

Water Resources Research



METHOD

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Key Points:

- An analytical solution is developed for oscillatory pumping tests with rate-dependent skin
- The effects of rate-dependent skin on the head response are evident in field data
- Neglecting rate-dependent skin can lead to biased estimates of aquifer parameters

Correspondence to:

Y.-F. Lin,
aar246860@gmail.com

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Author Contributions:

Formal analysis: Ali Mahdavi, Barret L. Kurylyk, Yin-Fang Lin

Methodology: Ali Mahdavi, Barret L. Kurylyk, Yin-Fang Lin

Software: Ali Mahdavi, Yin-Fang Lin

Validation: Yin-Fang Lin

Writing – original draft: Ali Mahdavi, Yin-Fang Lin

Writing – review & editing: Ali Mahdavi, Barret L. Kurylyk, Yin-Fang Lin

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Advanced Analytical Model for Interpreting Oscillatory Pumping Tests With Wellbore Skin and Rate-Dependent Skin Effects

Ali Mahdavi¹ , Barret L. Kurylyk² , and Yin-Fang Lin³ 

¹Department of Civil Engineering, Arak University, Arak, Iran, ²Department of Civil and Resource Engineering and Centre for Water Resources Studies, Dalhousie University, Halifax, NS, Canada, ³Department of Bioenvironmental Systems Engineering, National Taiwan University, Taipei, Taiwan

Abstract Oscillatory pumping tests presents a potential advancement in aquifer testing as they minimize hydraulic perturbations and result in no net water abstraction or injection. However, aquifer property estimates can be less accurate if they are compromised by well head losses during the pumping test. Currently, oscillatory pumping test models ignore these losses. The objective of this study is to develop and evaluate an analytical solution that can interpret oscillatory pumping test data affected by rate-dependent skin losses. Accordingly, the sensitivity of the drawdown response to small variations in aquifer and well parameters was analyzed. In addition, a previous field aquifer test was analyzed to evaluate the feasibility of the solution for aquifer parameter estimation. The results indicate that neglecting the rate-dependent skin effect can lead to substantial errors in the determination of aquifer parameters such as transmissivity and storativity. Additionally, the proposed solution effectively reproduces the sharp groundwater level peaks observed in field data, whereas the solution that does not consider the rate-dependent skin effect greatly underestimates the peak values.

Plain Language Summary This study looks at a new method called the oscillatory pump test for measuring aquifer properties. There can be problems with obtaining accurate results when testing aquifer properties, because there can be a loss of water at the top of the well. This study aims to solve this problem by creating a new solution that can account for this loss of water and still yield accurate results. The study found that ignoring this loss of water can lead to significant errors in the estimation of the aquifer properties. The study shows that when the value of the loss of water is over 10,000 min/m³, it can have a substantial effect on the accuracy of the results. Overall, this study improves our ability to estimate aquifer properties using the oscillatory pumping test method.

1. Introduction

Accurate estimation of subsurface hydraulic properties is crucial for groundwater resource management, contaminant remediation, and oil and gas exploration (ASTM, 2002). However, acquiring detailed knowledge about subsurface properties is challenging due to the high cost of drilling and the challenges of conducting aquifer hydraulic testing. Aquifer hydraulic tests such as constant-rate pumping tests and slug tests are commonly conducted to estimate aquifer parameters (Butler, 2019). Recently, oscillatory pumping tests (OPTs) have been increasingly recognized as a promising approach for characterizing sedimentary aquifers (Patterson & Cardiff, 2022). Compared to constant-rate pumping tests, OPTs offer several advantages, including no net water extraction or injection, reduced contaminant plume movement, and the ability to explore the spatial variability of aquifer properties in response to a range of perturbation frequencies imposed on the aquifer system (Bakhos et al., 2014; Cardiff & Barrash, 2015; Cardiff et al., 2013; Cardiff & Sayler, 2016; Rabinovich et al., 2015; Rasmussen et al., 2003). Furthermore, OPTs can effectively map the structure of fractured rock across multiple scales (Patterson & Cardiff, 2023). Specifically, low-frequency pumping signals yield insights into fracture scales comparable to those revealed by constant-rate pumping tests, whereas high-frequency pumping signals provide hydraulic information akin to that obtained from single-well slug tests (Cardiff et al., 2013). Employing standard signal processing methods allows for a deconvolution of the observed pressure signal from extraneous factors such as instrument drift, instrument noise, and various unrelated signals that may corrupt the data (Bakhos et al., 2014).

