Magnetic field

- There is an electric field in the space around the charge at rest

- There is a force acting on each charged object placed in this electric field

- The physical quantity that describes the electric field is the intensity of electric field $\vec{E}$

- When a charge moves, it is the source of another field in its surroundings - the magnetic field

- A force acts on a moving charged object placed in this magnetic field

- A magnetic field is described by the intensity of magnetic field $\vec{H}$ or by the magnetic induction $\vec{B}$

- These two quantities are related by $\vec{B} = \mu\vec{H}$, where μ is the permeability of the medium

Magnetic field

- SI units of these quantities:

- [H] = A/m, [B] = T (tesla) or Wb/m^2

$\mu = \mu_0 \mu_r$, [μ]= H/m

$\mu_0 = 4\pi \cdot 10^{-7}$ H/m – permeability of vacuum

μ_r – relative permeability of medium

- The magnetic field is represented by magnetic field lines, which show the direction of the force acting on the north pole of the permanent magnet at different points of this magnetic field - the direction of the vector $\vec{B}$

- The north pole of the magnet attracts the south pole and repels the north pole

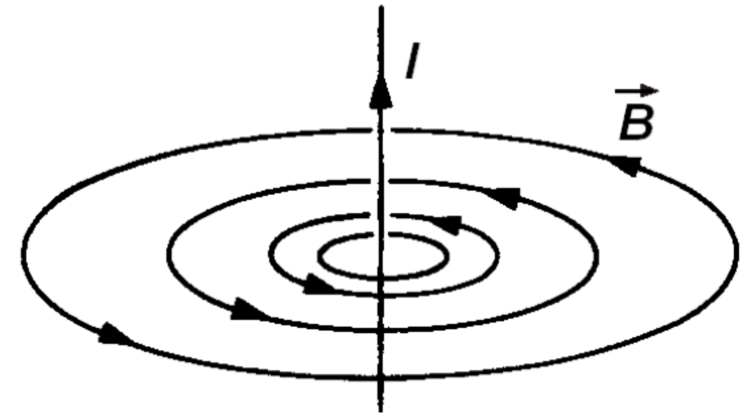
- The consonant poles of the magnets repel each other, the discordant poles attract each other

Magnetic field

- When a current passes through a conductor, a magnetic field is created around the conductor

- In a straight conductor, the induction lines form a system of concentric circles centred on the conductor – see Fig.

- The direction of the lines of force can be found by using the right-hand rule: If the thumb of the right hand points in the direction of the current, the fingers point in the direction of the magnetic field



Magnetic field around the wire with steady current

Magnetic field

Magnetic induction excited by a long straight wire

- The Biot-Savart law, which gives the relationship for the magnetic induction induced by an electric current, states that the magnitude of the magnetic induction *B* at a point having a distance *a* from the straight conductor through which the current *I* flows is: $B = \frac{\mu_0 I}{2\pi a}$

Example 9.2:

Determine the current *I*, which flows through a very long thin straight conductor when the magnitude of the magnetic induction $B = 5 \cdot 10^{-4}$ T has been found at a distance $a = 0.2$ m from it. Assume that the conductor is in air ($\mu_r \cong 1$), the permeability of the vakuum $\mu_0 = 4\pi \cdot 10^{-7}$ H/m.

[$I = 500$ A]

Magnetic field

Force on the moving charge in a magnetic field

- A magnetic force $\vec{F}_m = Q(\vec{v} \times \vec{B})$ acts on a particle with a charge Q moving with a velocity $\vec{v}$ in a magnetic field of induction $\vec{B}$

- $\vec{F}_m$ – vector product; the magnetic force vector $\vec{F}_m$ – always perpendicular to the vectors of induction $\vec{B}$ and velocity $\vec{v}$

- The magnitude of the force: $F_m = Q\,v\,B\,\sin\alpha$

(α – the angle between vectors $\vec{v}$ and $\vec{B}$)

- If $\alpha = 0°$ or $\alpha = 180°$ ($\vec{v} \uparrow\uparrow \vec{B}$, $\vec{v} \uparrow\downarrow \vec{B}$) – a charged particle moves parallel or antiparallel to the magnetic field $\Rightarrow$ the magnetic force = 0 $\Rightarrow$

$\Rightarrow$ the particle continues in its uniform straight-line motion ($\vec{v}$ = const.)

