

## Seminar 8

### Simultaneous econometric model - verification, interpretation and application

#### Exercises

1. Process an economic verification of the 1<sup>st</sup> equation of the model specified in seminar 6 (parameters estimated using TSLSM).

$$\hat{y}_{1t} = -1,07y_{3t} + 5868,9 - 43,42x_{3t} + 9,53x_{4t} - 142,12x_{10t} - 88,78x_{11t}$$

2. Process statistical verification of the equation specified above.

*Solution:*

- a) Calculation of adjusted residual variance  $\bar{S}_u^2$ :

$y_1$	$\hat{y}_1$	$y_1 - \hat{y}_1$	$(y_1 - \hat{y}_1)^2$	$y_1 - \hat{y}_1$	$(y_1 - \hat{y}_1)^2$
411,849	431,399	-710,179	504354,888	-19,550	382,221
531,725	637,251	-590,303	348458,193	-105,526	11135,713
592,715	676,245	-529,313	280172,755	-83,530	6977,214
761,867	778,093	-360,161	129716,289	-16,226	263,298
900,813	750,652	-221,215	48936,287	150,161	22548,415
983,544	1007,579	-138,484	19177,950	-24,035	577,677
1056,131	932,581	-65,897	4342,477	123,550	15264,683
1102,201	1280,652	-19,827	393,129	-178,451	31844,758
1181,920	1094,856	59,892	3586,995	87,064	7580,200
1255,014	1114,852	132,986	17685,150	140,162	19645,419
1288,474	1373,497	166,446	27704,113	-85,023	7228,846
1345,197	1518,818	223,169	49804,190	-173,621	30144,290
1464,086	1417,677	342,058	117003,350	46,409	2153,840
1488,689	1513,968	366,661	134439,940	-25,279	639,029
1622,135	1494,848	500,107	250106,536	127,287	16201,895
1966,096	1929,489	844,067	712449,327	36,607	1340,070

Total variance:  $S_y^2 = 165520,7$ .

Residual variance:  $S_u^2 = 10870,47$ .

Coefficient of determination:  $R^2 = 93,43 \%$ .

Degrees of freedom: 10.

Adjusted residual variance:  $\bar{S}_u^2 = 17392,76$ .

b) Calculation of  $S_{bi}$  and own statistical verification:

$$K^{-1} = \begin{vmatrix} 0,0002 & -0,1439 & -0,0002 & 0,0068 & 0,0035 & -0,0004 \\ -0,1439 & 141,1710 & 0,1359 & -5,9802 & -3,4004 & -0,2737 \\ -0,0002 & 0,1359 & 0,0052 & -0,0111 & -0,0051 & 0,0096 \\ 0,0068 & -5,9802 & -0,0111 & 0,3531 & 0,1320 & -0,0526 \\ 0,0035 & -3,4004 & -0,0051 & 0,1320 & 0,0854 & 0,0082 \\ -0,0004 & -0,2737 & 0,0096 & -0,0526 & 0,0082 & 0,0627 \end{vmatrix}$$

$S_{ii}$

$$S_{ii} = \begin{vmatrix} 3,07618 & & & & & & \\ & 2455353 & & & & & \\ & & 90,2258 & & & & \\ & & & 6141,155 & & & \\ & & & & 1485,498 & & \\ & & & & & 1090,937 & \\ & & & & & & \end{vmatrix}$$

	$\beta_{13}$	$\gamma_{11}$	$\gamma_{13}$	$\gamma_{14}$	$\gamma_{110}$	$\gamma_{111}$
$S_{bi}$	1,753904	1566,957	9,498724	78,36552	38,54216	33,02933
<b>parameter</b>	1,066446	5868,896	43,42332	9,528851	142,1154	88,77884
<b>t-value</b>	0,6080	3,7454	4,5715	0,1216	3,6873	2,687879
$\alpha=0,1$	I	S	S	I	S	S
$\alpha=0,05$	I	S	S	I	S	I

t-tab:  $t_{0,1} = 1,8121$ ;  $t_{0,05} = 2,2281$ .

