Dynamics

- **Dynamics** – explains HOW and WHY bodies move
- Dynamics explains a motion of a body by the action of a force
- **Force** – any kind of interaction between bodies
- Force $\vec{F}$ – a vector quantity
- To describe a force completely, we specify its magnitude and its direction
- Several forces acting on a body can be replaced by a single force – resultant of forces – given by vector addition of forces
- We want to know how the body will move if the force acts on this body
- It is described by **Newton's laws of motion** – three fundamental laws of classical mechanics

Dynamics

- **<u>Law of Inertia</u> – Newton's first law of mechanics:**
- **Every body remains in a state of rest or uniform straight-line motion unless acted upon by a force**
- **The force or resultant of forces is zero**
- **<u>Inertia</u> – the tendency of a body to maintain a resting or uniform straight-line motion**
- **<u>Law of Force</u> – Newton's second law of mechanics:**
- **The product of the mass of the body and its acceleration is equal to the total force acting on a body**
- **The direction of the acceleration is in the direction of the applied force (or resultant of forces)**
- $\vec{F} = m\vec{a}$
- **SI unit of force: [*F*] = N (newton), [*F*] = [*m*] . [*a*] = N = kg.m.s^{-2}**

Dynamics

- **<u>Law of Action and Reaction</u> - Newton's third law of mechanics:**
- **When a body A acts on a body B (*action force* $\vec{F}_{AB}$) then the body B acts on the body A (*reaction force* $\vec{F}_{BA}$)**
- **To every action force there is an equal and the opposite reaction force:** $\vec{F}_{AB} = -\vec{F}_{BA}$
- **These two forces $\vec{F}_{AB}$ and $\vec{F}_{BA}$ are equal in magnitude and are oppositely directed**

***Example*:**

The force exerted by the person on the floor (action) and the force exerted by the floor on the person (reaction). The result of these two forces is zero – the person is at rest.

Dynamics

- **<u>Momentum of particle</u> – defined as the product of its mass *m* and its velocity $\vec{v}$: $\vec{p} = m\,\vec{v}$**
- **SI unit of momentum [*p*] = [*m*] . [*v*] = kg.m.s^{-1}**
- **The change of the momentum of the body is equal to the total force acting upon it: $\vec{F} = m\vec{a} = m\frac{\mathrm{d}\vec{v}}{\mathrm{d}t} = \frac{\mathrm{d}\vec{p}}{\mathrm{d}t}$**
- **We assume the mass of the body as constant (*m* = constant) in classical physics**

Application:

The rocket works on the principle of action and reaction. Fuel is burned and gases are expelled from the rocket. At the same time, these gases exert an equal and opposite force on the rocket - the rocket accelerates. The rocket gains forward momentum, which is equal to the momentum of the gases pushed backwards.

Dynamics

- **Impulse of force**
- $\vec{F} = \frac{d\vec{p}}{dt} \Rightarrow \vec{F}dt = d\vec{p}$
- $\vec{I} = \int_{t_1}^{t_2} \vec{F}dt = \vec{p}_2 - \vec{p}_1 = \Delta\vec{p}$
- **Impulse $\vec{I}$ of force during time interval $\Delta t = t_2 - t_1$ changes the momentum of the body from $\vec{p}_1$ to $\vec{p}_2$**
- **SI unit of impulse of force: [*I*] = [*F*] . [*t*] = N.s = $kg.m.s^{-1}$**

- **Equations of motion**
- **Coordinates of vector $\vec{F}$ (from Law of force) give equations of motion for *x,y,z* axes:**
- $F_x = ma_x \Rightarrow a_x = \frac{F_x}{m}$, $F_y = ma_y \Rightarrow a_y = \frac{F_y}{m}$, $F_z = ma_z \Rightarrow a_z = \frac{F_z}{m}$

Dynamics

- **<u>Functions of motion</u>**

From equations of motion we determine functions of motion *x(t), y(t), z(t)*:

$$a_x = \frac{F_x}{m} \longrightarrow v_x = \int a_x \, \mathrm{d}t \longrightarrow x(t) = \int v_x \, \mathrm{d}t$$

$$a_y = \frac{F_y}{m} \longrightarrow v_y = \int a_y \, \mathrm{d}t \longrightarrow y(t) = \int v_y \, \mathrm{d}t$$

$$a_z = \frac{F_z}{m} \longrightarrow v_z = \int a_z \, \mathrm{d}t \longrightarrow z(t) = \int v_z \, \mathrm{d}t$$

