

# Oscillation and waves

- **Oscillation** – a repeated cycle motion of a particle about an equilibrium position

- Displacement along  $y$  axis for simple harmonic motion (SHM):

$$y = A \sin(\omega t + \varphi), \quad \text{where}$$

$A$  – amplitude of motion

$\omega$  – angular frequency

$(\omega.t + \varphi)$  – phase of motion

$\varphi$  – phase constant – phase angle for time  $t = 0$

$T$  – period - the time to complete one cycle,  $[T] = \text{s}$

$f = 1/T$  – frequency - number of cycles per second  $[f] = \text{s}^{-1} = \text{Hz}$

# Oscillation and waves

**Speed of SHM:**  $v = \frac{dy}{dt} = \omega A \cos(\omega t + \varphi)$

**Acceleration of SHM:**  $a = \frac{dv}{dt} = \frac{d^2y}{dt^2} = -\omega^2 A \sin(\omega t + \varphi) = -\omega^2 y$

- In SHM there is a constant interchange of energy between the kinetic and potential forms

- The kinetic energy is given by:  $E_k = \frac{1}{2} m v^2 = \frac{1}{2} m \omega^2 A^2 \cos^2(\omega t + \varphi)$

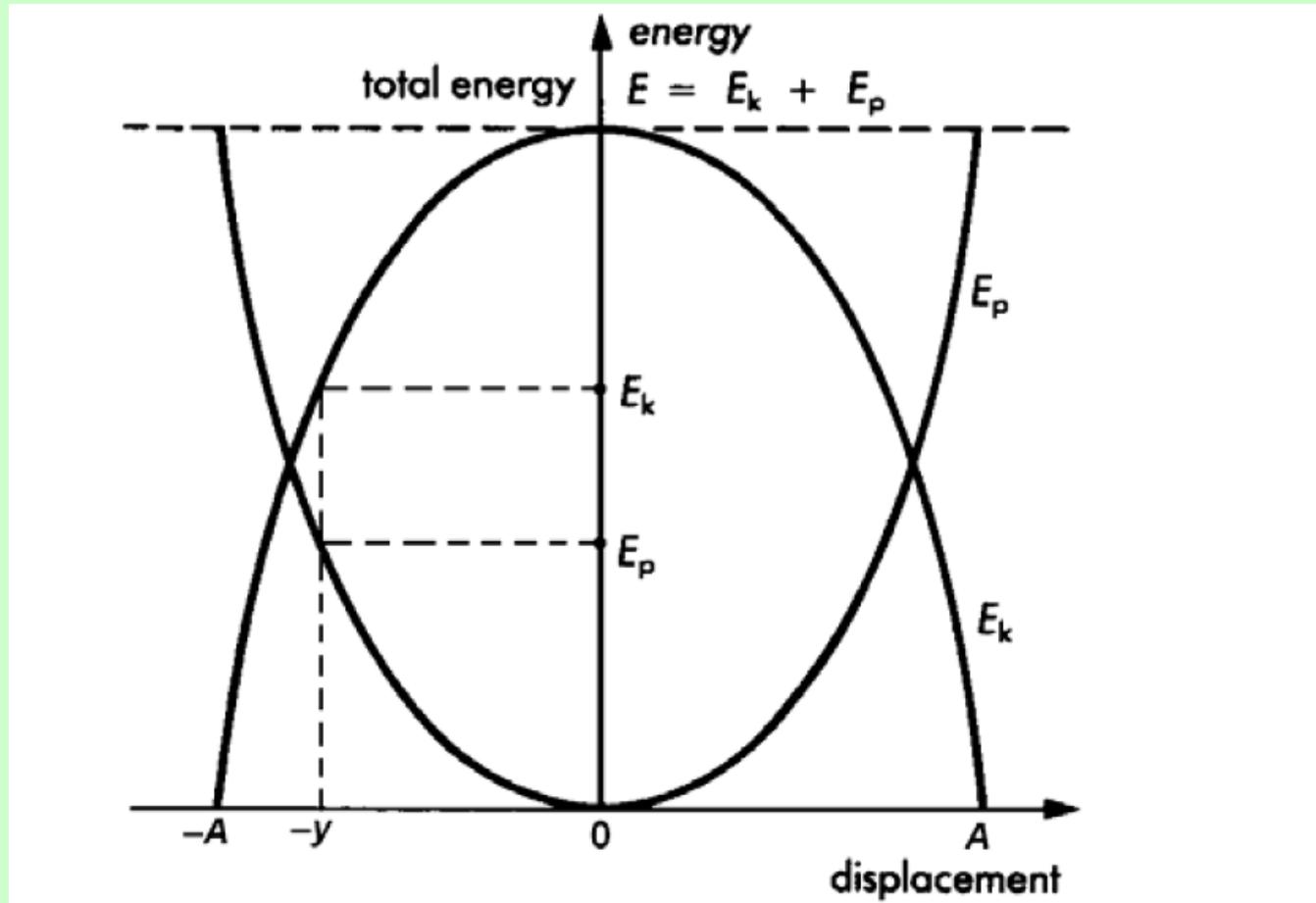
- The potential energy is given by:  $E_p = \frac{1}{2} m \omega^2 y^2 = \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \varphi)$

