Seminar 5

Simultaneous models

(1) Matrix B and matrix Γ
(2) Model identification
(3) Reduced form of the model, matrix M

5.2.1 Model construction

Based on the data table (see the last page of this material) following four-equations model was specified:

1) households consumption expenditures are influenced by foreign trade balance, inflation, interest rate of households, investments and unemployment;

2) fixed capital is influenced by foreign trade balance, interest rate of firms and number of employees;

3) Foreign trade balance is influenced by household consumption expenditures, exchange rate and foreign investment;

4) GDP is sum of household consumption expenditures, investments, foreign trade balance and government expenditure.

Economic model

$$y_1 = f(y_3, x_3, x_4, x_{10}, x_{11})$$
$$y_2 = f(y_3, x_5, x_{12})$$
$$y_3 = f(y_1, x_9, x_{15})$$
$$y_4 = y_1 + y_2 + y_3 + x_{13}$$

Econometric model

$$\beta_{11}y_{1t} = \beta_{13}y_{3t} + \gamma_{13}x_{3t} + \gamma_{14}x_{4t} + \gamma_{110}x_{10t} + \gamma_{111}x_{11t} + u_{1t}$$

$$\beta_{22}y_{2t} = \beta_{23}y_{3t} + \gamma_{25}x_{5t} + \gamma_{212}x_{12t} + u_{2t}$$

$$\beta_{33}y_{3t} = \beta_{31}y_{1t} + \gamma_{39}x_{9t} + \gamma_{315}x_{15t} + u_{3t}$$

$$\beta_{44}y_{4t} = \beta_{41}y_{1t} + \beta_{42}y_{2t} + \beta_{43}y_{3t} + \gamma_{413}x_{13t}$$

1. Define matrix Β and matrix Γ.

2. Identify the model.

5.2.4 Matrix of multipliers (M)

If the model is simple (has quite simple structure and small size), reduced form of the model might be derived using substitution.

An example of using substitution

Recursive econometric model:

$$y_{1t} = 2x_{2t} - x_{3t}$$
$$y_{2t} = -2y_{1t} - 3x_{1t} + 2x_{3t}.$$

Reduced form of them model:

$$y_{1t} = 2x_{2t} - x_{3t}$$
$$y_{2t} = -2\,(2x_{2t} - x_{3t}) -3x_{1t} + 2x_{3t} = -4x_{2t} +2x_{3t} -3x_{1t}+2x_{3t} = -3x_{1t} - 4x_{2t} +4x_{3t}$$

*Matrix **M** of four-equations simultaneous model is following (Try to verify it.):*

$$\begin{vmatrix} m_{13} & m_{14} & 0 & m_{19} & m_{110} & m_{111} & 0 & 0 & m_{115} \\ m_{23} & m_{24} & m_{25} & m_{29} & m_{210} & m_{211} & m_{212} & 0 & m_{215} \\ m_{33} & m_{34} & 0 & m_{39} & m_{310} & m_{311} & 0 & 0 & m_{315} \\ m_{43} & m_{44} & m_{45} & m_{49} & m_{410} & m_{411} & m_{412} & m_{413} & m_{415} \end{vmatrix}$$

Exercises

1. Define the type of the following models, identify them and define matrices **B**, **Γ** and matrix **M**.

a) $y_{1t} = \gamma_{13} x_{3t} + \gamma_{14} x_{4t} + u_{1t}$

b) $y_{1t} = \beta_{12} y_{2t} + \gamma_{11} x_{1t} + u_{1t}$
$y_{2t} = \beta_{21} y_{1t} + \gamma_{21} x_{1t} + u_{2t}$

c) $y_{1t} = 5 + 3y_{2t} + 4x_{1t} + u_{1t}$
$y_{2t} = 3 - y_{1t} + 0{,}5\, x_{2t} + 2y_{1(t-1)} + u_{2t}$
$y_{3t} = y_{1t} - y_{2t}$

