Fluids at rest – hydrostatics

- **The fundamental difference between solids and fluids: fluids can flow**

- **Fluids – liquids and gases**

- **The fluids – usually in a state of compression**

- **<u>Pressure</u> *p* in a fluid – the force *F* acting at right angle to surface *A***
- **The average pressure *p* over the area *A* – defined by:** $p = \frac{F}{A}$
- **SI unit of pressure – deduced from definition [*p*] = N.m^{-2} = Pa (pascal)**

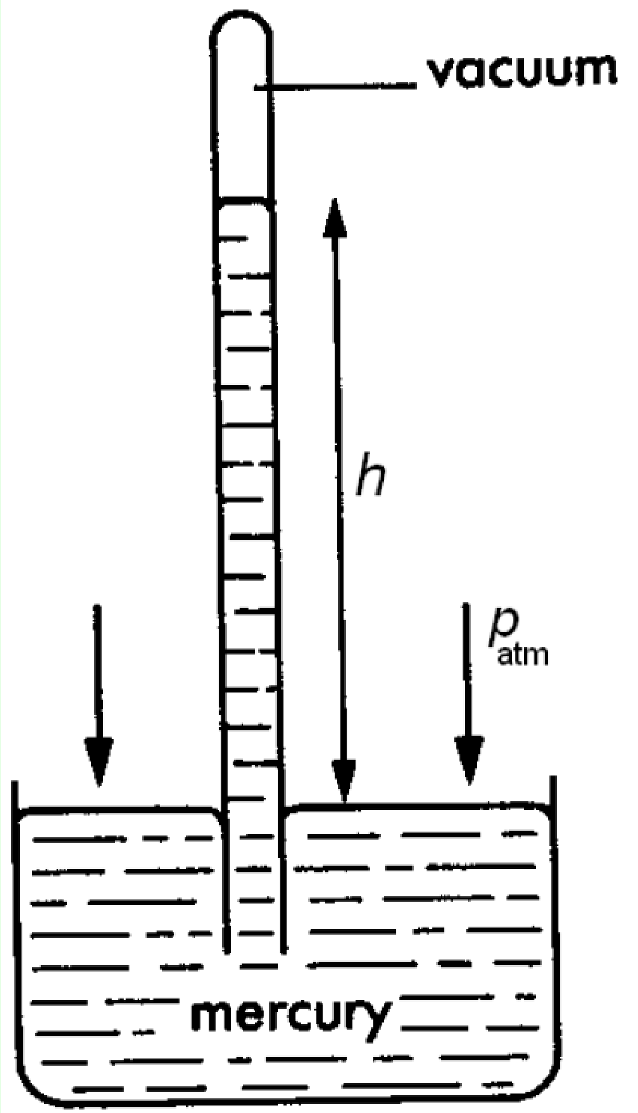
Fluids at rest – hydrostatics

- <u>Atmospheric pressure</u> – caused by weight of air

The standard atmospheric pressure at sea level:

$p_{atm} = 1.013 \cdot 10^5$ Pa

A barometer – a device for to measure atmospheric pressure. The mercury barometer constructed by Torricelli – see Fig. – a glass tube about 80 cm long, closed at one end and filled with mercury. The open end of the tube is immersed in a reservoir of mercury. The column of mercury in the tube is held up by the pressure of the atmosphere acting on the surface of the mercury in the reservoir. The height of the mercury at standard atmospheric pressure: h = 760 mm

