{S,24} = (0 \; \frac{2}{5} \; 0 \; \frac{3}{5} \; 0)$

$\pi_{S,24} = (\frac{2}{5} \; \frac{3}{5})$

$\pi_S = \alpha(0 \; 0 \; 1 \; 0 \; 0) + (1-\alpha)(0 \; \frac{2}{5} \; 0 \; \frac{3}{5} \; 0)$

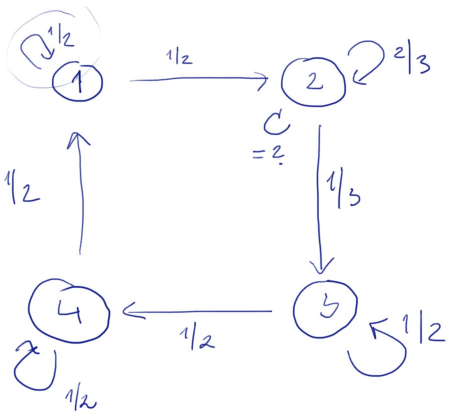
$= (0, (1-\alpha)\frac{2}{5}, \alpha, (1-\alpha)\frac{3}{5}, 0) \quad ; \quad \alpha \in \langle 0, 1 \rangle$

d) ...

13.5

$P =$

	1	2	3	4
1	1/2	1/2	0	0
2	0	2/3	1/3	0
3	0	0	1/2	1/2
4	1/2	0	0	1/2



posl. stavů : $(2, i, k, 3)$

$i, k = 2$

Uz. mn. st. $\emptyset$, $\{1,2,3,4\}$

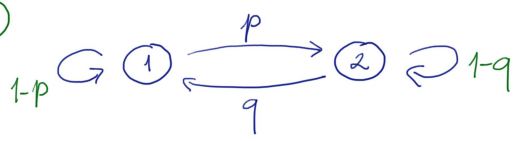
$L(2,2,2,3) = c \cdot \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{1}{3}$

$L(2,2,3,3) = c \cdot \frac{2}{3} \cdot \frac{1}{3} \cdot \frac{1}{2}$

$L(2,3,3,3) = c \cdot \frac{1}{3} \cdot \frac{1}{2} \cdot \frac{1}{2}$

$(2,2,2,3)$

14.1



$P =$

	1	2
1	1-p	p
2	q	1-q

1 2 2 1 1 1 2 1 1 1 2 2 1 1 1 1

ABS. ČET.

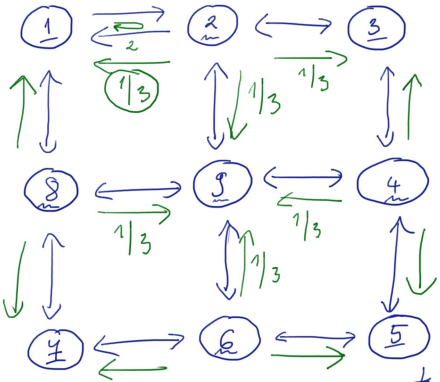
	1	2	Σ
1	4	3	10
2	3	2	5

REL. ČET.

$P =$

	1	2
1	4/10	3/10
2	3/5	2/5

14.2

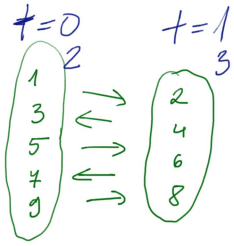


a) klas. stavy
a určit komp.

b) konverguje rozd. ke stac.?
kdy?