- Symbol $\int$ is indefinite integral (no limits of integration)

- To solve equations of motion as indefinite integrals, we need to know constants of integrations – *initial conditions* - coordinates of initial position vector $\vec{r}(0)$ and initial velocity $\vec{v}(0)$ in time *t* = 0: *x(0), y(0), z(0)* and $v_x(0)$, $v_y(0)$, $v_z(0)$

Dynamics

- **<u>Type of motion depends on the force</u> – Some examples:**

***A – The resulting force is zero*:**

- $\vec{F} = \vec{0} = m\vec{a} \Rightarrow \vec{a} = \frac{d\vec{v}}{dt} = \vec{0} \Rightarrow \vec{v} = const.$
- **1 - If *v* = 0 , body is <u>at rest</u>**
- **2 - a) *v* = constant ≠ 0 ⇒ uniform motion**
- **b) direction isn't changed ⇒ straight-line motion**

- **Uniform motion in one dimension – <u>uniform straight-line motion</u>**

Dynamics

- **We suppose this motion along *x* axis with initial conditions for speed $v_x(0)$ and for position $x(0)$:**
- $a_x = 0, a_x = \frac{dv_x}{dt} \Rightarrow v_x = const., v_x = v_x(0)$:
- **motion with constant speed along *x* axis**
- $v_x = \frac{dx}{dt} \Rightarrow x = \int v_x \, dt, x = v_x(0) \cdot t + x(0)$
- **Distance *d* in time *t*: $d(t) = x(t) - x(0)$**
- **If $x(0) = 0$ – the particle is at the origin at time $t = 0$ (at the beginning of motion), then $d(t) = x(t) = v_x \cdot t$**

Dynamics

***B – Constant force parallel with initial velocity*:**

- $\vec{F} = const. \Rightarrow \vec{a} = const.$

- $\vec{F} \parallel \vec{v}_0$, $m\vec{a} \parallel \vec{v}_0$, $\vec{a} = \frac{d\vec{v}}{dt} \parallel \vec{v}_0 \Rightarrow \vec{v} \parallel \vec{v}_0$

- **Particle (body) moves along a straight line with constant acceleration:**

$a = \frac{F}{m} \Rightarrow$ ***uniformly accelerated straight-line motion***

Law of force $\vec{F} = m\vec{a}$ for motion in one dimension (*x* - axis) with initial conditions for velocity $v_x(0)$ and for position $x(0)$:

$$F_x = ma_x \Rightarrow a_x = \frac{F_x}{m}$$

Dynamics

- **Then:**

- $\boldsymbol{a_x = \frac{dv_x}{dt} \Rightarrow v_x = \int a_x\, dt}$ **, if** $\boldsymbol{a_x}$ **= const., then** $\boldsymbol{v_x = a_x t + v_x(0)}$

- $\boldsymbol{v_x = \frac{dx}{dt} \Rightarrow x = \int v_x\, dt}$ **, if** $\boldsymbol{a_x}$ **= const., then** $\boldsymbol{x = \frac{1}{2} a_x \cdot t^2 + v_x(0) \cdot t + x(0)}$

- **The distance *d* the particle travels in time *t*:**

- $\boldsymbol{d(t) = x(t) - x(0) = \frac{1}{2} a_x \cdot t^2 + v_x(0) \cdot t}$

Dynamics

- **<u>Newton's Law of Gravitation</u>**
- **Any material bodies attract each other – gravitational attraction – <u>gravitational force</u>**
- **The magnitude of the gravitational force between two particles (mass points) of masses m_1 and m_2 separated by a distance d between them:**
- $F = G\frac{m_1 m_2}{d^2}$ **, G = 6.67 · 10^{-11} $N.m^2.kg^{-2}$ – Newton's gravitational constant**

- **<u>Force of Gravity</u> – gravitation caused by the Earth attraction:**
- **One body is the Earth with mass M = 5.98 · 10^{24} kg, the other body of mass m is on the surface of the Earth**
- **The distance between their centres of mass is equal to the radius of the Earth R = 6378 km**
- **then** $F_G = G\frac{Mm}{R^2} = g \cdot m \Rightarrow$ **calculation g = …**