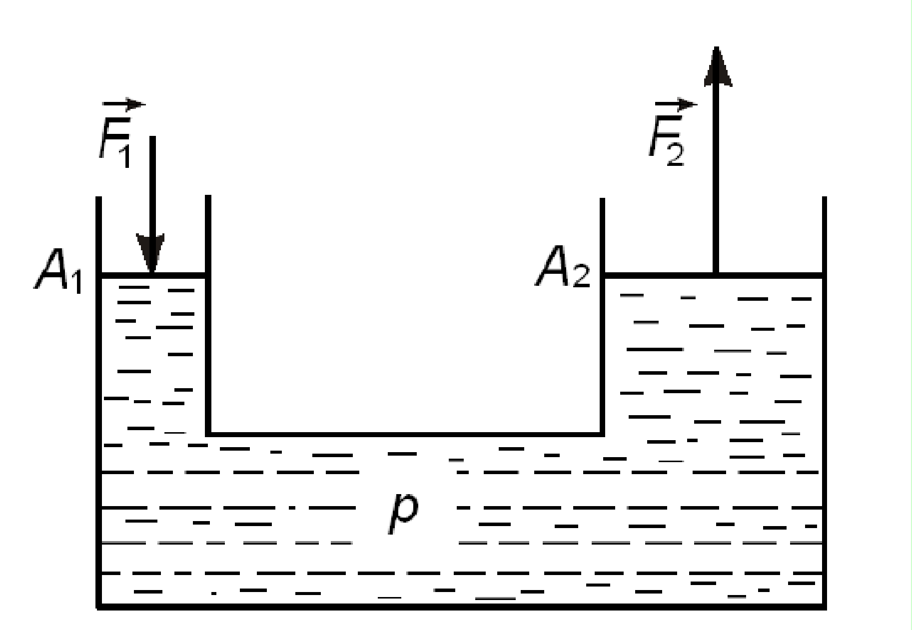


Fluids at rest – hydrostatics

- **Atmospheric pressure decreases approximately exponentially with increasing altitude**
- **<u>Pascal's principle</u>: Hydrostatic pressure in a fluid acts equally in all directions at any point**

- ***Application***
- **<u>Hydraulic press (jack)</u>**
- **This is the use of fluid pressure to exert a large force by applying a small force**
- **Consider a vessel with two cylinders, each containing a perfectly fitting piston – see Fig.**

Fluids at rest – hydrostatics



If a force F_1 is applied to a small piston of cross-section A_1, the pressure in the liquid (usually oil): $p = \frac{F_1}{A_1}$

- **This pressure p is transmitted through the liquid and acts on the piston in the larger cylinder of cross-section A_2**
- **The force F_2 acting on the larger piston:** $F_2 = pA_2 = \frac{F_1}{A_1} A_2 = F_1 \frac{A_2}{A_1}$

Fluids at rest – hydrostatics

- **The force that can be exerted by a hydraulic press can be of almost arbitrary when using a working piston of a suitable cross-section**

- **The hydraulic press – used to exert high forces when pressing metals, lifting loads, tipping, etc.**

Fluids in motion – hydrodynamics

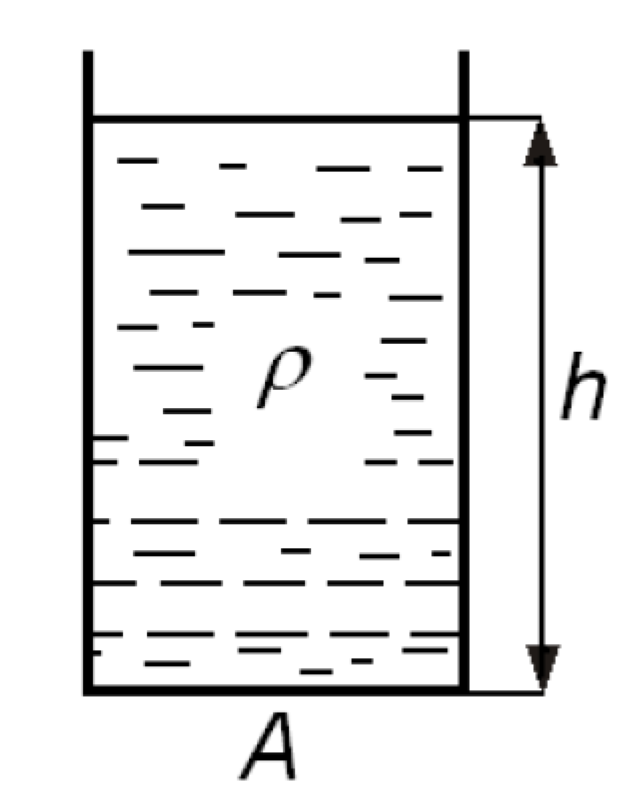
Example 5.7:

What is the force exerted by the large piston of a hydraulic jack (e.g. a jack for tilting a hull) when a force of 500 N is applied to the small piston? The diameter of the large (lifting) piston is 6 cm, the diameter of the small piston (pump piston) is 1 cm.

