$X \sim N(0,1)$

$P[X \leq 0.5] = \Phi(0.5) = 0.6915$

$P[X \leq -0.5] = \Phi(-0.5) = 1 - \Phi(0.5) = 1 - 0.6915 = 0.3085$

$\Phi(-x) = 1 - \Phi(x)$

$P[0.5 \leq X \leq 0.5] = \Phi(0.5) - \Phi(-0.5) = 0.6915 - 0.3085 = 0.383$

95% interval

$\Phi(a) = 0.95$

$a = \Phi^{-1}(0.95)

= 1.645$

$(-\infty, 1.645)$

$\Phi'(0.05) = -\Phi'(0.95) = -1.645$

$(-1.645, \infty)$

$\Phi'(a) = 0.025$

$a = \Phi^{-1}(0.025)

= -1.96$

$(-1.96, 1.96)$

$\Phi^{-1}(0.975) = 1.96$
\( \text{3) } X \sim \mathcal{N}(15, 16) \quad \mu \) 

\[
P[10 \leq X \leq 14] = P[X \leq 14] - P[X \leq 10] = 0.6915 - 0.10565 = 0.58585
\]

\[
P[X \leq 14] = \Phi \left( \frac{X - \mu}{\sigma} \right) = \Phi \left( \frac{14 - 15}{\sqrt{16}} \right) = \Phi \left( \frac{14 - 15}{4} \right) = \Phi \left( \frac{-5}{4} \right) = \Phi(-1.25) = 1 - \Phi(1.25)
\]

\[
P[X \leq 10] = 1 - \frac{0.8925 + 0.8362}{2} = 1 - 0.89435 = 0.10565
\]

90% down to naming int.

\[
P[X \leq a] = 5\%
\]

\[
P\left[ \frac{X - 15}{4} \leq \frac{a - 15}{4} \right] = 0.05
\]

\[
\Phi \left( \frac{a - 15}{4} \right) = 0.05
\]

\[
\frac{a - 15}{4} = \Phi^{-1}(0.05)
\]

\[
a = 4 \cdot \Phi^{-1}(0.05) + 15
\]

\[
a = 4 \cdot 1.645 + 15 = 21.58
\]

\[
P[X \leq b] = 0.95
\]

\[
P\left[ \frac{X - 15}{4} \leq \frac{b - 15}{4} \right] = 0.95
\]

\[
\Phi \left( \frac{b - 15}{4} \right) = 0.95
\]

\[
b = 4 \cdot \Phi^{-1}(0.95) + 15
\]

\[
b = 4 \cdot 1.645 + 15 = 21.58
\]

\[
\langle 8.142, 21.58 \rangle
\]
\[ f_X(x) = \begin{cases} \frac{1}{2} & 0 < x < 1 \\ 0 & \text{otherwise} \end{cases} \]

\[ F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{x^2}{2} & 0 \leq x < 1 \\ 1 & x \geq 1 \end{cases} \]

\[ q_X(x) = \sqrt{x} \]

\[ f_Y(y) = \begin{cases} 0 & y < 0 \\ 2 & 0 \leq y < 1 \\ 0 & y \geq 1 \end{cases} \]

\[ F_Y(y) = \begin{cases} 0 & y < 0 \\ \frac{(y-3)^2}{4} & 0 \leq y < 3 \\ 1 & y \geq 3 \end{cases} \]

\[ q_Y(y) = \sqrt{y+3} \]

\[ x = \sqrt{y} \]

\[ f_Z(z) = \begin{cases} 0 & z < 0 \\ \frac{2z}{\sqrt{2}} & 0 \leq z < \sqrt{2} \\ 0 & z \geq \sqrt{2} \end{cases} \]

\[ F_Z(z) = \begin{cases} 0 & z < 0 \\ \left(\frac{z}{\sqrt{2}}\right)^2 & 0 \leq z < \sqrt{2} \\ 1 & z \geq \sqrt{2} \end{cases} \]

\[ q_Z(z) = 2 \cdot \sqrt{z} \]

\[ u = -X \]

\[ f_U(u) = \begin{cases} 0 & u < 0 \\ -2u & 0 \leq u < 1 \\ 0 & u \geq 1 \end{cases} \]

\[ F_U(u) = \begin{cases} 0 & u < 0 \\ 1 - \left(\frac{u}{2}\right)^2 & 0 \leq u < 1 \\ 1 & u \geq 1 \end{cases} \]

\[ q_U(u) = \sqrt{1-u} \]

\[ y = \frac{u}{1-u} \]

\[ q_U(u) = -\sqrt{1-u} = -q_X(u-x) \]
\[ W = X^2 \]

\[ F_W(x) = P[W \leq x] = P[X^2 \leq x] = P[X \leq \sqrt{x}] = F_X(\sqrt{x}) \]

\[ F_X(x) = \begin{cases} 
0 & x < 0 \\
\frac{x^2}{2} & 0 \leq x \leq 1 \\
1 & x \geq 1 
\end{cases} \]

\[ F_W(x) = \begin{cases} 
0 & x < 0 \\
\frac{x^2}{2} & 0 \leq x^2 \leq \sigma^2 \\
1 & x^2 \geq \sigma^2 
\end{cases} \]