Dynamics

- **g – acceleration due to gravity (gravitational acceleration) conventionally - g = 9.81 m.s^{-2}**

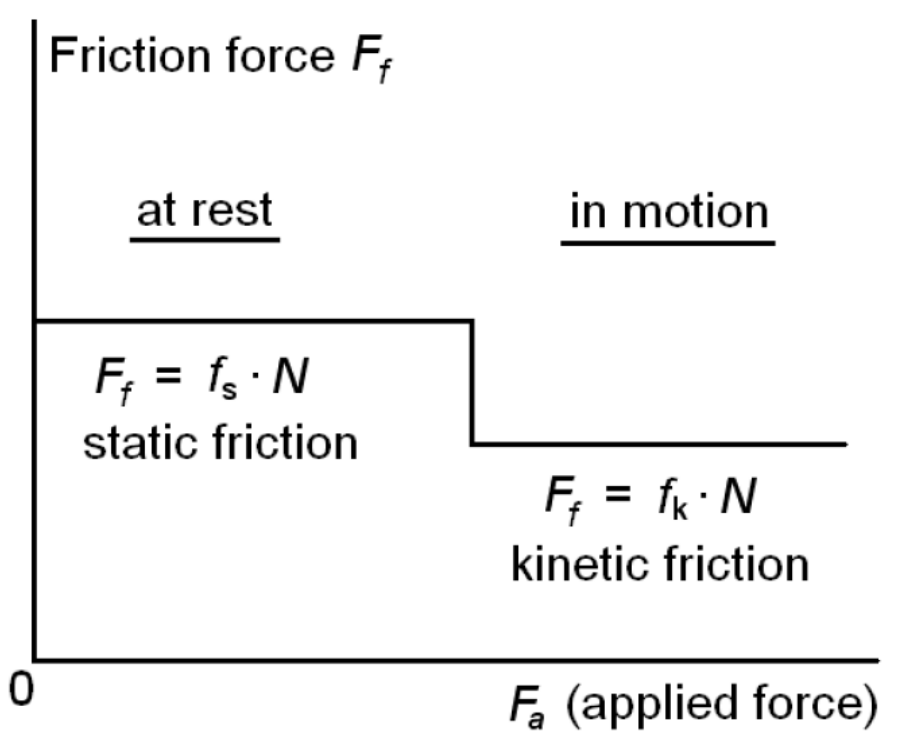
- **<u>Weight</u>: $W = m\,g$ – the gravitational force between Earth and a body of mass m on its surface**
- **This force is always directed downwards**
- **<u>Free fall</u>**
- **Due to the force of gravity W, all objects fall with uniform gravitational acceleration g**
- **Then the body moves uniformly in an accelerated straight-line motion**
- **If the initial velocity is zero, then acceleration a , speed v and distance (height) h are:**
- $a = g,\ v = g\ .\ t,\ \ h = ½\ g\ .\ t^2$

Dynamics

- **<u>Friction</u>**
- **The frictional force F_f acts against the motion or tendency of the body to move and acts parallel to the surface**
- **The frictional force opposes the movement of one surface relative to another**
- **Magnitude of the frictional force $F_f = f \cdot N$**
- **N – normal force (perpendicular to the surface)**
- **f – the coefficient of friction depends on the surfaces of the body and the plane on which the body moves**

