Electrostatics

- <u>Electrostatics</u> deals with the interaction of electric charges at rest and the electric fields associated with them

- Each atom consists of a positively charged nucleus containing protons and neutrons, surrounded by negatively charged electrons (neutrons are neutral)

- In an electrically neutral atom, there are just as many electrons (*e*-) outside the nucleus as there are protons in the nucleus (*e*+)

- The smallest charge (charge of electron or proton): the elementary charge $e = 1.6 \cdot 10^{-19}$ C (SI unit of charge is coulomb – C)

- Every electric charge is quantized - all charges are integer multiples of the elementary charge

Electrostatics

- **Ion** - an atom that has either lost one or more electrons, making it positively charged (+*Q*), or gained one or more electrons, making it negatively charged (-*Q*)

Coulomb's Law

The magnitude of force between two point charges is directly proportional to the product of charges Q and Q_0 , and is inversely proportional to the square of the distance d between them: $F_e = \frac{1}{4\pi\varepsilon} \cdot \frac{QQ_0}{d^2}$, where

ε – permittivity of medium: $\varepsilon = \varepsilon_0 \varepsilon_r$

$\varepsilon_0 = 8.854 \cdot 10^{-12}$ F/m – the permittivity of vacuum

ε_r – relative permittivity

Electrostatics

- Between any charges acts force:

- consonant charges repel each other - *repulsive force*

- discordant charges attract each other - *attractive force*

<u>Electric field</u>

- There is a space around the charge in which a force field - an electric field - can be detected

- The electric field around a charge at rest – the electrostatic field

- A force is exerted on a charged object placed in an electric field

The intensity of electric field $\vec{E}$ around charge *Q* at any point at distance *d*:

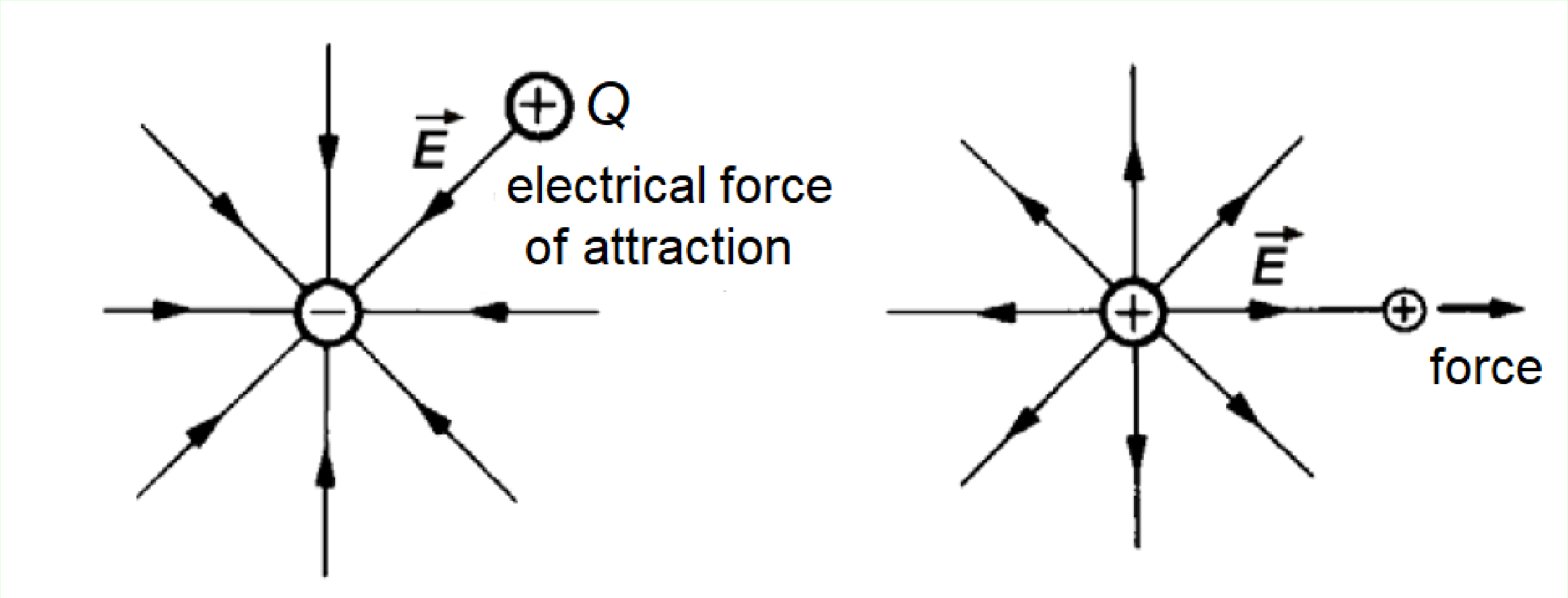
$\vec{E} = \frac{\vec{F}}{Q_0}$ **– the force $\vec{F}$ acting on the unit positive charge (1C)**