3. Process and economic verification of the equation specified above and propose modifications to its improvement.

4. Calculate confidence interval for all estimated parameters.

5. Process an economic and statistical verification of the following equation (1<sup>st</sup> equation of the model specified in previous seminars) estimated using MVRM.

$$y_{1t} = -13.93y_{3t} + 16337 - 29.4x_{3t} + 483x_{4t} - 394x_{10t} - 61.7x_{11t} + u_{1t}$$

$y_1$	$\hat{y}_1$	$y_1 - \bar{y}_1$	$(y_1 - \bar{y}_1)^2$	$y_1 - \hat{y}_1$	$(y_1 - \hat{y}_1)^2$
411,8					
531,7					
592,7					
761,9					
900,8					
983,5					
1056,1					
1102,2					
1181,9					
1255,0					
1288,5					
1345,2					
1464,1					
1488,7					
1622,1					
1966,1					
Průměr					
Suma					
Délka časové řady					
Počet stupňů volnosti					
Reziduální rozptyl					
Korigovaný reziduální rozptyl					
Celkový rozptyl					
Koeficient vícenásobné determinace					
Korigovaný koeficient vícenásobné determinace					

Matrix X contains vectors  $x_1, x_3, x_4, x_{10}, x_{11}, y_3$  in this order.

$$(X^T X)^{-1} \begin{pmatrix} 80,565 & 0,055 & -3,127 & -1,944 & -0,431 & -0,069 \\ 0,055 & 0,005 & -0,007 & -0,003 & 0,009 & 0,000 \\ -3,127 & -0,007 & 0,219 & 0,063 & -0,045 & 0,003 \\ -1,944 & -0,003 & 0,063 & 0,050 & 0,012 & 0,002 \\ -0,431 & 0,009 & -0,045 & 0,012 & 0,062 & 0,000 \\ -0,069 & 0,000 & 0,003 & 0,002 & 0,000 & 0,00009 \end{pmatrix}$$

	$\gamma_{11}$	$\gamma_{13}$	$\gamma_{14}$	$\gamma_{110}$	$\gamma_{111}$	$\beta_{13}$
$S_{bi}$						
t-value						
$\alpha=0,1$						
$\alpha=0,05$						

6. Transform the following structural econometric model into the reduced form:

$$\hat{y}_{1t} = -1,07y_{3t} + 5868,9 - 43,42x_{3t} + 9,53x_{4t} - 142,12x_{10t} - 88,78x_{11t}$$

$$\hat{y}_{2t} = -0,17y_{3t} - 1020,8 - 40,7x_{5t} + 408,42x_{12t}$$

$$\hat{y}_{3t} = 0,06y_{1t} - 99,27 + 0,08x_{15t}$$

$$\hat{y}_{4t} = y_{1t} + y_{2t} + y_{3t} + x_{13t}$$

## Individual exercises

1. Verify the statistical significance of the 2<sup>nd</sup> equation of the model estimated using TSLSM.
2. Verify the statistical significance of the 3<sup>rd</sup> equation of the model estimated using TSLSM.
3. Process statistical verification of the 1<sup>st</sup> equation of the following econometric model:

$$y_{1t} = f(y_{2t}, x_{1t}, x_{2t}) + u_{1t}$$

$$y_{2t} = f(y_{1t}, x_{1t}, x_{3t}) + u_{2t}$$

if data set,  $K^{-1}$  and parameters estimated using TSLSM are following:

Period	$y_1$	$y_2$	$x_1$	$x_2$	$x_3$
1	4	1,5	1	0,5	1,1
2	3	1,8	1	0,6	2,1
3	5	1,4	1	0,4	2
4	3,5	1,3	1	0,7	3
Average	3,875	1,5	1	0,55	2,05

$$K^{-1} = \begin{vmatrix} 34,9481 & -52,4221 & 2,5E-13 \\ -52,422 & 84,9332 & -11 \\ 2,6E-13 & -11 & 20 \end{vmatrix}$$

$$\begin{vmatrix} -3,0882 \\ 11,5324 \\ -5,5 \end{vmatrix} \begin{matrix} \beta_{12} \\ \gamma_{11} \\ \gamma_{12} \end{matrix}$$

4. Transform the following structural econometric model into the reduced form:

$$\hat{y}_{1t} = 3y_{2t} + 0,5x_{1t} + 2x_{2t}$$

$$\hat{y}_{2t} = 0,3x_{1t} + 0,7x_{3t}$$

5. Transform the following structural econometric model into the reduced form:

$$\hat{y}_{1t} = 3y_{2t} + 1,5x_{1t} + 2x_{2t}$$

$$\hat{y}_{2t} = 0,5y_{1t} + 0,3x_{1t} + 0,7x_{3t}$$