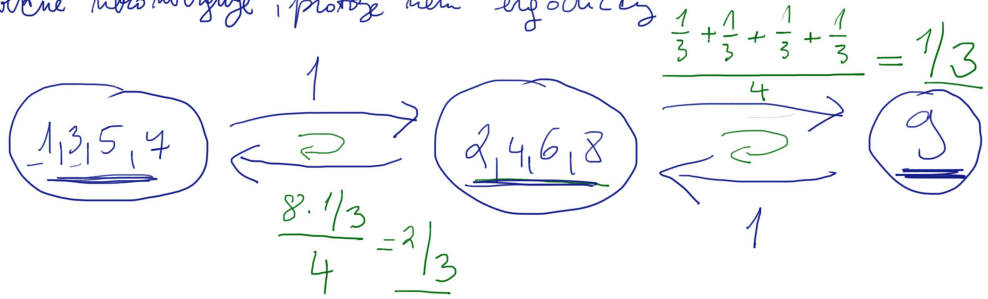
c) $p_0 = (1\,0\,0\,0\,0\,0\,0\,0\,0)$
rozdělení p_0 10^5 zrocích

a) 5. stavy trvalé
1 komponenta
$\pi = 2 \Rightarrow$ není erg.



$\Rightarrow \pi = 2$

b) řetězec nekonverguje, protože není ergodický



	1357	2468	9
1357	0	1	0
2468	$2/3$	0	$1/3$
9	0	1	0

$$\begin{pmatrix} 0-1 & 1/3 & 0 \\ 1 & 0-1 & 1 \\ 0 & 1/3 & 0-1 \end{pmatrix}$$

$$\begin{pmatrix} -1 & 2/3 & 0 \\ 1 & -1 & 1 \\ 0 & 1/3 & -1 \end{pmatrix} \sim \begin{pmatrix} -1 & 2/3 & 0 \\ 0 & -1/3 & 1 \\ 0 & 1/3 & -1 \end{pmatrix} \to y = 3, z = 1$$

$-x + \frac{2}{3} \cdot 3 = 0$
$x = 2$

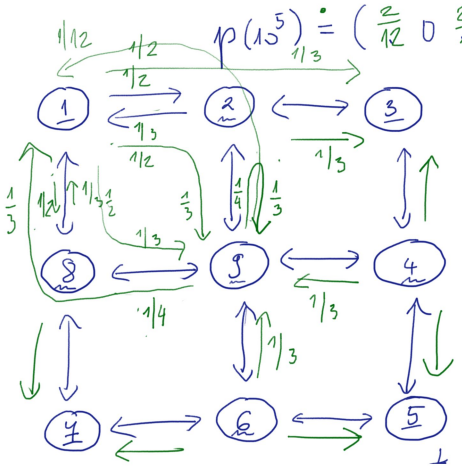
$(xyz) = \alpha(2\ 3\ 1);\ \alpha \in R$
$\frac{1}{6}$ $\frac{6}{6}$

$$p_S = \left(\frac{2}{24}\ \frac{3}{24}\ \frac{2}{24}\ \frac{3}{24}\ \frac{2}{24}\ \frac{3}{24}\ \frac{2}{24}\ \frac{3}{24}\ \frac{1}{6}\right)$$

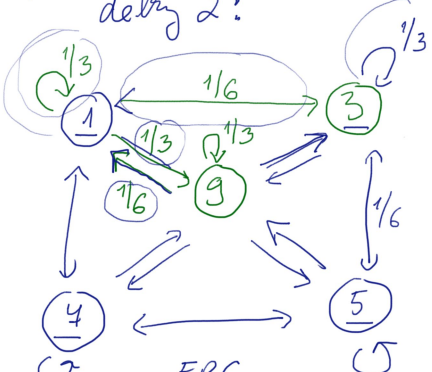
$\hat{p}_S = \left(\frac{2}{6}\ \frac{3}{6}\ \frac{1}{6}\right)$

c) p_0 10^5 zrocích

$p(10^5) \doteq \left(\frac{2}{12}\ 0\ \frac{2}{12}\ 0\ \frac{2}{12}\ 0\ \frac{2}{12}\ 0\ \frac{1}{3}\right)$

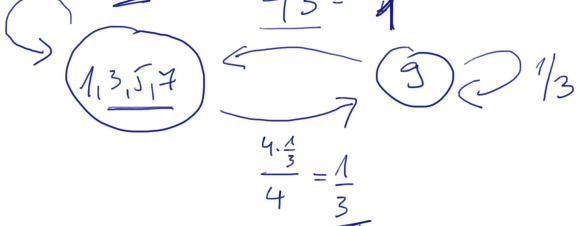


projdeme ke EroEnim
délky 2:



$\frac{4 \cdot \frac{1}{3} + 8 \cdot \frac{1}{6}}{4} = \frac{2}{3}$

$2/3 = \frac{4 \cdot \frac{1}{6}}{1}$



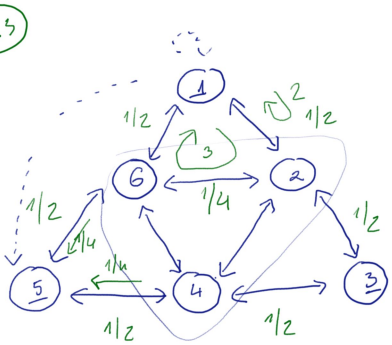
	1357	9
1357	$2/3$	$1/3$
9	$2/3$	$1/3$

$\hat{p}_S = \left(\frac{2}{3}\ \frac{1}{3}\right)$

$$\begin{pmatrix} 2/3-1 & 2/3 & -1/3 & 2/3 \\ 1/3 & 1/3-1 & 1/3 & -2/3 \end{pmatrix} \sim$$

$\alpha(2\ 1);\ \alpha \in R$
$\frac{1}{3}$ $\frac{3}{3}$

14.3



a) klas. stav. + komp.

b) $p(1 \to \cdot \to \cdot \to 4)$

c) $p(t=0) = (1\ 0\ 0\ 0\ 0\ 0)$

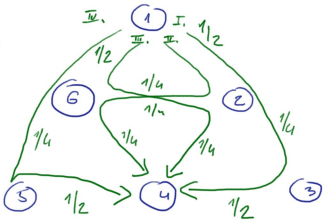
$p(t=1000) = ?$

d) konv. ?

a) 00. stavy trvalé
1 komponenta
perioda = 1
$\Rightarrow$ ERGODICKÁ

$nsd(2,3) = 1!$
$\Rightarrow \pi = 1$

b)

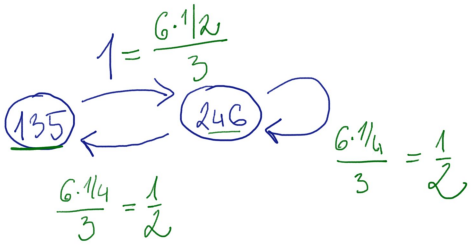


I. $\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{16}$
II. $\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{32}$
III. $\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{4} = \frac{1}{32}$
IV. $\frac{1}{2} \cdot \frac{1}{4} \cdot \frac{1}{2} = \frac{1}{16}$

$\Big\} \frac{6}{32}$

d) ANO

c)



$\hat{P}$ =

	135	246
135	0	1
246	1/2	1/2

$\hat{P}^T - E =$

$$\begin{pmatrix} 0-1 & 1/2 \\ 1 & 1/2-1 \end{pmatrix} = \begin{pmatrix} -1 & 1/2 \\ 1 & -1/2 \end{pmatrix}$$

(2) (1)

$\alpha(2\ \ 1)$; $\alpha \in R$

$\frac{1}{3}$ 3

$$p_S = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ \frac{2}{9} & \frac{1}{9} & \frac{2}{9} & \frac{1}{9} & \frac{2}{9} & \frac{1}{9} \end{pmatrix}$$

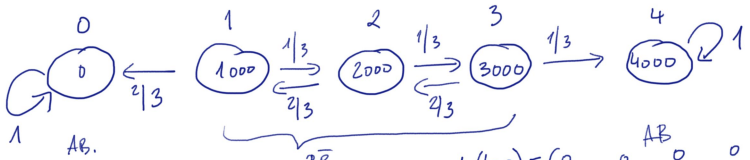
$\hat{p}_S = (\frac{2}{3}\ \ \frac{1}{3})$