Magnetic field

- For the charge moving perpendicular to the magnetic field (α = 90°, $\vec{v} \perp \vec{B}$)

$\Rightarrow$ The maximum value of the force $F_m = Q\,v\,B\sin\alpha = Q\,v\,B$

- $\vec{F}_m$ also perpendicular to velocity $\vec{v}$ $\Rightarrow$ so it must be a centripetal force $\vec{F}_c$ (is directed to the centre of the circle):

$$F_m = F_c \Leftrightarrow QvB = \frac{mv^2}{r}$$

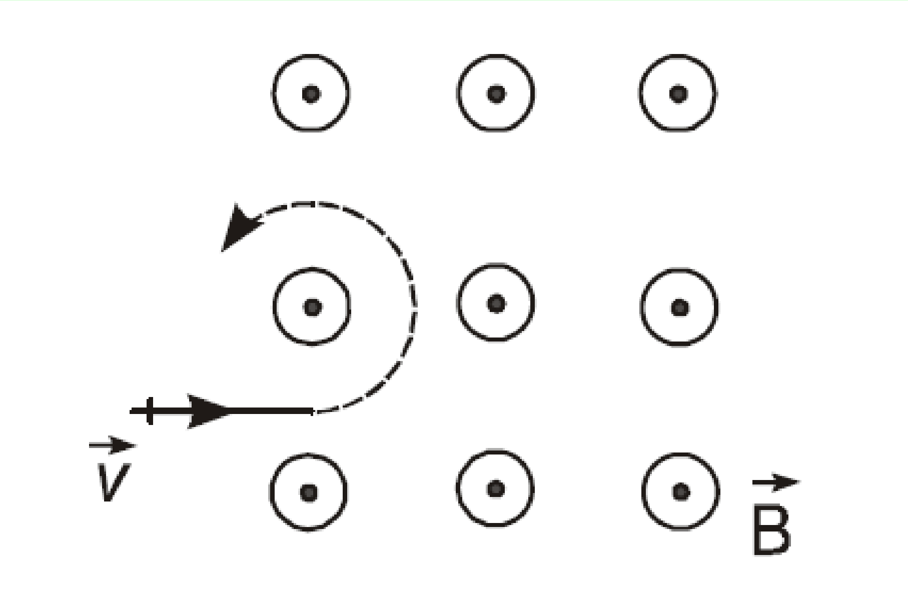
- The trajectory of a charged particle entering the magnetic field perpendicular to the direction of the induction lines is a circle of radius *r*

Magnetic field

<u>**Example 9.6:**</u>

The electron enters a homogeneous magnetic field of induction B = 1 μT perpendicular to the direction of the induction lines - see Fig. - with speed v = 1000 m/s and starts moving in a circle. What is the radius r of this circle? The charge of the electron $e = 1.6 \cdot 10^{-19}$ C, the mass of the electron $m = 9.1 \cdot 10^{-31}$ kg.

[r = 5.7 mm]



Magnetic field

Application

- Mass spectrometer is used to determine the mass of atoms or molecules – see Fig.