[*F* = 18 kN]

Fluids at rest – hydrostatics

- **<u>Hydrostatic pressure</u> p_h – pressure in a liquid due to gravity**
- **We suppose a liquid of density ρ in a closed cylinder of base area A – see Fig.**



The pressure p – the weight G of the liquid in the vessel above the bottom of area A:

$$p = \frac{F}{A} = \frac{G}{A} = \frac{mg}{A} = \frac{\rho V g}{A} = \frac{\rho A h g}{A} = \rho g h \,,$$

$m = \rho V$ – mass of the liquid in a cylinder vessel

$V = A h$ – volume of the liquid in a vessel

h – height of the liquid in a vessel

- The total pressure at the bottom of a vessel p_{total} – made by atmospheric pressure p_{atm} and hydrostatic pressure p_h:

$$p_{total} = p_{atm} + p_h = p_{atm} + \rho g h$$

Fluids at rest – hydrostatics

- **<u>Archimedes' Principle:</u>**
- **The buoyant force acting on a body immersed in a liquid (fluid) is equal to the gravity of the liquid (fluid) displaced by the body: $F_A = \rho g V$,**
- **ρ – the density of liquid (fluid)**
- **V – the volume of the submerged part of the body**
- **The buoyant force F_A – directed upwards $\Rightarrow$ the opposite direction to the weight of body G**
- **The apparent weight of the body G_A submerged in liquid is smaller than the weight of the body G in the air: $G_A = G - F_A$**
- **The body of density ρ_b in the liquid of density ρ :**
- **$\rho_b > \rho \Rightarrow G > F_A \Rightarrow$ body will sink,**
- **$\rho_b = \rho \Rightarrow G = F_A \Rightarrow$ body will float in the liquid (resultant force is zero),**
- **$\rho_b < \rho \Rightarrow G < F_A \Rightarrow$ body will float on the liquid**

Fluids at rest – hydrostatics

- Flotation
- When a body is floating on the surface of a liquid, the weight of the liquid displaced by the submerged part of the body is equal to the weight of the whole body

- *Application*
- Submarine
- All of the above cases may involve a submarine
- It can change its average density by pumping water into or out of large ballast tanks

Fluids at rest – hydrostatics

<u>**Example 5.16:**</u>

***Alloy components*:**

The Greek king asked Archimedes for help. He gave the goldsmith the gold and asked for a golden royal crown. The challenge for Archimedes was to prove whether this crown was actually made of pure gold or an alloy with silver. A crown weighing 14.7 kg has an apparent weight equivalent to 13.4 kg when immersed in water. Is the crown pure gold? If not, what is the weight of the gold in the crown?

Density of gold $\rho_g = 19.3 \cdot 10^3$ kg/m^3, density of silver $\rho_s = 10.5 \cdot 10^3$ kg/m^3.

Solution – see paper

Fluids at rest – hydrostatics

- **<u>Balloon</u>**
- **Archimedes' Principle of buoyancy is also applicable to gases**

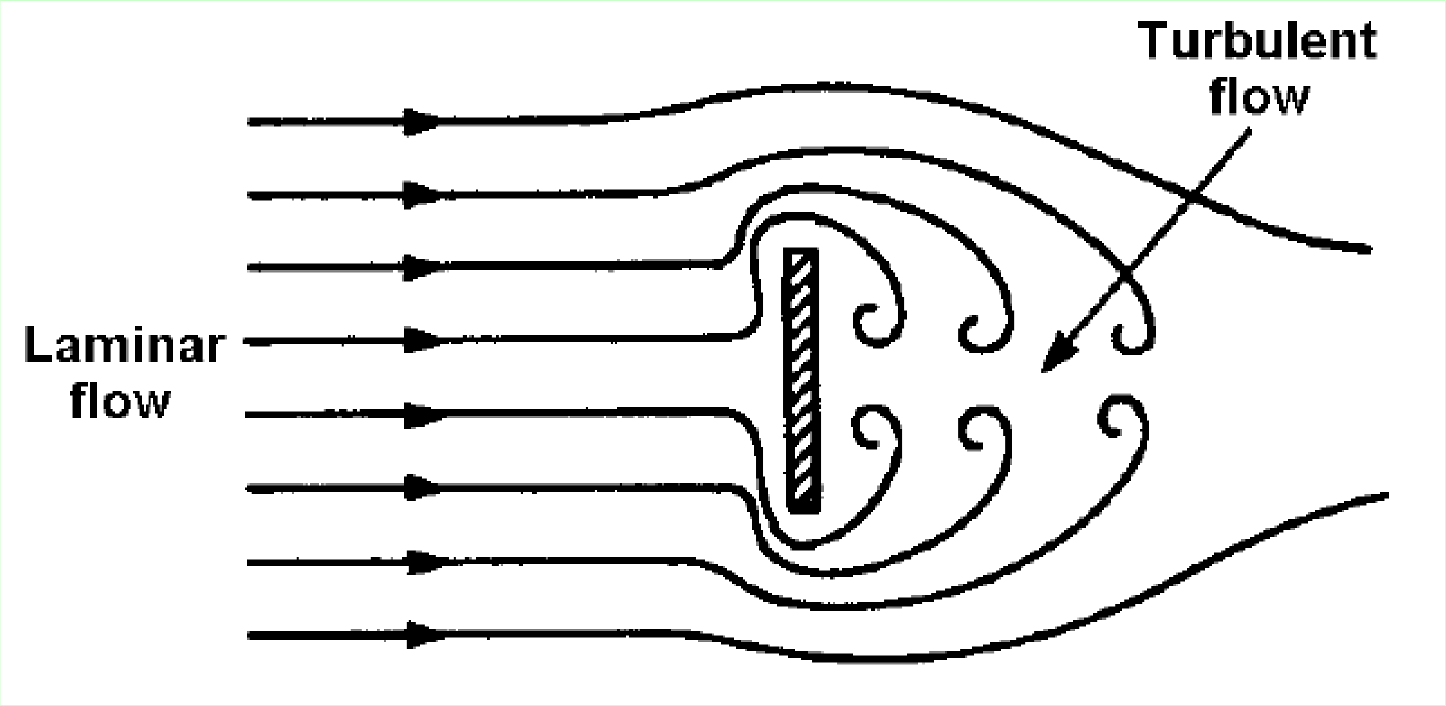
- **A balloon can displace more air than its own weight and can thus float in the air**