2. Set up two-equations simultaneous exactly identified econometric model.

3. Set up a general three equations simultaneous econometric model with symmetric matrix *B*, in which elements β_{13} and β_{23} are the only zero values above the leading diagonal. Identify this model

4. Quantify reduced form of the following econometric model.

$y_{1t} = 3y_{3t} - 2x_{6t} + u_{1t}$
$y_{2t} = 4x_{2t} - 3x_{3t} + x_{5t} + u_{2t}$
$y_{3t} = -2y_{2t} + 1 - 3x_{3t} + u_{3t}$

Individual exercises

1. What is the meaning of explanatory endogenous variable?
2. What is the reason for including identity equation into the model?
3. What is the content of matrices **B**, **Γ**, **M** and what is the procedure of their construction?
4. What is the procedure of vectors u_t a v_t quantification?

Data table

	Households consumption expenditure [billion. CZK]	Fixed capital [billion CZK]	Foreign trade balance [billion CZK]	GDP [billion CZK]	UV	Inflation [%]	Interest rate of households [%]	Interest rate of firms [%]	CZK/EUR	Investments [%]	Unemployment [%]	Number of employees [mil.]	Government expenditure [billion CZK]	Foreign investments [billion CZK]
Year	**y1**	**y2**	**y3**	**y4**	**x1**	**x3**	**x4**	**x5**	**x9**	**x10**	**x11**	**x12**	**x13**	**x15**
1992	411,8	285,9	-20,3	846,8	1	11,1	5,4	15,6	28,9	33,7	2,7	4,9	169,4	28,4
1993	531,7	289,6	-19,5	1002,3	1	20,8	7,2	14,6	30,0	28,4	4,3	4,9	200,5	16,6
1994	592,7	361,2	-39,5	1143,0	1	10,0	7,6	13,9	28,8	31,6	4,3	4,9	228,6	24,8
1995	761,9	461,8	-63,5	1466,5	1	9,1	7,2	13,5	26,5	31,5	4,0	5,0	306,3	67,9
1996	900,8	540,4	-98,3	1683,3	1	8,8	7,1	13,1	27,1	32,1	3,9	5,0	340,4	38,8
1997	983,5	542,1	-93,8	1811,1	1	8,5	8,7	13,7	31,7	29,9	4,8	4,9	379,3	41,3
1998	1056,1	562,4	-21,7	1996,5	1	10,7	9,4	13,3	32,3	28,2	6,5	4,9	399,7	81,9
1999	1102,2	562,3	-24,3	2080,8	1	2,1	9,1	9,0	36,9	27,0	8,7	4,8	440,6	168,7
2000	1181,9	612,5	-66,1	2189,2	1	3,9	9,0	7,3	35,6	28,0	8,8	4,7	460,9	129,8
2001	1255,0	659,3	-58,8	2352,2	1	4,7	9,0	6,8	34,1	28,0	8,1	4,7	496,7	214,6
2002	1288,5	677,8	-51,4	2464,4	1	1,8	8,8	5,9	30,8	27,5	7,3	4,8	549,5	277,7
2003	1345,2	687,5	-58,8	2577,1	1	0,1	8,2	4,5	31,9	26,7	7,8	4,7	603,2	59,3
2004	1464,1	727,2	1,9	2814,8	1	2,8	8,0	4,8	31,9	25,8	8,3	4,7	621,6	114,7
2005	1488,7	746,1	94,7	2987,7	1	1,9	7,2	4,2	29,8	24,9	7,9	4,8	658,2	263,2
2006	1622,1	812,9	111,2	3231,6	1	2,5	6,8	4,5	28,3	25,2	7,1	4,8	685,4	134,7
2007	1966,1	850,2	29,9	3557,7	1	2,8	8,5	2,1	27,8	23,9	5,3	4,9	711,5	218,0
Average	**1122,0**	**586,2**	**-23,7**	**2137,8**	**1,0**	**6,4**	**7,9**	**9,2**	**30,8**	**28,3**	**6,2**	**4,8**	**453,2**	**117,5**

Source: CZSO