- The total energy  $E$  – constant ; always the sum of  $E_k$  and  $E_p$ :

$$E = E_k + E_p = \frac{1}{2} m \omega^2 A^2$$

# Oscillation and waves

Fig. shows the energy as the oscillator goes through a complete cycle:



# Oscillation and waves

- **Wave motion** – a means of transferring energy from one point to another without there being any transfer of matter between the points
- ***Mechanical waves*** (e.g. sound waves) require a material medium to propagate
- ***Electromagnetic waves*** - electrical oscillations in which voltage, charge or current cycles around an equilibrium value - can propagate through a vacuum
- **Mechanical waves** – the propagation of the oscillatory motion of particles through space due to the elasticity of the environment
- ***Transverse waves*** – particles oscillate perpendicular to the motion of the wave (e.g., water rippling on the surface)
- ***Longitudinal waves*** – the particles of the environment oscillate parallel to the wave motion (e.g. sound waves in air)

# Oscillation and waves

- The source of the waves oscillates harmonically  $\Rightarrow$  the plot of the displacement of the particles from the equilibrium position at a given time versus the distance from the source is also sinusoidal see Fig.

- The amplitude  $A$  of a single particle oscillation; also the amplitude of the wave

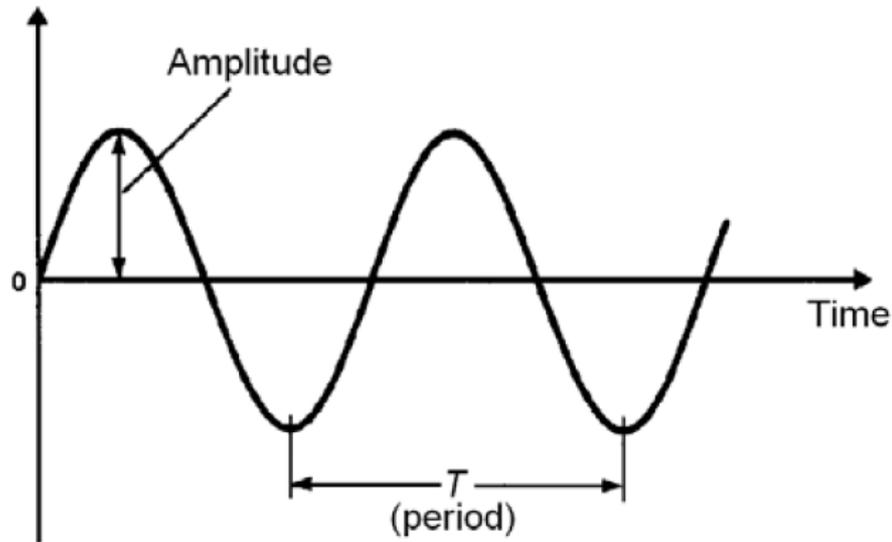
The wavelength  $\lambda$  – the distance between any nearest particles at the same phase of their motion  $[\lambda] = \text{m}$

- The period  $T$  – the time it takes any particle to complete one oscillation; also the time it takes a wave to travel one wavelength