- **The balloon only floats when the weight of the displaced air equals its own weight with the basket**

- **Balloons are usually filled with hot air, hydrogen or helium**
- **These gases provide a large buoyant force**

Fluids in motion – hydrodynamics

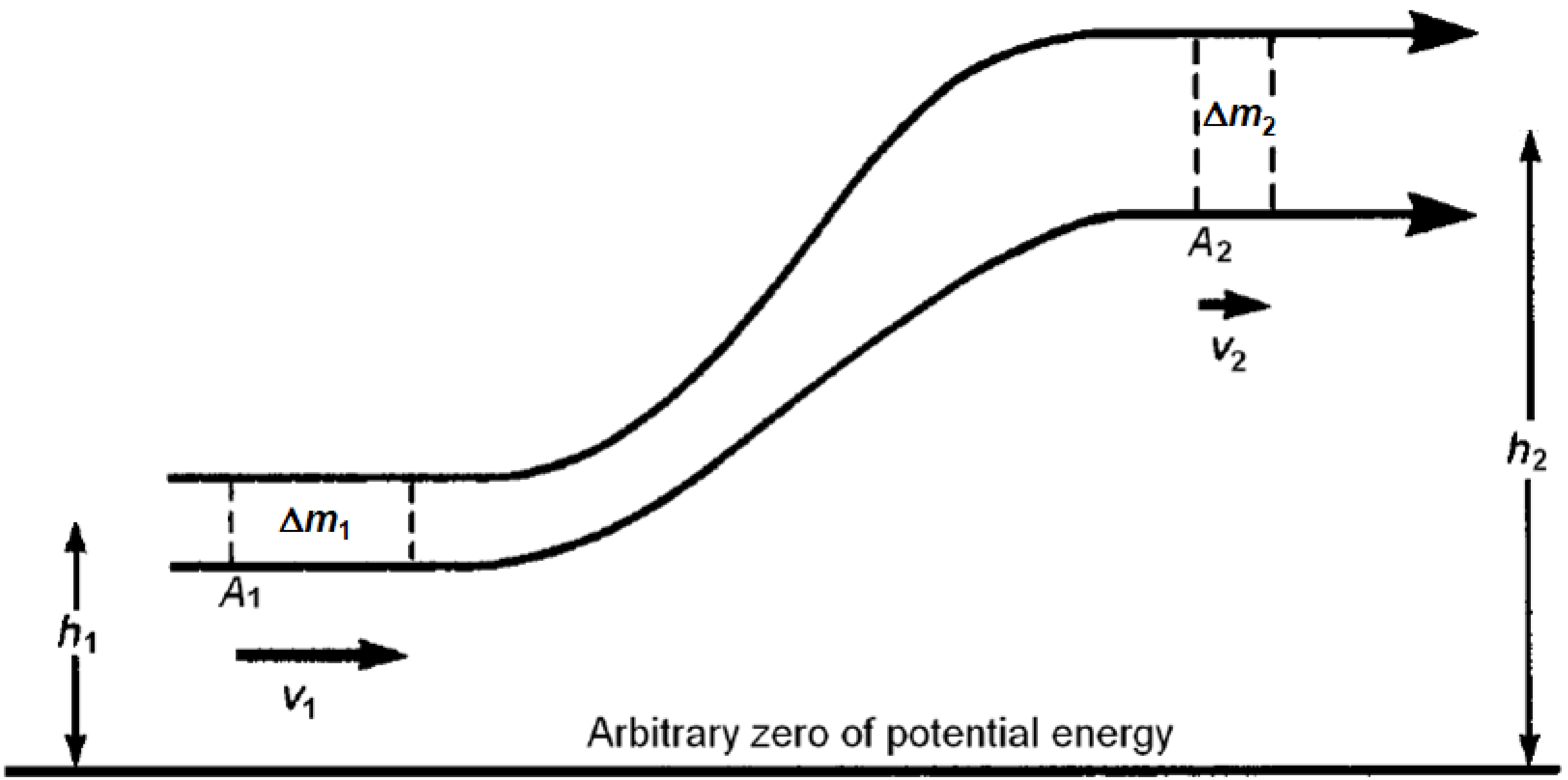
- **<u>Fluid flow</u> – the movement of small elements (particles) of a fluid**
- **<u>Laminar flow</u> – each fluid particle follows a smooth path (streamline), these paths do not cross each other – see Fig.**
- **<u>Turbulent flow</u> – fluid particles move along irregular paths (vortices)**