- SI unit of intensity of electric field [*E*] = N/C or V/m (volt per metre)

Electrostatics

- From Coulomb's law, the magnitude of the intensity of electric field *E* around a point charge *Q* at distance *d*: $E = \frac{1}{4\pi\varepsilon} \cdot \frac{Q}{d^2}$

- The intensity of electric field $\vec{E}$ – a vector quantity, the direction of this vector at any point is the direction of the force that would act on a positive charge located at that point – see Fig.



Electrostatics

<u>Electric potential</u>

- The *electric potential* V_A at point A is defined as the work done by the electric field in moving a unit positive charge from this point to infinity:

$$\boldsymbol{V_A = \frac{W_{A\infty}}{Q} = \int_A^\infty \frac{\vec{F}\cdot d\vec{r}}{Q} = \int_A^\infty \frac{\vec{E}\cdot Q}{Q}\cdot d\vec{r} = \int_A^\infty \vec{E}\cdot d\vec{r}}$$

- The electric potential of a point charge at a distance *d*: $\boldsymbol{V(d) = \frac{1}{4\pi\varepsilon}\cdot\frac{Q}{d}}$

- Electric potential: a scalar quantity, positive for positive charge and negative for negative charge

<u>Voltage</u>: difference in electric potentials between two points:

$U_{AB} = V_A - V_B$

- Unit of electric potential and potential difference – volt [*V*] = [*U*] = J/C = V

Electrostatics

Conductors and insulators

- Insulators or non-conductors: materials such as glass, rubber, porcelain, etc. – do not allow the free passage of charge

- Conductors: metals, carbon and some liquids – charge can flow through them

- The best conductors: copper, silver and gold

- There is also a category "between" conductors and non-conductors: *semiconductors*

Electrostatics

<u>Capacitance and capacitors</u>

- If a charge is given to a conductor, the electric potential of the conductor is raised

- The *capacitance C* of a conductor for storing charge: the ratio of its charge *Q* to the potential difference *U* between two conductors or between a conductor and ground: $C = Q / U$

- SI unit of capacitance: [*C*] = C/V = F (farad)

- A capacitor – the device for storing charge

- The simplest example of a capacitor: parallel plate capacitor – see Fig.