- Positive ions are produced in the source *S*

- Only those ions whose velocity is $v = E / B$ will pass through the selector in an electric field of intensity $\vec{E}$ and a magnetic field of induction $\vec{B}$

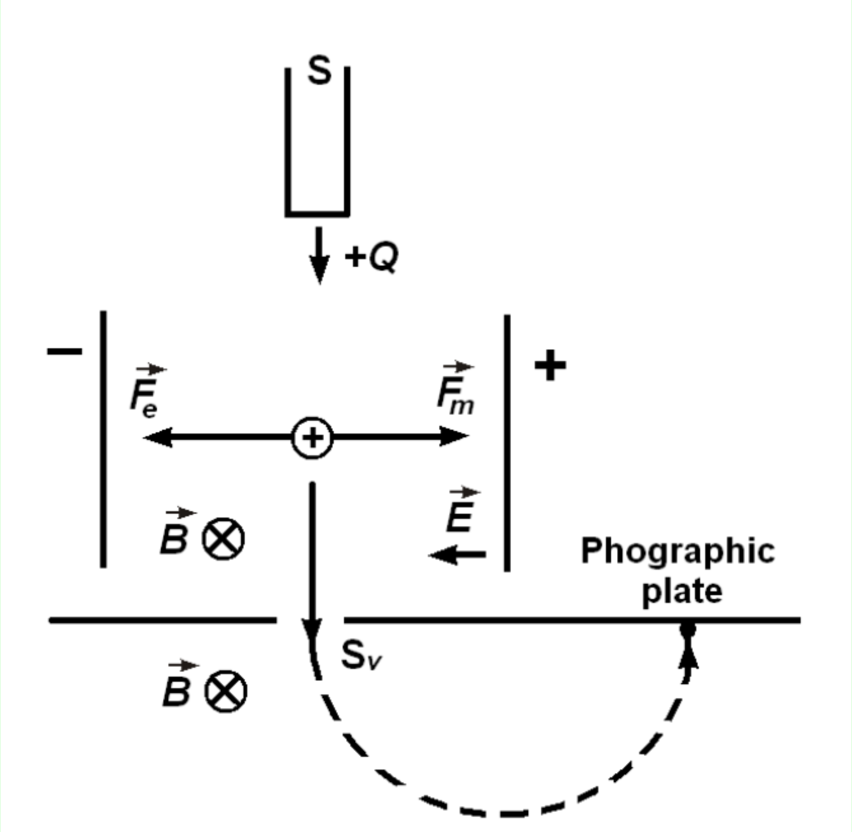
- These ions pass through the divider S_v

- Furthermore, there is only a magnetic field of induction $\vec{B}$, which is perpendicular to the ion velocity ($\vec{B} \perp \vec{v}$) $\Rightarrow$ the ions move in a circular path

- The ions illuminate the photographic plate at the point where they fall (the radius of the path *r* is measured)

- By measuring from a mark on the photographic plate, the mass of any particle can be accurately determined

Magnetic field



Mass spectrometer

- We can determine the mass of ions *m* as function of radius *r* :

$$\frac{mv^2}{r} = QvB \Rightarrow m(r) = \frac{QBr}{v} = \frac{QB^2r}{E}$$

Magnetic field

Force on the wire conductor in a magnetic field

- A conductor with a constant current placed in a magnetic field ⇒ a force is applied to it