- The frequency  $f$  – the number of cycles of any particle in one second; also the number of wavelengths that pass through a fixed point in one second  $[f] = \text{Hz}$

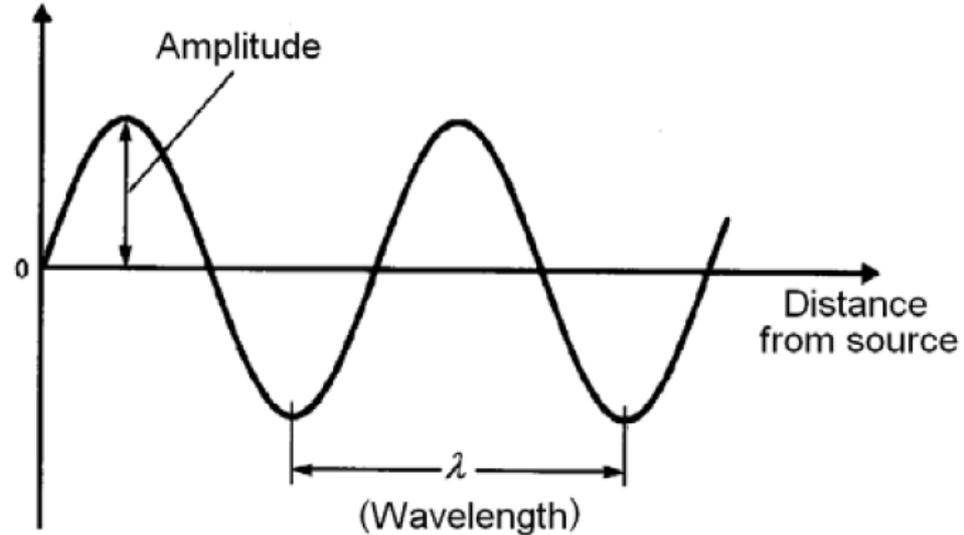
# Oscillation and waves

Displacement from equilibrium position



Displacement against time for a single particle in harmonic wave

Displacement from equilibrium position



Displacement against distance for particles in a harmonic wave

# Oscillation and waves

The speed of wave  $v$  – the distance the wave travels in one second

- The product of the number of cycles per second (frequency)  $f$  and the wavelength  $\lambda$ :  $v = f \lambda$

- The energy transferred by an oscillator is proportional to the square of its amplitude  $\Rightarrow$  a proportionality between the square of the amplitude of the wave and its intensity

- The intensity  $I$  at a given point – the energy per second passing through the unit surface in the direction along the normal

- The intensity  $I$  at a distance  $r$  from a point source of power  $P$  which radiates waves in all directions in a non-absorbing medium:

$$I = \frac{P}{4\pi r^2}, \quad [I] = \text{W/m}^2 \quad (4\pi r^2 - \text{the area of a sphere of radius } r)$$

- The intensity  $I(r)$  of emitting waves – inversely proportional to the distance  $r$  from the source:  $I(r) \sim 1 / r^2$

# Acoustics

- Sound waves – longitudinal pressure waves travelling through a medium
- Sounds – produced by vibrating objects (strings, pipes, loudspeakers)
- The hearing range for humans – sounds of frequencies 20 Hz - 20 kHz

## Example:

We assume that the speed of sound in air is about 333 m/s. If a lightning flash in a thunderstorm is followed by sound after 6 seconds, what is the distance between the lightning and the observer? Assume that light travels much faster than sound.

[ $d = 2 \text{ km}$ ]