- It consists of two metal plates of area *A*, distance *d* separating them

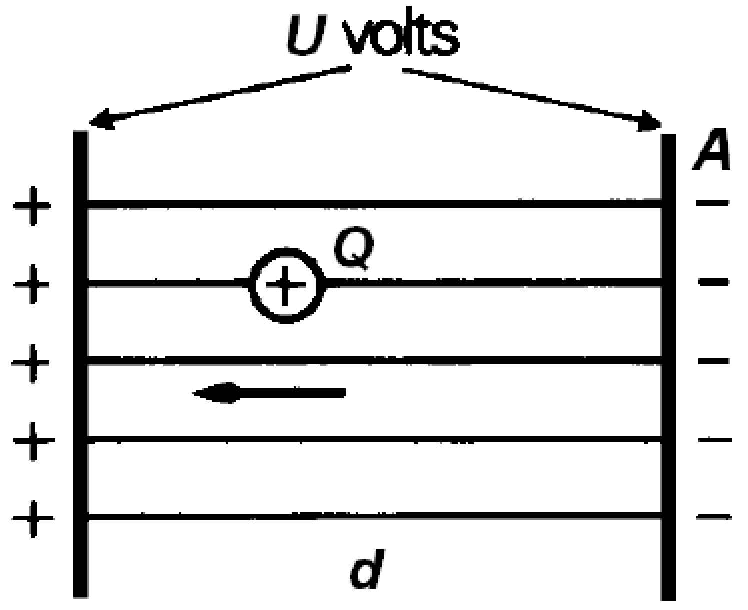
- The space between the plates is filled by dielectric (insulating material) of permittivity ε

Electrostatics

- The capacitance *C* of parallel plate capacitor: $C = \frac{\varepsilon A}{d}$

- The capacitance – constant for a given capacitor

- The value of the capacitance depends on area of metal plates, on their relative position and on the material that separates them



Parallel plate capacitor

Electrostatics

Application

- A charged capacitor stores electrical energy

The energy stored in the capacitor is equal to the work done in charging it

- The work required to add a small charge d*Q* from one plate to the other plate: d*W* = *U* d*Q*, where *U* is the potential difference across the plates

- Total work done: $W = \int_0^Q U\, dQ = \frac{1}{C}\int_0^Q Q\, dQ = \frac{1}{2}\frac{Q^2}{C}$

- Energy stored in capacitor: $E = \frac{1}{2}\frac{Q^2}{C} = \frac{1}{2}CU^2 = \frac{1}{2}QU$

- It gives for a parallel plate capacitor: $E = \frac{1}{2}CU^2 = \frac{1}{2}\left(\frac{\varepsilon A}{d}\right)E^2 d^2 = \frac{1}{2}\varepsilon A E^2 d$

- This storing a large amount of energy can be released during a very short time – for example flashlight of camera

Electrostatics

Example 8.4:

Two point electric charges act on each other in paraffin from a distance d_1 = 2 cm with a force of magnitude F. To act on each other in air with a force of the same magnitude F, they must be separated by d_2 = 2.9 cm. What is the relative permittivity of paraffin ε_{r1}? The relative permittivity of air ε_{r2} = 1.

[ε_{r1} = 2.1]

Electric current

- Electric current – the ordered movement of electric charges

- An electric force is required to move the charge

- The force on the electrons in the conductor produces a current

- In a battery (potential difference source), one plate has a positive potential and the other plate has a negative potential