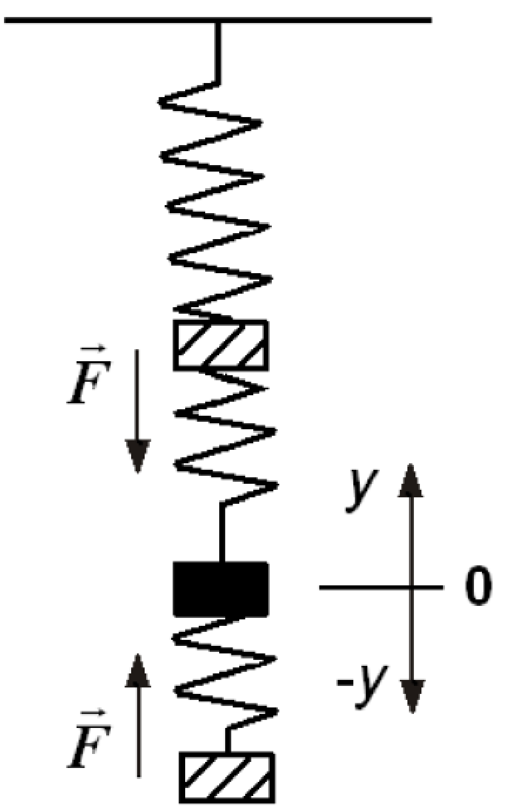
Dynamics

- [*f*] – dimensionless quantity
- For body moving along a horizontal surface, the normal force is equal to its weight $N = W$
- f_s – coefficient of static friction, f_k – coefficient of kinetic friction ($f_s > f_k$)
- It is easier to keep an object moving than to start its motion – see Fig.



Dynamics

- **<u>Simple harmonic motion (SHM)</u>**
- **When solving the mass-spring system (see Fig.), the return force is proportional to the displacement, but has the opposite sign: $F = -k\,y$**
- **k – spring constant $[k]$ = N/m**



Linear harmonic oscillator

Dynamics

- The force $\vec{F}$ is directed to the equilibrium position 0, opposite to the displacement *y*
- We use Newton's law of force $\vec{F} = m\vec{a}$ for motion along *y* axis:

F* = *m.a* = - *k.y $\Rightarrow$ **equation of motion (second-order differential equation):**

$$m\frac{d^2y}{dt^2} + ky = 0 \Rightarrow \frac{d^2y}{dt^2} + \frac{k}{m}y = 0$$

- The solution – a displacement along *y* axis – is a simple harmonic motion (SHM): *y* = *A* sin(*ωt* + *φ*)
- *A* – amplitude of motion
- (*ω.t* + *φ*) – phase of motion
- *φ* – phase constant – phase angle for time *t* = 0

Dynamics

- ***Angular frequency***: $\omega = \frac{2\pi}{T} = 2\pi f$, $\omega = \sqrt{\frac{k}{m}}$, where
- ***Period T*** – the time to complete one cycle [*T*] = s,
- ***Frequency f = 1/T*** is number of cycles per second [f] = s^{-1} = Hz (hertz)

- **Period of the simple harmonic motion (SHM):** $T = 2\pi\sqrt{\frac{m}{k}}$
- ***Speed*** **of SHM – the first derivative of** ***y*** **with respect to time** ***t*****:**
- $v = \frac{\mathrm{d}y}{\mathrm{d}t} = \omega A cos(\omega \mathrm{t} + \varphi)$

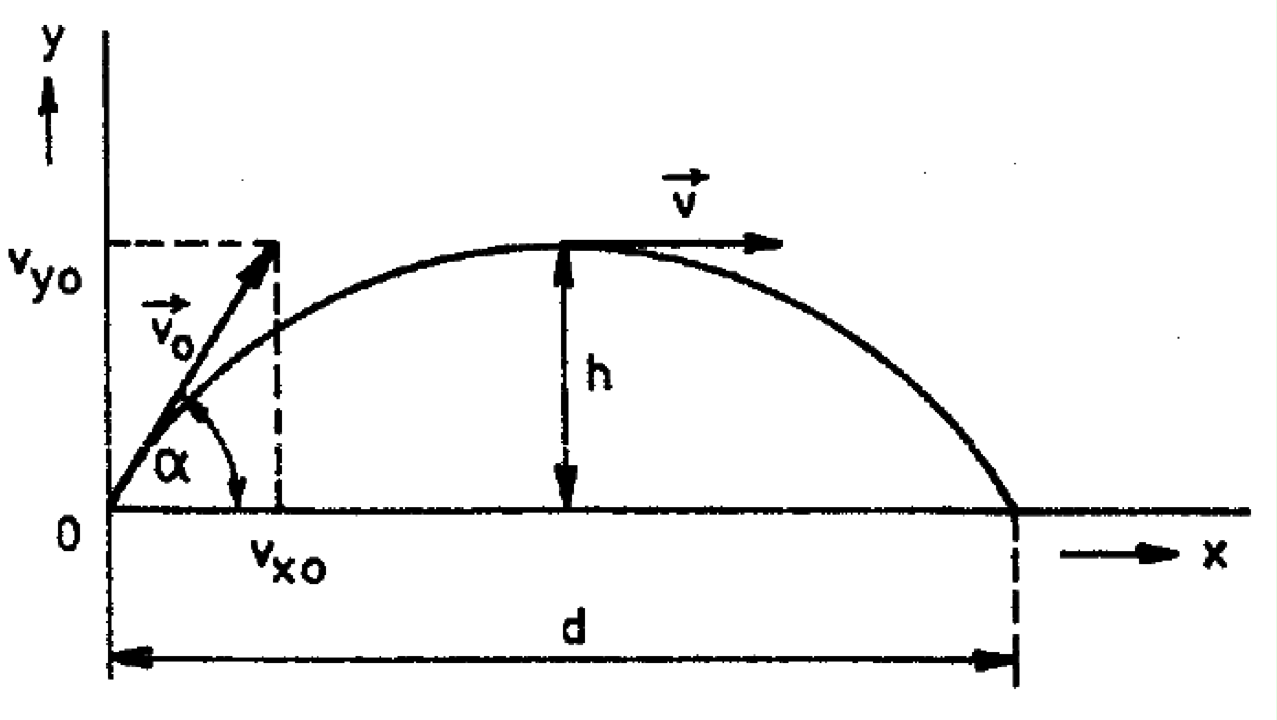
- ***Acceleration*** **of SHM – the first derivative of speed** ***v*** **with respect to time** ***t*** **or the second derivative of displacement** ***y*** **with respect to** ***t*****:**
- $a = \frac{\mathrm{d}v}{\mathrm{d}t} = \frac{d^2y}{dt^2} = -\omega^2 A sin(\omega \mathrm{t} + \varphi) = -\omega^2 y$