The magnitude of the magnetic force: $F_m = I\,l\,B\,sin\,\alpha$ **,**

where: ***I*** **– magnitude of current flowing through the conductor,**

l **– length of conductor**

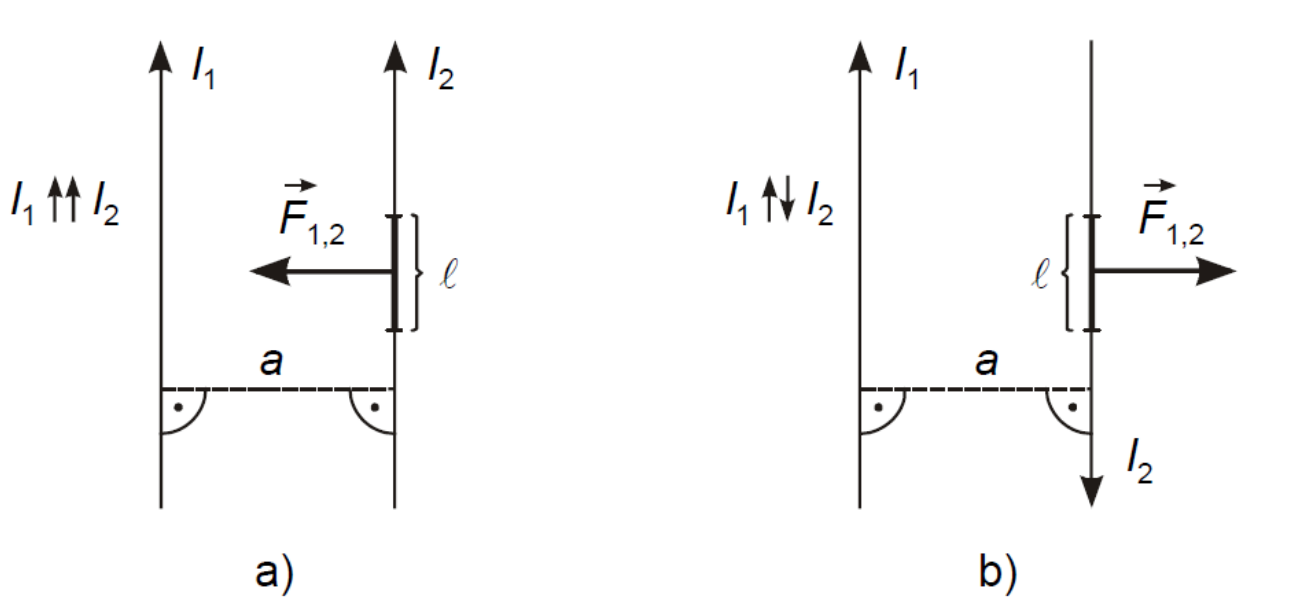
B **– magnitude of magnetic induction**

α **– angle between the conductor and magnetic induction**

Magnetic field

Force between two parallel conductors

- If two conductors through which a current flows are placed side by side, then each of them is in the magnetic field of the conductor and is subject to a force perpendicular both to its own current and to the magnetic field of the other conductor



Attractive and repulsive forces between two parallel conductors

- If the currents are in the same direction, the force is attractive

- If the currents are in opposite directions, the force is repulsive – see Fig.

Magnetic field

Force magnitude $F_{1,2}$ per length l of conductor: $F_{1,2} = \frac{\mu_0 I_1 I_2 l}{2\pi a}$

Application

- This equation is used to define the ampere (A), one of the seven SI base units – if $I_1 = I_2 = 1$ A, and the wires are 1 m apart ($\mu_0 = 4\pi \cdot 10^{-7}$ H/m)

<u>One ampere</u> – the current flowing in each two long parallel wires, placed in a vacuum 1 metre apart, which produces a force between them $2 \cdot 10^{-7}$ newtons per metre length

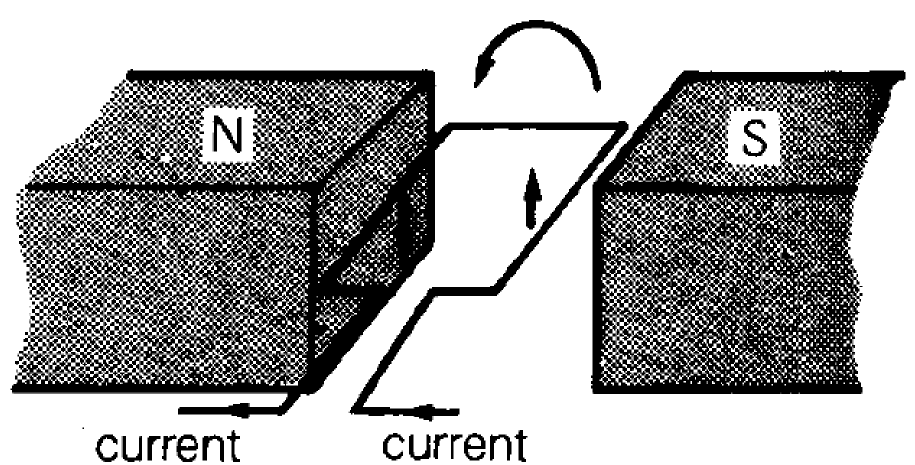
Magnetic field

<u>Electric motor</u> – a machine for converting electrical energy into mechanical energy