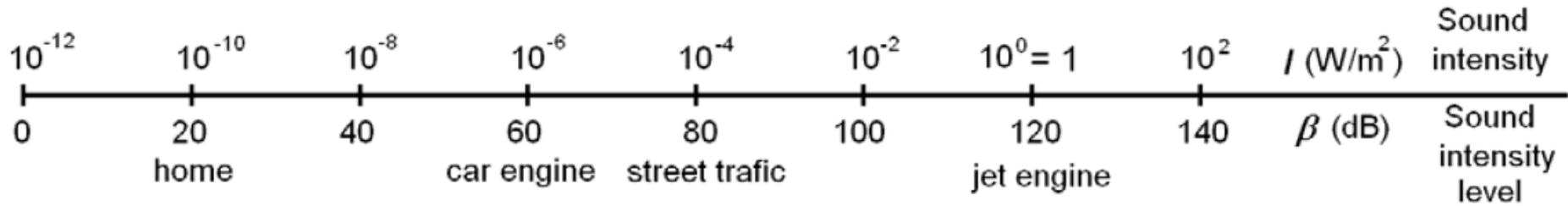
# Acoustics

- The speed of sound as any mechanical waves depends on the elastic and inertial properties of the medium:
- The speed of sound – in water about 1450 m/s, in steel 5900 m/s
- The intensity of a sound  $I$  – the energy of a sound wave per unit area per second:  $[I] = \text{J.m}^{-2}.\text{s}^{-1} = \text{W.m}^{-2}$
- Average sound intensity range for human hearing:  
*hearing threshold* ( $I_0 = 10^{-12} \text{ W/m}^2$ ) - *pain threshold* ( $I_{\text{max}} = 10^2 \text{ W/m}^2$ )  $\Rightarrow$   
 $\Rightarrow$  huge range of sound intensity values -  $10^{14}$
- Better to use a logarithmic scale:

The sound intensity level of the sound intensity  $I$ :  $\beta = 10 \log \frac{I}{I_0}$

$[\beta] = \text{dB}$  – decibel

# Acoustics



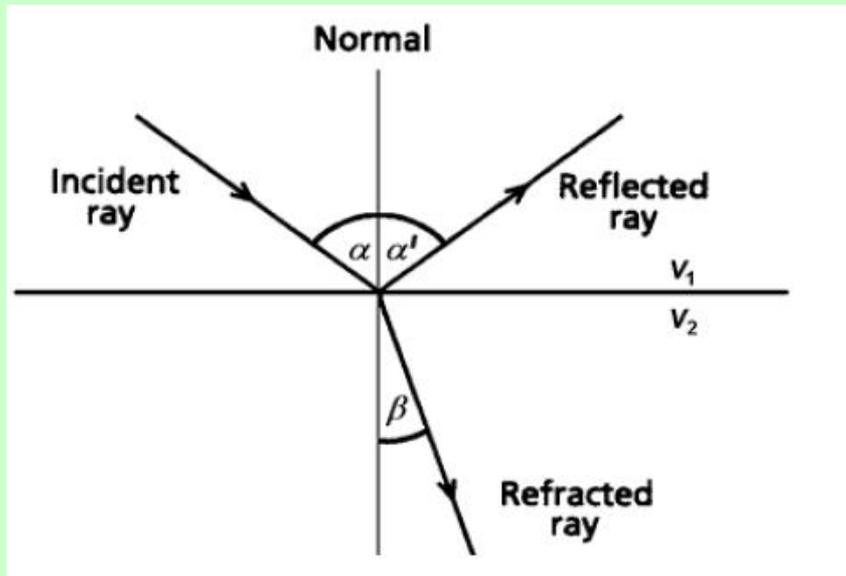
## Example 6.15:

What sound intensity level corresponds to a sound intensity of  $10^{-4} \text{ W/m}^2$ ?

$[\beta = 80 \text{ dB}]$

# Geometrical (Ray) Optics

- The rays show the direction of light propagation
- Geometric optics assumes that light moves in straight lines and presents laws that describe the reflection and refraction of light rays
- When rays strike a plane surface, their direction changes
- Light is reflected from the plane surface, and some light may pass from one medium to another transparent medium in which the speed of propagation changes - light refracts – see Fig.



# Geometrical (Ray) Optics

Law of reflection:  $\alpha = \alpha'$  (alpha with a line)

- The angle of incidence  $\alpha$  always equals the angle of reflection  $\alpha'$

Law of refraction - Snell's law:

- The ratio of the sine angle of incidence  $\alpha$  to the sine angle of refraction  $\beta$

is constant:  $\frac{\sin \alpha}{\sin \beta} = \frac{v_1}{v_2} = \frac{n_2}{n_1}$ ,

where  $v_1$  – the speed of light in the medium 1

$v_2$  – the speed of light in the medium 2

$n_1 = \frac{c}{v_1}$  – *index of refraction* of a medium with the speed of light  $v_1$