Fluids in motion – hydrodynamics

- <u>**Equation of continuity**</u>
- **The law of conservation of mass for fluid flow:**
- **The ideal fluid is incompressible, the fluid cannot rise or fall along the flow tube**
- **The same mass of fluid must pass through any cross-section of the tube in the same time interval**
- **The mass Δm_1 in time Δt flowing through surface A_1 is equal to the mass Δm_2 in time Δt flowing through surface A_2 – see Fig.**

Fluids in motion – hydrodynamics



Fluid flow in the tube

Fluids in motion – hydrodynamics

- $\Delta m_1 = \Delta m_2 \Rightarrow \rho\,\Delta V_1 = \rho\,\Delta V_2 \Rightarrow \rho\,A_1\,v_1\,\Delta t = \rho\,A_2\,v_2\,\Delta t \Rightarrow A_1\,v_1 = A_2\,v_2$,

- ΔV_1 , ΔV_2 – volumes of fluid per time Δt
- ρ – density of fluid is constant (incompressible fluid)
- A_1 , A_2 – cross-section areas of tube
- v_1 , v_2 – speeds of fluid

- Volume flow rate Q (volume per second) at any cross-section area of tube is constant:
- $Q = A\,v$ = const. , $[Q]$ = m^3/s

Fluids in motion – hydrodynamics

- <u>Bernoulli's equation</u>
- Bernoulli's equation expresses the principle of the conservation of mechanical energy for an ideal fluid
- Consider a steady flow of an ideal fluid in a tube – see Fig.
- Work done on fluid = Increase in kinetic energy + Increase in potential energy: $\frac{1}{2}\rho v_1^2 + \rho g h_1 + p_1 = \frac{1}{2}\rho v_2^2 + \rho g h_2 + p_2$

<u>At each point in the tube through which the fluid flows, the sum of the pressure energy, kinetic energy and potential energy per unit volume of fluid is constant</u>: $\frac{1}{2}\rho v^2 + \rho g h + p = const.$, where