- It works on the principle that current passing through a loop in a magnetic field exerts a force that can rotate the loop

- Since the currents in the loop have opposite directions, the directions of the forces induced by the magnetic field are opposite

- One side of the loop moves up and the other down - see Fig. The result is torque and rotation of the loop



Loop in magnetic field

Alternating current

<u>Electromagnetic induction</u>

- If we move a conductor in a magnetic field, a current flows through it and an electromotive force is induced at the ends of the conductor

- The electric current in the conductor is also induced by the changing magnetic field

- We speak of *electromagnetic induction*

<u>Faraday's law of electromagnetic induction</u>: $u = -\frac{d\Phi}{dt}$

- The induced e.m.f. (voltage) = the negative change in magnetic flux over time

- The direction of the induced voltage is indicated by the minus sign - <u>Lenz's law:</u> The direction of the induced e.m.f. is such that it acts against the change that induces it

Alternating current

<u>Magnetic flux</u>: $\Phi = \vec{B} \cdot \vec{A} = B \cdot A\, cos\alpha$, where

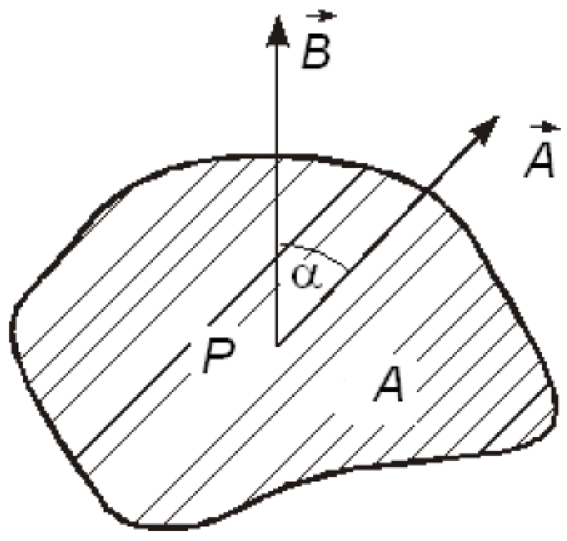
$\vec{A}$ is the vector perpendicular to the area *A* through which the magnetic field of induction $\vec{B}$ passes,

α – the angle between vectors $\vec{A}$ and $\vec{B}$ – see Fig.

- The magnetic flux Φ changes over time if the magnitude of the magnetic induction *B* or the size of the area *A* or the angle α change

- SI unit of magnetic flux – weber:

$[\phi] = [B].[A] = T.m^2 = Wb$



Magnetic flux

Alternating current

<u>Inductance</u>

- The changing current flowing through the coil ⇒ the changing magnetic field around the coil ⇒ e.m.f. in this coil

- If a varying current *i*(*t*) passes through the coil, the coil induces a voltage (e.m.f.) at its ends – *self inductance L*

[*L*] = H (henry)

$\phi \sim i$, $\phi = L\, i$ (magnetic flux ϕ is proportional to current *i*)