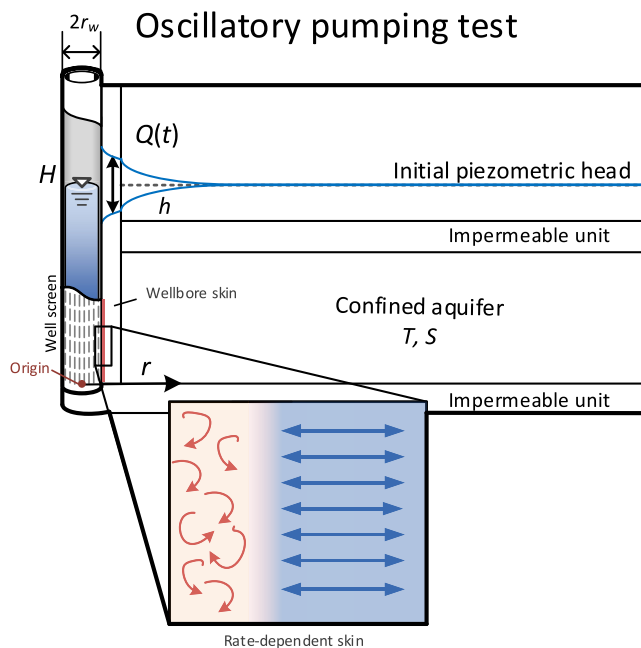


Figure 1. Schematic diagram of an oscillatory pumping test conducted in a confined aquifer by considering the wellbore skin and rate-dependent skin.

The wellbore skin is a small region surrounding a wellbore that has different permeability than the surrounding formation. The skin is formed during the well installation process (Yeh & Chang, 2013) and can increase or decrease the head response during aquifer testing, depending on whether its permeability is lower or higher than the ambient permeability. Studies have considered the wellbore skin by assuming that it is either infinitesimally thin or has a finite thickness with its own flow equation that is connected to the aquifer flow equation (Barker & Herbert, 1982; Barua & Bora, 2010; Jia et al., 2017; Lin et al., 2016; Novakowski, 1989; Wang et al., 2018; Yang & Yeh, 2005). The skin effect is important in the analysis of aquifer tests because it can affect flow behavior induced by pumping.

Non-Darcian flow can occur in high-velocity water flowing through well screens and gravel packs, causing a rate-dependent effect on groundwater flow. In petroleum engineering, this effect is accommodated by introducing a rate-dependent factor. The rate-dependent factor can account for non-Darcian flow by expressing it as the product of a non-Darcian flow constant and the wellbore flow rate. Henry and Ramey (1982) showed that combining a model boundary condition with a rate-dependent factor yields an equivalent form of the well-loss equation proposed by Jacob (1947). Beidokhti et al. (2011) derived the rate-dependent factor from the Forchheimer equation and found it to have a similar form to the boundary condition with a rate-dependent factor. The rate-dependent factor has been widely accepted in pressure models appearing in petroleum engineering (Beidokhti et al., 2011; Ramey, 1965). In groundwater hydraulics, however, the Forchheimer (1901) and Izbash (1931)

laws and mass conservation are still commonly combined to develop the groundwater flow equation. This approach implies that flow in the entire aquifer system is non-Darcian, which may not be reasonable even during pumping since flow further from the well is typically Darcian. Accordingly, Lin and Yeh (2020a) proposed an analytical framework accounting for non-Darcian flow through the well screen boundary condition and found that non-Darcian flow due to pumping at the well screen results in head loss, known as the rate-dependent skin, that is proportional to the pumping rate and can affect aquifer parameter estimation.