$n_2 = \frac{c}{v_2}$  – *index of refraction* of a medium with the speed of light  $v_2$

( $c = 3 \cdot 10^8$  m/s – speed of light in vacuum)

# Geometrical (Ray) Optics

## Total reflection

- If light travels from the optically denser medium of  $n_1$  into the optically thinner medium of  $n_2$  ( $v_1 < v_2$ ), then  $\beta > \alpha$
- The *critical angle*  $\alpha_m$  is reached when the refraction angle of the ray  $\beta = 90^\circ$
- At angles  $\alpha$  greater than the critical angle  $\alpha > \alpha_m$  the ray does not pass to the second medium and it is *totally reflected*

## Example 10.17:

The light ray passes from the glass into the air. Calculate the angle of refraction if the angle of incidence is  $30^\circ$  and the refractive index of the glass is 1.5. Calculate the speed of light in the glass.

$$[\beta = 48^\circ 35', v_1 = 2 \cdot 10^8 \text{ m/s}]$$

# Geometrical (Ray) Optics

## Image creation

- Reflection or refraction of light can be used to create an image of a physical object using mirrors and lenses

Mirror – a surface that reflects most of the light that falls on it

- When rays of light fall on a mirror, the law of reflection tells how the rays are reflected

- *A plane mirror* – a flat reflecting surface

- The image formed by a plane mirror is always the same size as the object; is as far behind the mirror as the object is in front of the mirror

- The image is laterally inverted

- When you look at yourself in a plane mirror, you do not see yourself exactly as you appear to others, but with the right and left sides reversed

# Geometrical (Ray) Optics

- ***Spherical mirrors*** – made up of the surfaces of spheres

**Concave mirrors** have a reflecting surface in the inner parts of the sphere

**Convex mirrors** have a reflecting surface in the outer parts of the sphere

- **Position of the image compared to the position of the object:**

- The image can be *upright* or *inverted*

- **Height of the image compared to the height of the object:**

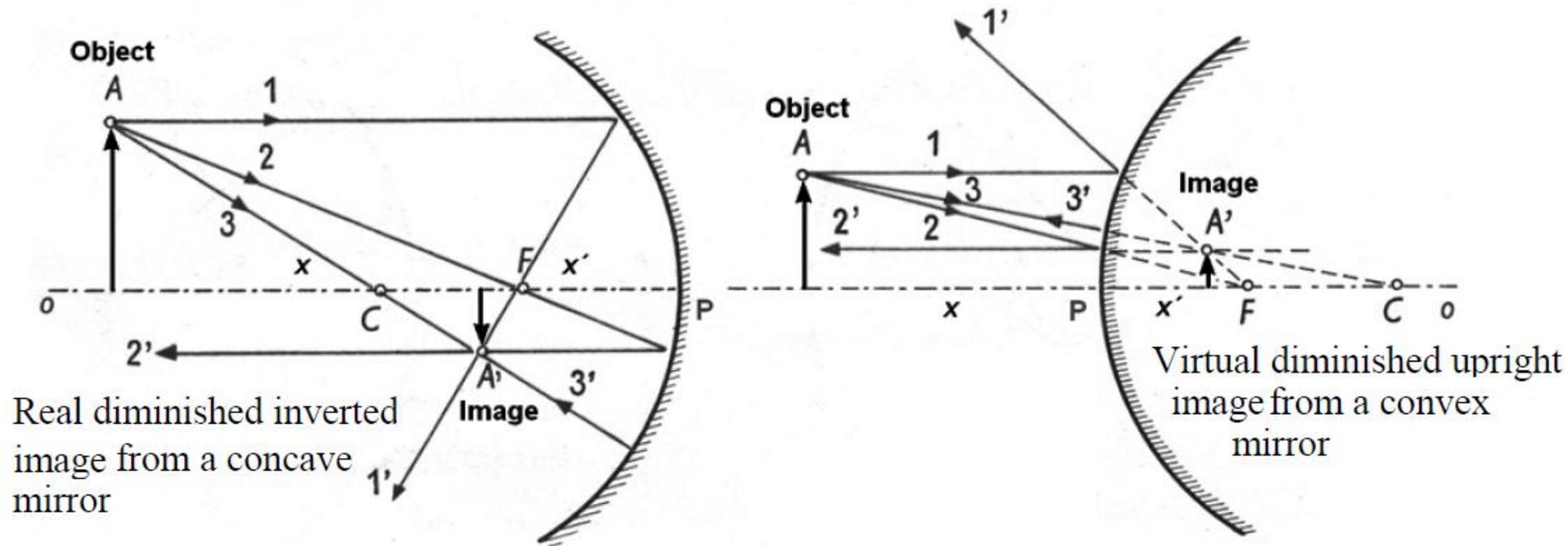
- The image can be *enlarged*, *diminished* or the *same size* as the object