$p(t=1000) \doteq p_S$

14.5

1/3 nyhmje

$1000



$p(t=0) = (0 \quad 0 \quad \overset{AB}{0} \quad 0 \quad 0 \quad 1)$

AB. \ FR.	0	1	2	3	4
0	1	0	0	0	0
1	2/3	0	1/3	0	0
2	0	2/3	0	1/3	0
3	0	0	2/3	0	1/3
4	0	0	0	0	1

$P =$

	0	4	1	2	3
0	1	0	0	0	0
4	0	1	0	0	0
1	2/3	0	0	1/3	0
2	0	0	2/3	0	1/3
3	0	1/3	0	2/3	0

$F = (E - Q)^{-1}$

$$F \cdot R \doteq \begin{bmatrix} 0{,}93 & 0{,}07 \\ 0{,}80 & 0{,}20 \\ 0{,}53 & 0{,}47 \end{bmatrix}$$

$$P^{\infty} = \begin{bmatrix} E & 0 \\ \hline FR & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ \hline 0{,}93 & 0{,}07 & 0 & 0 & 0 \\ 0{,}80 & 0{,}20 & 0 & 0 & 0 \\ 0{,}53 & 0{,}47 & 0 & 0 & 0 \end{bmatrix}$$

$$p(t=0) = \overset{0\ 4\ 1\ 2\ 3}{(0 \ 0 \ 0 \ 0 \ 1)} \cdot P^{\infty} = (0{,}53 \quad 0{,}47 \quad 0 \quad 0 \quad 0)$$

14.7

$$P = \begin{array}{c} \\ \to A \\ \to C \\ \to B \end{array} \begin{array}{c} \begin{array}{ccc} \downarrow A & \downarrow C & \downarrow B \\ C & A & \end{array} \\ \begin{bmatrix} 1/3 & 1/3 & 1/3 \\ 0.1 & 0.8 & 0.1 \\ 0.5 & 0.5 & 0 \end{bmatrix} \end{array}$$

(A) (B) (C)

poz:

B C C C A A B A C

a)

	A	C	B
A	1/3	1/3	1/3
C	0.1	0.8	0.1
B	0.5	0.5	0

$L(ACB) = 0.5 \cdot 0.8 \cdot 0.8 \cdot 0.1 \cdot 1/3 \cdot 1/3 \cdot 0.5 \cdot 1/3$

$L(CAB) = 0.5 \cdot 1/3 \cdot 1/3 \cdot 1/3 \cdot 0.8 \cdot 0.1 \cdot 0.5 \cdot 0.1$

	C	A	B
C	1/3	1/3	1/3
A	0.1	0.8	0.1
B	0.5	0.5	0

A C B

1. ... C A C C B
2. ... C C C B A ✓
3. ... C B A A C

14.8

$1 \to \cdot \to \cdot \to \cdot \to 3$

$L(3) =$

$k = 3$:

1 → 2 ⇄ 3 (1, 1/2, 1/2)

$1 \cdot \frac{1}{2} \cdot 1 \cdot \frac{1}{2}$

+

1 ⇄ 2 → 3 (1, 1/2, 1)

$1 \cdot \frac{1}{2} \cdot 1 \cdot \frac{1}{2}$

$k = 4$:

1 → 2 ⇄ 3 → 4 (1, 1/2, 1/2, 1)

$L(4) = 1 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

$+ 1 \cdot \frac{1}{2} \cdot 1 \cdot \frac{1}{2}$ ← nejvíce nahodilých!