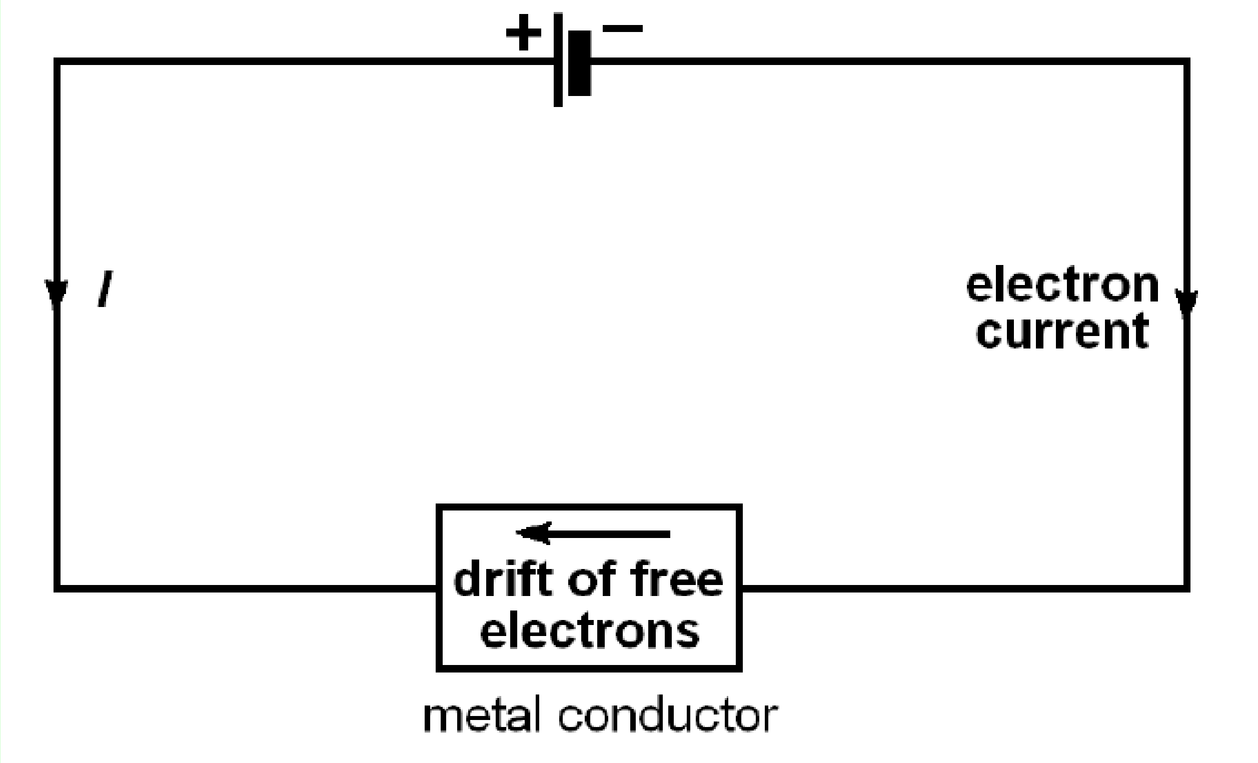
- The battery provides the *electromotive force* (e.m.f.) for electrons to flow through the metal conductor

- When the conductor is placed in a circuit containing the battery, the movement of free electrons is directed toward the positive plate

- The flow of electrons is directed from the negative to the positive plate

- The speed of the free electrons in random thermal motion becomes directional – *drift speed* – see Fig.

Electric current



Conduction in a metal

- Conventional electric current *I* – the direction of positive charge in an electric field; from a point of positive (higher) potential to a point of negative (lower) potential

Electric current

- The electric current *I* – the amount of charge *Q* which passes through the cross-section of wire per unit of time *t*: $I = \frac{Q}{t}$
- Unit of current – coulomb per second – ampere, basic unit of SI system: [*I*] = C/s = A
- Current can flow in a circuit if there is a potential difference *U* - the terminals of the potential source (battery) are connected by conductors (metallic wire) and form an electrical circuit
- The electric current in the metallic conductor is proportional to the potential difference applied to its ends (*I* ~ *U*)
- The ratio of the potential difference to the current = constant
- The value of this constant – the *resistance R* of the conductor:

R = *U* / *I* or *U* = *R I* – <u>Ohm's law</u>; the SI unit of resistance: ohm [*R*] = Ω

Electric current

- The resistance *R* of a homogeneous isotropic metal wire is directly proportional to its length l and inversely proportional to the cross-sectional area *A* of the wire: $R = \frac{\rho\, l}{A}$ **,**

ρ **– resistivity - the material constant of wire, [*ρ*] = Ω.m**

- The resistance of materials depends on the temperature

- The resistance of conductors – directly proportional to the actual temperature: $\Delta R = R_0\, \alpha\, \Delta t$ **, where**

$\Delta R = R - R_0$ **,** R **– the resistance for final temperature** t**,**

R_0 **– resistance for initial temperature** t_0**,**

$\Delta t = t - t_0$ **– change of temperature,**

α **– the temperature coefficient of resistance [*α*] =** K^{-1}

Electric current

Example 8.19:

Calculate the length of a constantan wire (60 % Cu, 40 % Ni) of diameter d = 0.1 mm if its resistance is to be R = 1000 Ω. The resistivity of the constantan ρ = 0.49 μΩ·m.