From Faraday's law: $u = -\frac{d\Phi}{dt} = -L\frac{di}{dt}$

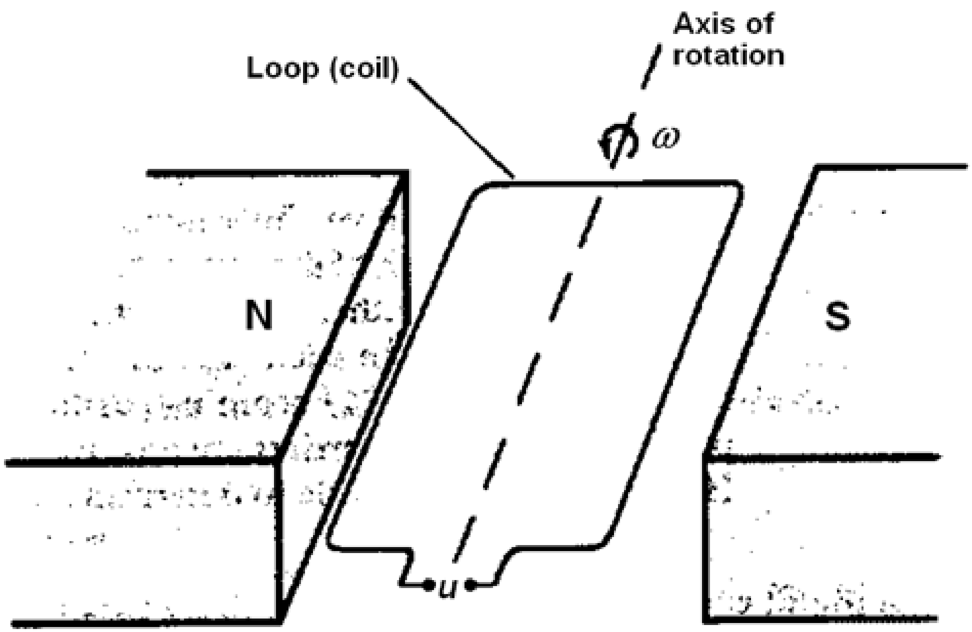
- If two coils are placed close together, the changing current in one coil will cause e.m.f. in the other – *mutual inductance M*

Alternating current

Application

Alternating current

- The most important practical result of Faraday's discovery – the generation of the alternating current (symbol a.c. or AC)



Electric generator

- An electric generator transforms mechanical energy into electric energy

- An alternating-current generator – a loop (coil) of area *A* that rotates in the uniform magnetic field $\vec{B}$ produced by permanent magnet – see Fig.

Alternating current

- The frequency *f* of the alternating current (voltage) produced =

= the frequency *f* at which the loop rotates (angular velocity $\omega = 2\pi f$)

- From Faraday's law of electromagnetic induction:

Induced voltage $u = -\frac{d\Phi}{dt}$ **, where** $\Phi = \vec{B} \cdot \vec{A} = B \cdot A\, cos\alpha$

- We assume: for time $t = 0$ the angle α between the vectors $\vec{B}$ and $\vec{A} = 0 \Rightarrow$