- ***A virtual image*** – an image from which light rays seem to pass and which cannot be created on a screen

- ***A real image*** – one through which light rays actually pass and can be displayed on a screen

# Geometrical (Ray) Optics

Graphical construction of image – see Fig.



**C** – the centre of curvature of the sphere from which the mirror's part of surface is formed,

**P** – the top of the mirror,

**CP** – the radius of curvature  $r$  of the mirror

# Geometrical (Ray) Optics

- The distance from the focus  $F$  to the top of the mirror  $P$  – the *focal length*  $f$
- The focal length  $f$  is related to the radius of curvature  $r$  of the mirror by the relation:  $f = r / 2$
- For each type of spherical mirror there are three main rays whose paths can be determined
  - 1 - the incident ray parallel to the optical axis is reflected to the focus  $F$ ,
  - 2 - the incident ray passing through focus  $F$  is reflected back parallel to the optical axis,
  - 3 - the incident ray passing through the centre of curvature  $C$  is reflected back along its original path

# Geometrical (Ray) Optics

## Calculation of image properties

- To calculate image position, the mirror formula (equation) is used:

$$\frac{1}{x} + \frac{1}{x'} = \frac{1}{f} \quad (f = r / 2) , \text{ where}$$

$x$  – object-mirror distance,

$x'$  – image-mirror distance,

$f$  – focal length of mirror,

$r$  – radius of curvature of mirror

*Lateral magnification*  $m$  – the ratio of the image height  $h_i$  to the object

height  $h_o$ : 
$$m = \frac{h_i}{h_o} = -\frac{x'}{x}$$

# Geometrical (Ray) Optics

Sign convention for these two equations:

- All distances are measured to the top P of the mirror
- The distances in front of the mirror are of positive algebraic sign
- The distances behind the mirror are of negative algebraic sign
- This is also true for focal distances:  $f > 0$  concave mirror,  $f < 0$  convex mirror
- Positive values  $x' > 0$  – for real images
- Negative values of  $x' < 0$  – for virtual images
- The object and image heights,  $h_o$  and  $h_i$  – positive for points above the optical axis, negative for points below the optical axis

# Geometrical (Ray) Optics

**Lens** – a curved, transparent material used to refract the light that falls on it

- Light is refracted from the first material (usually air) into the lens, passes through the lens, and then refracts back into the first material
- ***Converging lens*** - incident light rays parallel to the optical axis converge to the ***focus (focal point) F*** after refraction on the lens surfaces
- ***Diverging lens*** - incident light rays parallel to the optical axis diverge from the ***focus (focal point) F*** after refraction on the lens surfaces
- ***Focal length f*** – the distance between the optical centre O and the focal point F