[l = 16.03 m]

Electric current

Resistors

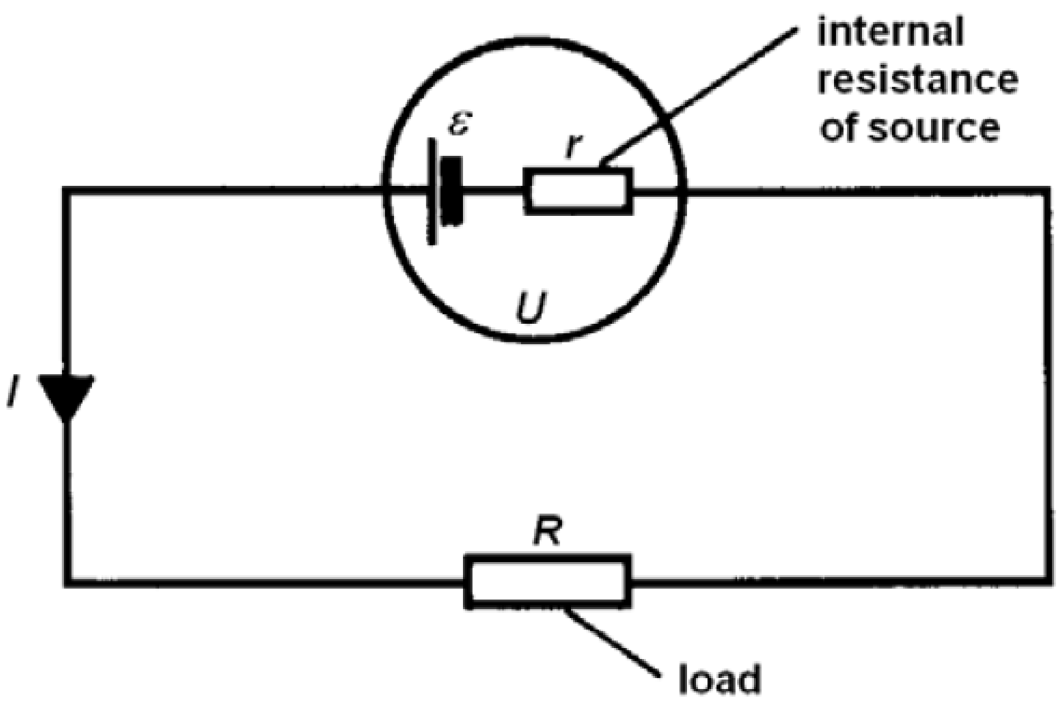
- All electric components in a circuit have a resistance

- The internal resistance *r* is inside the power source, for example a battery - see Fig.

- If the circuit is disconnected, no current flows through it $\Rightarrow$ the electromotive force *ε* (e.m.f.) is across the terminals of the power supply

- If the circuit is connected, the source energizes the circuit, current *I* flows in the circuit, and the potential loss $I \cdot r$ is converted to heat inside the source $\Rightarrow$ the potential difference (voltage) *U* across the source terminals is less: $U = \varepsilon - I \cdot r$