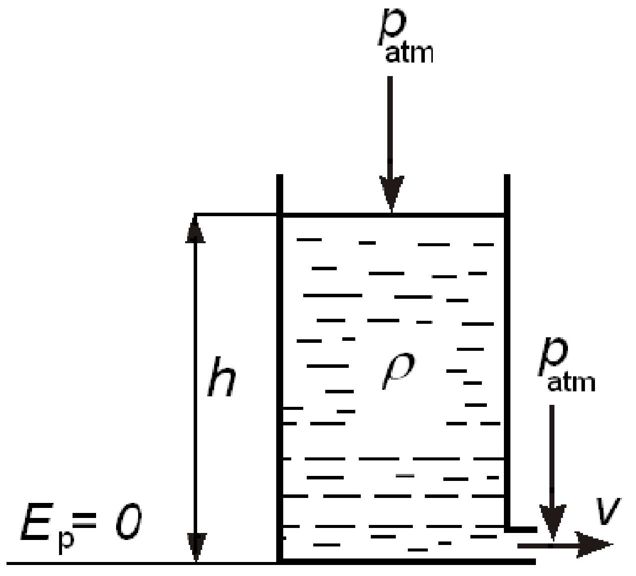
v – speed of fluid,

h – height measured from a reference level (where potential energy = 0),

p – pressure at this point

Fluids in motion – hydrodynamics

- *Application:*
- <u>Outlow of liquid from a wide vessel</u>
- A wide vessel with a small hole just above bottom contains liquid of density ρ - *see* Fig
- We can suppose that the surface of the liquid is still at the same height
- Pressures on the surface of the liquid and also in the hole = p_{atm}



Bernoulli's equation:

mechanical energy at the surface =

= mechanical energy in the hole:

$$p_{atm} + 0 + \rho g h = p_{atm} + \frac{1}{2}\rho v^2 + 0 \ ,$$

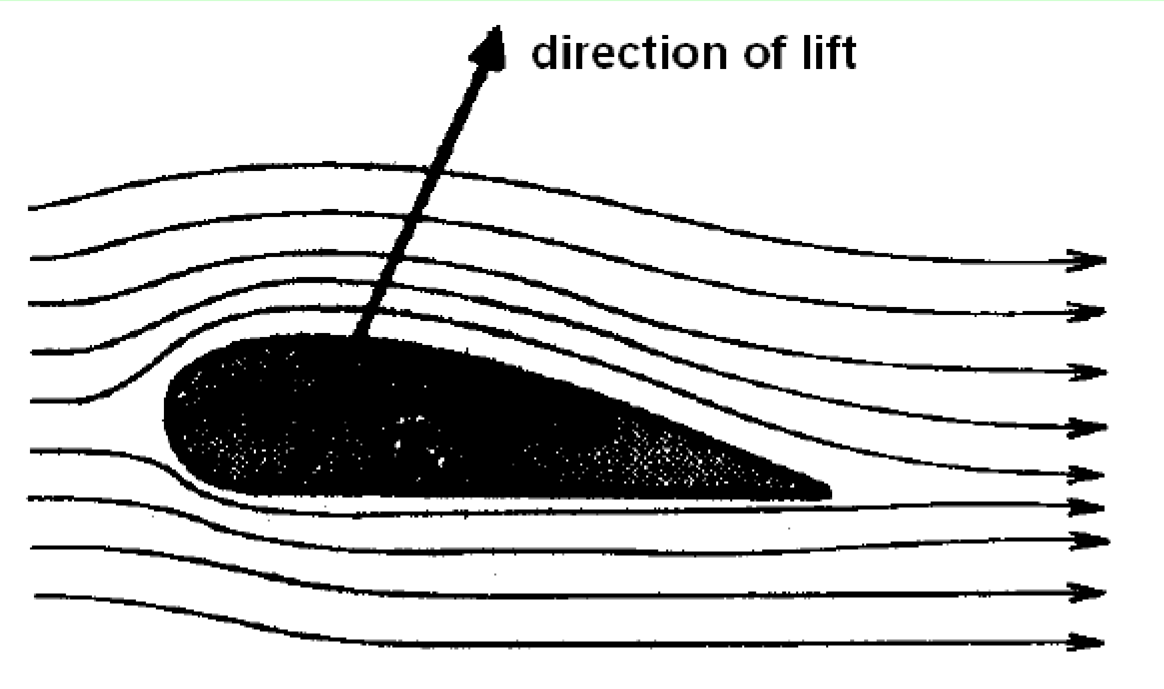
then the speed *v* of liquid from the hole is:

$v = \sqrt{2gh}$ – <u>Torricelli's theorem</u>

Fluids in motion – hydrodynamics

<u>Airplane wing</u>

Since the air flowing over the upper surface of the wing must travel a longer distance than the air flowing under the lower surface of the wing, the air speed above the wing must be greater than the air speed below the wing. Therefore, there is less pressure on the upper side of the wing than on the lower side. The result is a lift force acting on the wing – see Fig.



Fluids in motion – hydrodynamics

Example 5.18:

Calculate the mass of gas flowing per hour through the constriction point of a horizontal tube if the density of the gas is ρ = 1.4 kg/m^3, the inner diameter of the tube is d_1 = 50 mm, the inner diameter at the constriction point is d_2 = 40 mm.

The pressure difference at the two points is Δp = 120 Pa. Assume that the gas flows as an ideal fluid.

[m = 107.7 kg]