$\Rightarrow \phi(0) = B A$

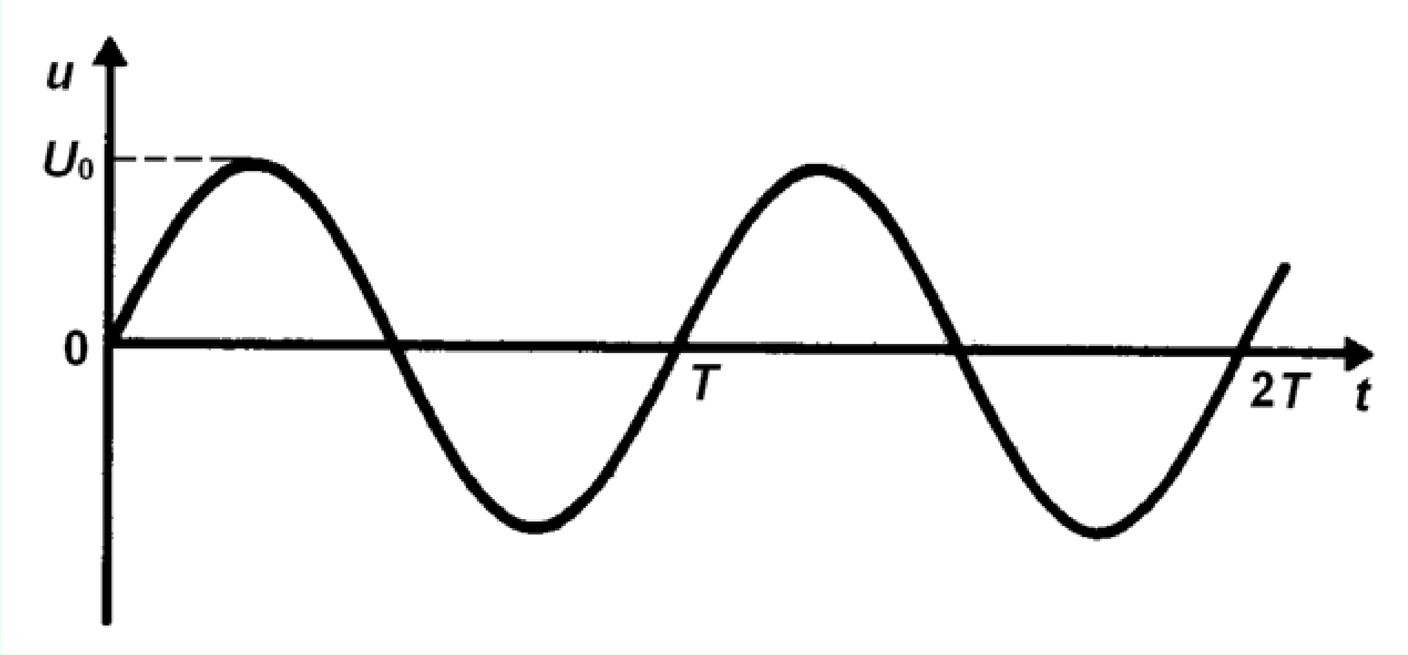
$\alpha = \omega t \Rightarrow$ magnetic flux $\phi(t) = B A \cos\alpha = B A \cos \omega t \Rightarrow$

$\Rightarrow$ the instantaneous value of voltage at time *t*:

$$u = -\frac{d\Phi}{dt} = -\frac{d(BA\, cos\, \omega t)}{dt} = \omega BA\, sin\, \omega t = U_0\, sin\, \omega t$$

Alternating current

- The output voltage $u(t)$ – sinusoidal with amplitude $U_0 = \omega BA$ and angular frequency $\omega = 2\pi f = \frac{2\pi}{T}$ (T is the period)
- The graph of voltage versus time – a sinusoidal function – see Fig.

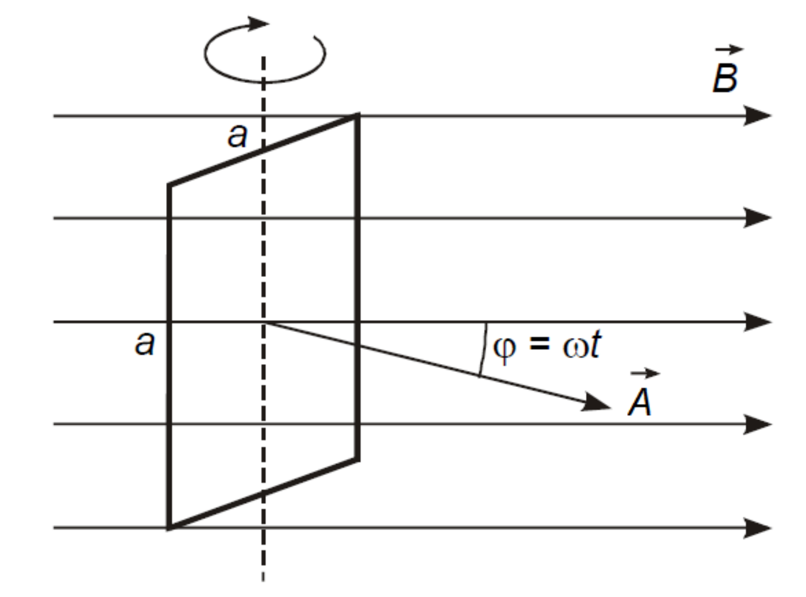


Alternating current

Example 9.16:

A flat square coil with side a = 0.12 m has N = 10 turns of very thin wire. The coil rotates uniformly in a homogeneous magnetic field of induction magnitude B = 0.25 T about an axis passing through its centre (parallel to the side of the square) and lying in the plane of the coil perpendicular to the vector $\vec{B}$ - see Fig. Determine the magnitude of the angular velocity of rotation of the coil ω , if the maximum value of the voltage induced in the coil is U_0 = 20 V.

[ω = 556 rad/s]

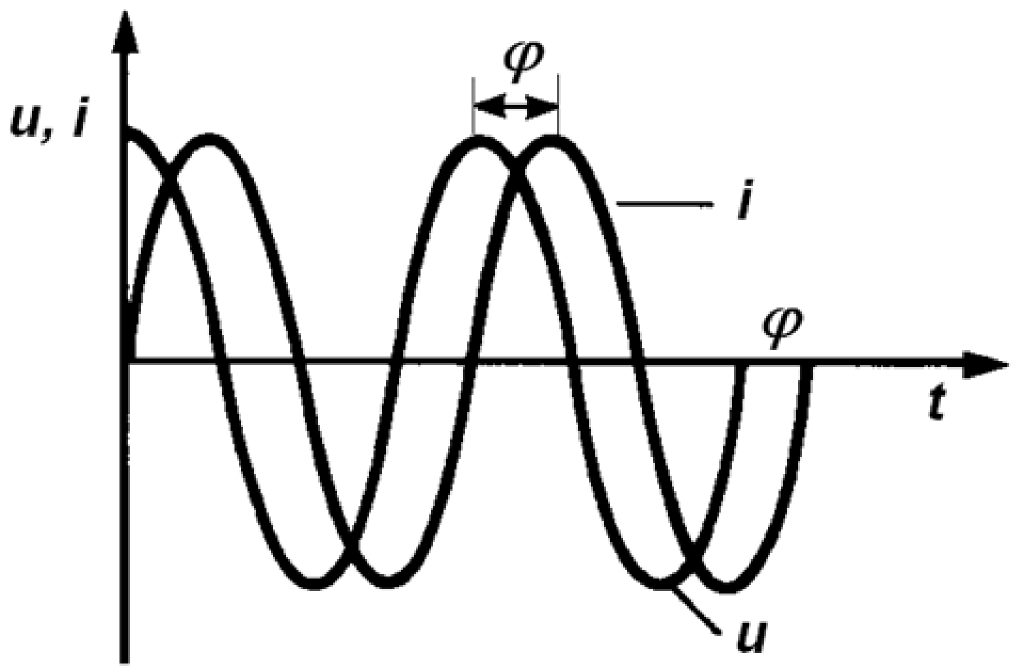


Alternating current

- Unlike direct current (DC), the value of alternating current (and voltage) (AC) varies continuously

- Therefore, we define the effective value of voltage and current

- For a sinusoidal waveform: $U_{ef} = \frac{U_0}{\sqrt{2}}$, $I_{ef} = \frac{I_0}{\sqrt{2}}$



Phase difference between voltage and current

- In AC circuits, the voltage and corresponding current are generally not in phase
- They are shifted by the phase angle φ – see Fig.

Alternating current

Electric power

The average power $\overline{P}$ for AC current:

$\overline{P} = U_{ef}\, I_{ef}\, cos\varphi$ **(*P* = *U I* for DC current) ,**

where U_{ef} and I_{ef} – the effective values of voltage and current,

φ – phase angle,

cosφ – power factor of the circuit