The hydraulic response in OPTs can also be substantially influenced by wellbore skin and changes in flow direction due to rate-dependent skin effects. However, existing OPT models do not incorporate the joint effects of wellbore skin and rate-dependent skin. This study aims to incorporate both types of skin effects into a new OPT analytical model, while accommodating a variable pumping rate. The effect of rate-dependent skin on the hydraulic response to an OPT is investigated by varying its value in the model within reasonable bounds. A sensitivity analysis is performed to investigate the influence of aquifer and well parameters on the hydraulic response. The new OPT model is also applied to analyze previously collected OPT field data to assess the model's performance under non-idealized conditions. The impact of considering rate-dependent skin effects for parameter estimation is evaluated through the application of the inverse model.

2. Methods

2.1. Model Setup

Figure 1 shows a confined aquifer subject to an OPT bounded by impermeable units above and below. The center of the pumping well serves as the origin of the coordinate system, with the initial piezometric head as the reference datum. The well screen is installed throughout the thickness of the aquifer and has a radius of r_w [L]. The OPT allows for a variable pumping rate, $Q(t)$ [$L^3 T^{-1}$], and the hydrogeological parameters of interest are transmissivity, T [$L^2 T^{-1}$] and the storage coefficient, S [–]. In general, the counteracting effects of T and S on the aquifer response to the OPT can be combined by considering the aquifer diffusivity (T/S [$L^2 T^{-1}$]) as the governing parameter; however we prefer to separate out the transmissive and storage controls. The aquifer is assumed to have an infinite lateral extent and be confined, two-dimensional, homogeneous, and isotropic. The hydraulic head in the well $H(t)$ [L] and the aquifer $h(r, t)$ [L] differ from each other due to the well storage effect. The parameter r [L] is the radial distance from the pumping well, and t [T] is the operation time. The wellbore skin and

rate-dependent skin near the well screen affect the aquifer flow caused by pumping. By combining Darcy's Law and the continuity equation, the aquifer head for radially symmetric flow produced by the OPT can be described by Cardiff and Saylor (2016):

$$S \frac{\partial h(r, t)}{\partial t} = T \left(\frac{\partial^2 h(r, t)}{\partial r^2} + \frac{1}{r} \frac{\partial h(r, t)}{\partial r} \right), (r, t) \in [r_w, \infty) \times (0, \infty). \quad (1)$$

By assuming that the groundwater is undisturbed at the beginning of the test, the initial condition can be expressed as:

$$h(r, t) = 0, t = 0 \quad (2)$$

The boundary condition at the well surface considering the effect of wellbore storage is represented by the equation proposed by Papadopoulos and Cooper (1967):

$$2\pi T r \frac{\partial h(r, t)}{\partial r} = -Q(t) + \pi r^2 \frac{\partial H(t)}{\partial t}, r = r_w \quad (3)$$

The operating pumping rate $Q(t)$ can be defined as $Q_{\text{peak}} \text{Re}[\exp(i\omega t)]$ ($Q_{\text{peak}} \cos(\omega t)$) or $Q_{\text{peak}} \text{Im}[\exp(i\omega t)]$ ($Q_{\text{peak}} \sin(\omega t)$) where Q_{peak} [$\text{L}^3 \text{T}^{-1}$] is the maximum pumping rate achieved during a pumping cycle, ω [T^{-1}] is the angular pumping frequency, $i = \sqrt{-1}$, and $\text{Im}(\cdot)$ represents the imaginary part of the function.

To include the rate-dependent skin effects, an additional condition is required (Lin & Yeh, 2020a):

$$h(r, t) - r(S_f + D|Q(t)|) \frac{\partial h(r, t)}{\partial r} = H(t), r = r_w \quad (4)$$

in which S_f is the skin factor [–] accounting for the formation damage at the interface between the wellbore surface and the aquifer, and $DQ(t)$ is the rate-dependent skin [–] representing the head loss caused by the high velocity at the screen with the rate-dependent factor D [$\text{L}^{-3} \text{T}$]. In addition, the far-field aquifer conditions are assumed to be unaffected by the OPT:

$$h(r, t) = 0, r \rightarrow \infty \quad (5)$$

2.2. Analytical Solution

A closed-form solution to the boundary value problem described in Section 2.1 is derived by means of the direct Fourier method. Considering sinusoidal pumping, the transformation $h(r, t) = \text{Im}[\Phi(t, r)e^{i\omega t}]$ and $H(t) = \text{Im}[\Phi^*(t)e^{i\omega t}]$, with the wave phasors designated by Φ and Φ^* , results in the following expression:

$$i\omega S \Phi(r, t) = T \left(\frac{d^2 \Phi(r, t)}{dr^2} + \frac{1}{r} \frac{d\Phi(r, t)}{dr} \right) \quad (6)$$

subject to the conditions:

$$2\pi T r \frac{d\Phi(r, t)}{dr} = Q_{\text{peak}} + \pi r^2 i\omega \Phi^*(t), r = r_w \quad (7a)$$

$$\Phi(r, t) - r(S_f + DQ_{\text{peak}} |\sin(\omega t)|) \frac{d\Phi(r, t)}{dr} = \Phi^*(t), r = r_w \quad (7b)$$

$$\Phi(r, t) = 0, r \rightarrow \infty. \quad (7c)$$

By combining Equations 7a and 7b to eliminate Φ^* , an updated inner boundary condition can be obtained as:

$$\Omega_1 r \frac{d\Phi(r, t)}{dr} - \Omega_2 \Phi(r, t) = Q_{\text{peak}}, r = r_w \quad (8)$$

where

$$\Omega_1(t) = \Gamma_1 + \Gamma_2(t) \quad (9a)$$

$$\Omega_2 = \pi r_w^2 i \omega \quad (9b)$$

$$\Gamma_1 = 2\pi T \quad (9c)$$

$$\Gamma_2(t) = \Omega_2 (S_f + DQ_{\text{peak}} |\sin(\omega t)|) \quad (9d)$$

Thus, the solution of Equation 6 takes the form:

$$\Phi(r, t) = -\frac{Q_{\text{peak}} K_0(\lambda r)}{\Omega_2 K_0(\lambda r_w) + \lambda r_w \Omega_1(t) K_1(\lambda r_w)} \quad (10)$$

where $\lambda = \sqrt{i\omega/\kappa}$, $\kappa = T/S$ (diffusivity) and $K_0(\cdot)$ and $K_1(\cdot)$ are the zero-order and first-order modified Bessel functions of the second kind, respectively. Consequently, the head solution is expressed as

$$h(r, t) = \text{Im} \left[\frac{-Q_{\text{peak}} K_0(\lambda r) e^{i\omega t}}{\Omega_2 K_0(\lambda r_w) + \lambda r_w \Omega_1 K_1(\lambda r_w)} \right] \quad (11)$$

For sinusoidal pumping, the solution can be modified by using a sine function in Γ_2 and taking the imaginary part of the h solution.

2.3. Sensitivity Analysis

A sensitivity analysis was performed to identify the effects of the aquifer parameters on the model output (Dell'Oca et al., 2020; Knabe et al., 2021; Mahdavi et al., 2022). Kabala (2001) defined the normalized sensitivity coefficient as:

$$X_k(r, t; P_k) = P_k \frac{\partial \Phi(r, t; P_k)}{\partial P_k} \quad (12)$$

where X_k is the normalized sensitivity of the output (i.e., the wave phasor Φ here) to the k th input aquifer parameters P_k . A positive value of X means that an increase in an aquifer parameter causes an increase in h . Conversely, a negative X implies that an increase in a parameter yields a decrease in h .

A differential importance measure (DIM) method has also been proposed to classify the importance of the model parameters (Borgonovo & Apostolakis, 2001; Lin et al., 2022; Lin & Yeh, 2020b):

$$\text{DIM}_k(r, t; P_k) = \frac{|X_k(r, t; P_k)|}{\sum |X_k(r, t; P_k)|} \quad (13)$$

where $\text{DIM} \in [0, 1]$. Equation 13 quantifies the contribution of a parameter's sensitivity to the sensitivities of all parameters in response to a phasor change. Thus, a higher value of DIM means a higher sensitivity of the given parameter.