Electric current



Internal resistance of source

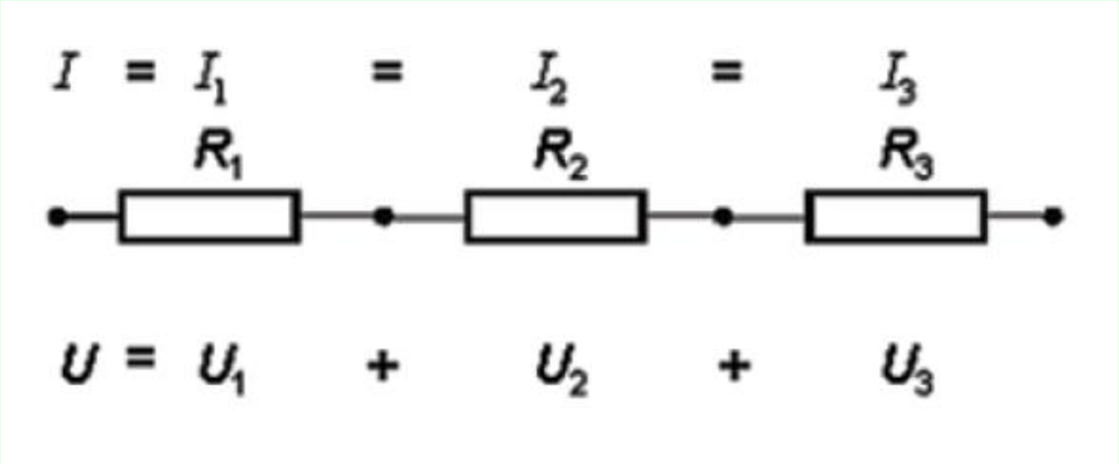
The current *I* which flows in a circuit is determined with a *total resistance* $R_{total} = R + r$ and with e.m.f. *ε* of the power supply:

$$I = \frac{\varepsilon}{R + r}$$

Electric current

<u>Combination of resistors</u>

Resistors connected in series **– see Fig.**



- The current *I* must be the same throughout the circuit because it has only one path

- If *R* is the total resistance of the combination of resistors and *U* is the total potential difference across them, then $U = I\,R$, the total potential difference *U* is the sum of the potential differences across resistors R_1, R_2, R_3

Electric current

- Thus:

$U = U_1 + U_2 + U_3$

and U = $I\,R_1 + I\,R_2 + I\,R_3$

therefore $I\,R = I\,R_1 + I\,R_2 + I\,R_3$

and hence $R = R_1 + R_2 + R_3$

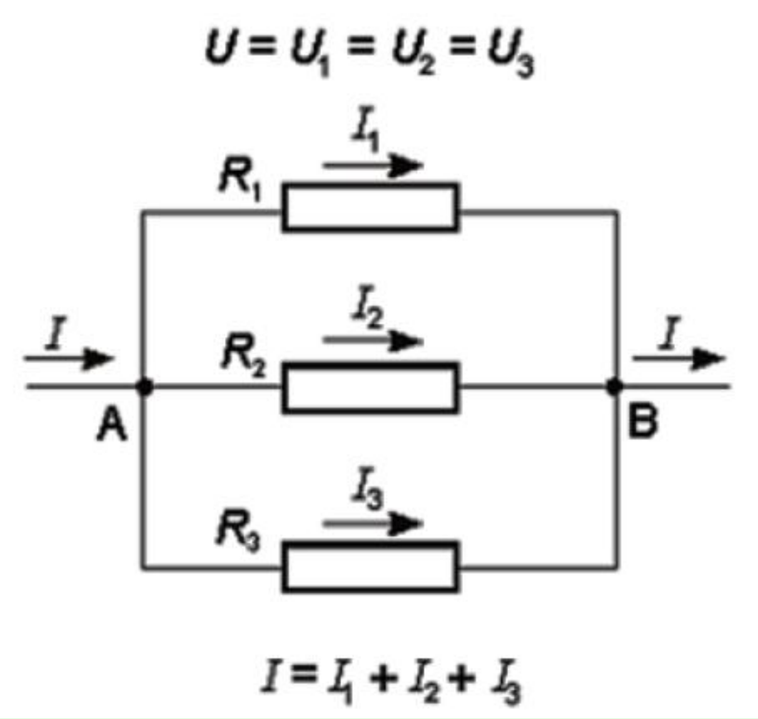
- The total resistance of the series connected resistors R is the sum of the individual resistances R_1, R_2, R_3