Dynamics

***C – Constant force is not parallel with initial velocity*:**

- $\vec{F} = const., \vec{F} \nparallel \vec{v}_0$
- **<u>Projectile motion</u> (air resistance is neglected):**

Particle is projected with initial velocity $\vec{v}_0$ at an angle α to the horizontal plane – the motion is in the *xy* plane – two dimension motion – see Fig.:



Dynamics

- It is plane motion – Law of force $\vec{F} = m\vec{a}$ for coordinates *x* and *y* are equations of motion:
- $\vec{F} = \vec{W}$ (weight) $= 0\vec{i} - W\vec{j} + 0\vec{k}$:
- $F_x = m\,a_x = 0 \Rightarrow a_x = 0$, $F_y = m\,a_y = -W = -mg \Rightarrow a_y = -g$
- Initial conditions: particle is in origin at the beginning of motion:
- $t = 0$; $x_0 = 0$; $y_0 = 0$
- $v_x(0) = v_0 \cos\alpha$; $v_y(0) = v_0 \sin\alpha$

- From equation of motion $a_x = 0$:
- $v_x = \text{const.} = v_x(0) = v_0 \cos\alpha$, $x = (v_0 \cos\alpha) \,.\, t$
- From equation of motion $a_y = -g$:
- $v_y = -g\,t + v_0 \sin\alpha$, $y = -\frac{1}{2}\,g \,.\, t^2 + (v_0 \sin\alpha) \,.\, t$

Dynamics

- **The highest point of the path *h* (in which $v_y = 0$) occurs at time t_h:**

- $$v_y(t_h) = \frac{dy}{dt} = 0 \Rightarrow v_0 sin\alpha - g t_h = 0 \Rightarrow t_h = \frac{v_0 sin\alpha}{g}$$

- **and for maximum height is:**

- $$h = y(t_h) = -\frac{1}{2} g \left(\frac{v_0 sin\alpha}{g}\right)^2 + v_0 sin\alpha \cdot \frac{v_0 sin\alpha}{g} = \frac{1}{2}\frac{(v_0 sin\alpha)^2}{g}$$

Dynamics

- **The distance *d* (horizontal range) of the point where the particle hits the ground (*y* = 0) occurs at time t_d:**

- $y(t_d) = 0 = -\frac{1}{2} g t_d^2 + v_0 sin\alpha \cdot t_d$ **(quadratic equation – two roots)**

- **from it t_{d1} = 0 (time of the beginning of motion – *y*(0) = 0 – the origin)**

- **and** $t_{d2} = \frac{2 v_0 sin\alpha}{g}$ **, then:**

- $d = x(t_{d2}) = v_0 cos\alpha \cdot t_{d2} = v_0 cos\alpha \cdot \frac{2 v_0 sin\alpha}{g} = 2 \frac{v_0^2 cos\alpha \cdot sin\alpha}{g}$