$+ 1 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot 1$

1 ⇄ 2 → 3 → 4 (1, 1/2, 1/2, 1)

1 → 2 → 3 ⇄ 4 (1, 1/2, 1/2, 1)

$k = 5$:

a nice

1 → 2 ⇄ 3 → 4 → 5 (1, 1/2, 1/2, 1/2)

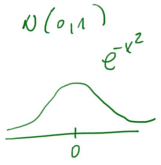
$L(5) = 1 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

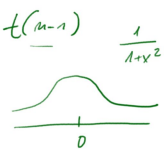
$+ 1 \cdot \frac{1}{2} \cdot 1 \cdot \frac{1}{2}$

$+ 1 \cdot \frac{1}{2} \cdot \frac{1}{2} \cdot \frac{1}{2}$

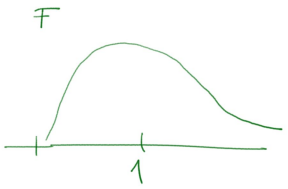
1 ⇄ 2 → 3 → 4 → 5 (1, 1/2, 1/2, 1/2)

1 → 2 → 3 ⇄ 4 → 5 (1, 1/2, 1/2, 1/2)





$T = \frac{S_x^2}{S_y^2}$



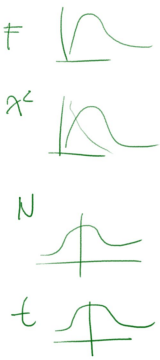
$T = \frac{\bar{X} - c}{\sqrt{\frac{S^2}{n}}}$

$T = \frac{\bar{X} - c}{\sqrt{\frac{S_x^2}{n}}}$

	0	1	2	3	4	5	6
n_i	29	15	10	5	3	0	2
p_i	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{1}{8}$	$\frac{1}{16}$	....		
$n \cdot p_i$	32	16	8	4	....		

$q = \frac{1}{2}$

$n = 64$



$T = \sum \frac{(n_i - np_i)^2}{np_i} \dots \chi^2_{(n-1)}$

25 J, 45 H

$P(S|J) = 40\%$

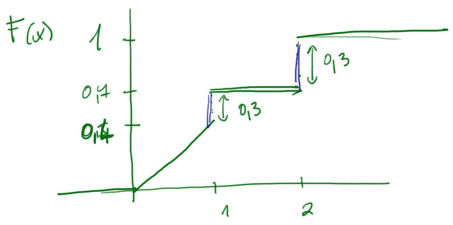
$P(S|H) = 20\%$

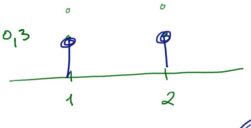
$$P(J|S) = \frac{P(J) \cdot P(S|J)}{P(J) \cdot P(S|J) + P(H) \cdot P(S|J)} = \frac{0{,}1}{0{,}25} = \frac{10}{25}$$

($P(J) \cdot P(S|J) + P(H) \cdot P(S|J) = P(S)$)

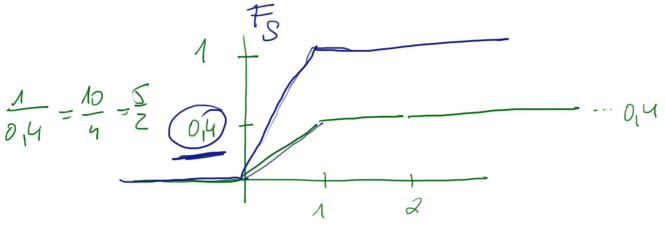
$$P(S) = \frac{25}{100} \cdot 0{,}4 + \frac{45}{100} \cdot 0{,}2 = 0{,}25 \cdot 0{,}4 + 0{,}75 \cdot 0{,}2$$
$$= 0{,}1 + 0{,}15 = 0{,}25$$

$$P(H|S) = \frac{15}{25}$$

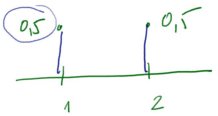




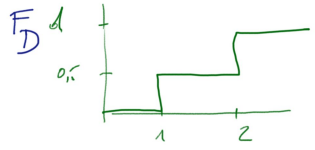
0,3 + 0,3 = 0,6



$\frac{1}{0,6} = \frac{10}{6} = \frac{5}{3}$



$X = Mix_{0,4;\,0,6}(S, D)$



$F_X(x) = 0{,}4 \cdot F_S(x) + 0{,}6 \cdot F_D(x)$