- The same result can be applied to n resistors in series:

$$R = \sum_{i=1}^{n} R_i$$

Electric current

Resistors connected in parallel **– see Fig.**



- The total current *I* in the main circuit is equal to the sum of the currents flowing in the parallel branches

- In a parallel circuit, the total current entering the junction must equal the total current leaving the junction

Electric current

- The potential difference *U* across all parallel resistors is the same
- If *R* is total resistance of the combination and *I* is the total current, then:

$$I = \frac{U}{R},$$

but *I* is the sum of the currents flowing in resistors R_1, R_2, R_3

Thus $I = I_1 + I_2 + I_3$, therefore $\frac{U}{R} = \frac{U}{R_1} + \frac{U}{R_2} + \frac{U}{R_3}$ and hence $\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$

- The reciprocal value of the total resistance of the resistors connected in parallel is the sum of the reciprocal values of the individual resistors R_1, R_2, R_3
- The same result can be applied to *n* resistors in parallel:

$$\frac{1}{R} = \sum_{i=1}^{n} \frac{1}{R_i}$$

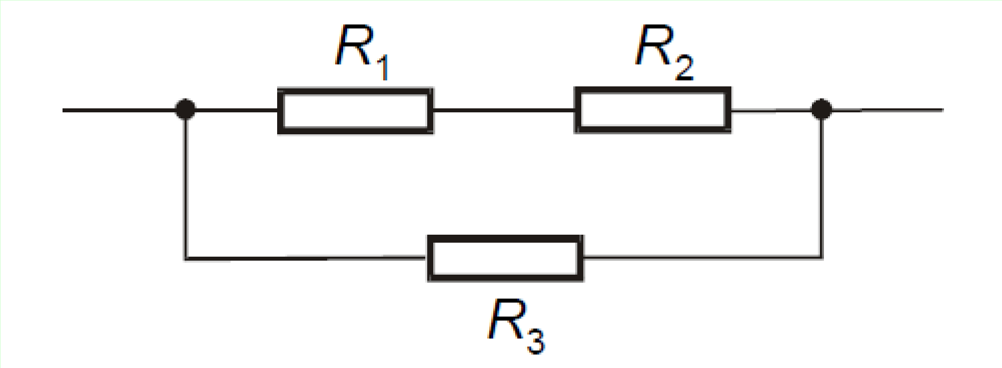
Electric current

Example 8.22:

Three resistors with resistances R_1 = 100 Ω, R_2 = 200 Ω and R_3 = 400 Ω are connected according to the diagram.

Calculate their resulting resistence R.

[R = 171 Ω]



Electric current

Electric energy

- From the definition of potential difference (voltage) and electric current it follows that if a potential difference *U* is applied to the ends of a conductor and a charge *Q* passes through it, then the work *W* done in time *t*: $W = U I t$

Application

- The work of an electric current - electrical energy - is converted into other forms of energy

- In an electric heater it is converted into heat $Q = U I t$, in an electric bulb into light and heat, in an electric motor into mechanical energy of rotation, etc.

Electric current

<u>**Electric power**</u>

- The definition of power *P* is the rate of work *W* done and time *t* taken

- Using the equations for electric energy and Ohm's law:

$$P = \frac{W}{t} = UI = RI^2 = \frac{U^2}{R}\text{, } [P] = \text{W (watt)}$$

Application

From the definition of power $P = \frac{W}{t} \Rightarrow W = P\,t \Rightarrow$

$\Rightarrow$ **unit of work (electric energy)** $[W] = [P] \cdot [t] = \text{W}\cdot\text{s}$

- The commercial unit of electric energy – kilowatt-hour (kWh):

1 kWh = 1000 . 3600 W.s (J) = 3.6 MJ

- Electricity is billed using the unit kilowatt-hour

- Total energy consumed (number of kWh) x price per unit = total electricity charge