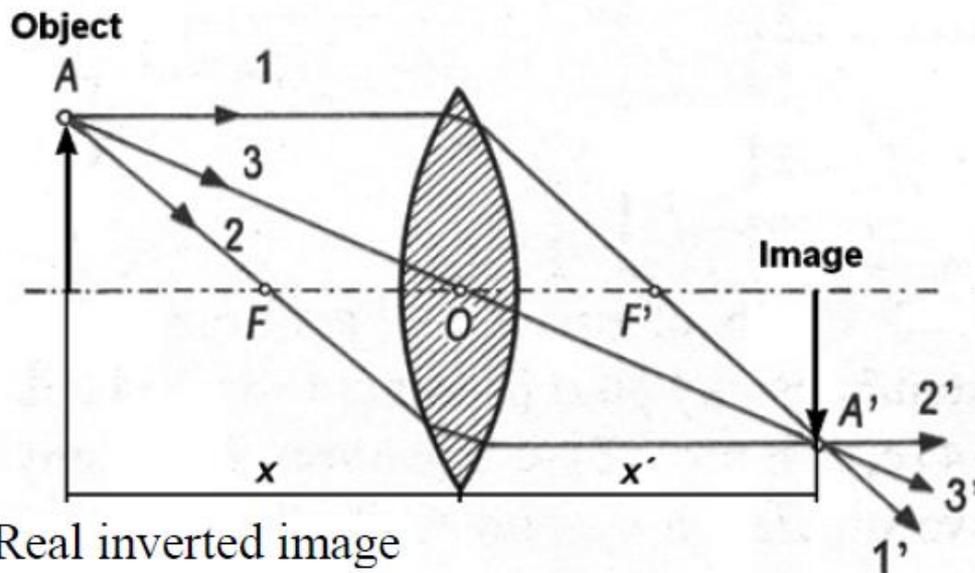
***The power of lens P:***  $P = \frac{1}{f}$  ( Power =  $\frac{1}{\text{focal length in metres}}$  )

**$[P] = \text{m}^{-1} = \text{D (diopetre)}$**

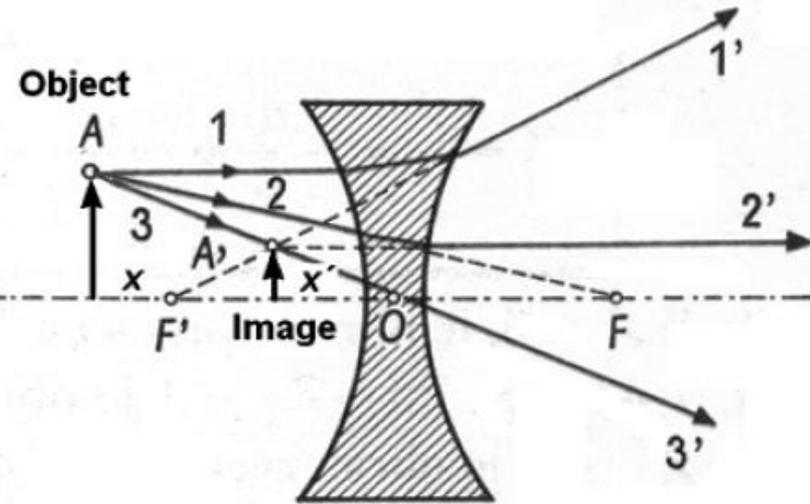
# Geometrical (Ray) Optics

Graphic construction of the image – see Fig.

- If the paths of two arbitrary rays from one point on an object are known, the position of its image (the intersection of the two refracted rays) can be determined
- For each type of lens there are three main rays whose paths can be determined



Real inverted image  
from a converging lens



Virtual upright image  
from a diverging lens

# Geometrical (Ray) Optics

1 - the incident ray parallel to the optical axis is refracted to the focus  $F'$  on the other side of the lens,

2 - the incident ray passing through the  $F$  focus is refracted parallel to the optical axis,

3 - the incident ray passing through the optical centre  $O$  does not change its direction

## Calculation of image properties

- To calculate image position, the lens formula (equation) is used:

$$\frac{1}{x} + \frac{1}{x'} = \frac{1}{f}, \text{ where}$$

$x$  – object-lens distance,

$x'$  – image-lens distance,

$f$  – focal length of lens

# Geometrical (Ray) Optics

**Lateral magnification  $m$**  – the ratio of the image height  $h_i$  to the object

height  $h_o$ : 
$$m = \frac{h_i}{h_o} = -\frac{x'}{x}$$

**Sign convention for these two equations:**

- All distances are measured from the optical centre O of the lens,
- The object distance – positive if the object is in front of the lens,
- The image distance – positive if the image is behind the lens and negative if the image is in front of the lens (positive values of  $x' > 0$  for real images, negative values of  $x' < 0$  for virtual images),
- The focal length – positive for a converging lens  $f > 0$  and negative for a diverging lens  $f < 0$ ,
- The object and image heights  $h_o$  and  $h_i$  – positive for points above the principal axis and negative for points below the principal axis