Dynamics

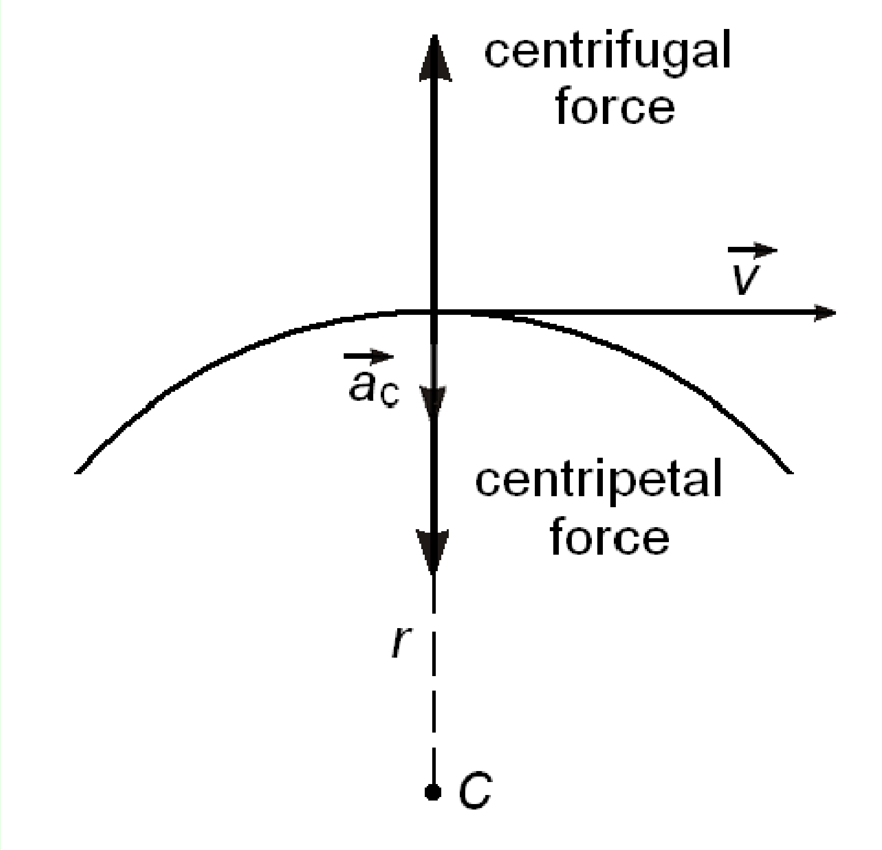
- **Uniform circular motion**
- ***F* = constant (magnitude of force is constant)**
- **The force is perpendicular to the velocity $(\vec{F} \perp \vec{v})$ directed to the centre of the circle – centripetal force**
- **The body moves in a circle of radius *r* with constant speed *v* and tangential acceleration $a_t = \frac{dv}{dt} = 0$**
- **The body is only accelerated toward the centre of the circle – centripetal acceleration:**
- $a_c = const.$; $a_c = \frac{v^2}{r}$; $\vec{a}_c \neq const.$; $\vec{a}_c \perp \vec{v}$

Dynamics

- **Uniform circular motion**
- **speed is constant $v = d/t$ (distance d divided by time t)**
- **and $v = \frac{2\pi r}{T} = 2\pi f r = \omega r$, ($d = 2\pi r$ – circumference of a circle), where**
- **period T – the time to complete one cycle $[T] = s$,**
- **frequency $f = 1/T$ – the number of cycles per second $[f] = s^{-1} = Hz$**
- **angular velocity $\omega = 2\pi f$ – the angle in radians per second $[\omega] = rad.s^{-1}$**
- **Magnitude of the centripetal force: $F_c = m a_c = m\frac{v^2}{r}$**

Dynamics

- The centripetal force acts if the body moves in a circle (generally along any curved path)
- <u>Centrifugal force</u> is an apparent force, which has the same magnitude as centripetal force but in the opposite direction – see Fig.



Dynamics

Example 3.5:

A 250 kg cage is lowered into the mining pit. It travels a distance of 35 m in 10 seconds. Determine the force *F* that stresses the weighting rope, assuming that the motion is uniformly accelerated.

[*F* = 2 277.5 N]