2.4. Parameter Estimation

Rabinovich et al. (2015) carried out field OPTs within an aquifer system near the Boise River at the Boise Hydrogeophysical Research Site, ID, USA. The delineated aquifer encompasses four predominantly cobble-dominated strata overlain by a sand channel that tapers off across the center of the wellfield (Rabinovich et al., 2015). A total of 14 OPTs were conducted, each with varying pumping/injection rates and frequencies. To evaluate the impact of the rate-dependent skin, data from one particular test (0717 Test 8) was selected for parameter estimation as it was the sole test for which water level time series were visually provided in their study. Data from this test were digitally extracted from the plot (Figure 4 in Rabinovich et al., 2015). The pumping well had a radius of 0.05 m. The pumping rate was characterized by the expression $Q_{\text{peak}} \sin(\omega t)$, where $Q_{\text{peak}} = 5.8 \times 10^{-5} \text{ m}^3/\text{s}$ and $\omega = 2\pi/24 \text{ s}^{-1}$. The head data were recorded in the pumping well B3. Three distinct models were deployed to analyze the head data and estimate aquifer parameters: Solution 1 retains the original configuration of the present model (Equation 11); Solution 2 modifies the present model by omitting the rate-dependent skin; and Solution 3 further alters the present model by excluding both the wellbore skin and the rate-dependent skin. The solutions are converted to sinusoidal pumping.

The objective function OB is defined as minimizing the sum of the squared prediction errors:

$$OB : \min_{T, S, S_f, D} \sum_{j=1}^M e_j^2 \quad (14)$$

where M is the total number of observations and e_j is the j th prediction error, calculated as the difference between the observed and predicted heads for the j th data. During parameter estimation, initial values for the parameters T and S were considered, where T is 10^{-5} m²/min, and S is 10^{-3} , which are typical values for various permeable geologic materials, such as sandstone, limestone, fractured rock, and unconsolidated sediments (Freeze & Cherry, 1977). A logarithmic transformation was applied to T , S , S_f , and D to avoid generating a negative estimate. Parameter values were estimated in accordance with Equation 11, employing a combined approach of the differential evolution method (Storn & Price, 1997) and the Levenberg-Marquardt method (Gill et al., 1981) in the post-processing phase. The estimation procedure was executed using the Mathematica routine `NonlinearModelFit` (Wolfram Research, Inc, 2023), limiting the maximum iterations to 1000.

The goodness of fit was evaluated with two statistics: the standard error of estimation (SEE) and the mean error (ME) (Yeh, 1987). The SEE and ME were respectively calculated as the square root of the mean squared prediction error and mean prediction error:

$$SEE = \sqrt{\frac{1}{M - v} \sum_{j=1}^M e_j^2} \quad (15a)$$

and

$$ME = \frac{1}{M} \sum_{j=1}^M e_j \quad (15b)$$

where $M - v$ is the degrees of freedom, with v being the number of uncertain parameters.

3. Results and Discussion

3.1. Solution Behavior

Unless otherwise stated, default representative values assigned to the model parameters are $Q_{\text{peak}} = 1 \times 10^{-3}$ m³/min, $r_w = 0.05$ m, $r = 10$ m, $T = 1 \times 10^{-6}$ m²/min, $S = 1 \times 10^{-4}$ (i.e., the diffusivity is $\kappa = 1 \times 10^{-2}$ m²/min), $\omega = 2\pi/500$ min⁻¹, $S_f = 10$, and $D = 0.01$ min/m³, where the value of 0.01 m³/min is chosen to minimize its interference with the effects of other parameters while still having an impact during the sensitivity analysis.

3.1.1. Effects of the Aquifer Parameters and Wellbore Skin on the Aquifer Hydraulic Response

It is critical to analyze how the parameters T , S , and S_f affect the head behavior caused by OPT to gain a comprehensive understanding of the rate-dependent skin effects and how these effects may influence parameter estimation. While acknowledging the extensive research that has been conducted on these parameters in the general context of well hydraulics, this study aims to investigate their specific influence in relation to the rate-dependent skin and during OPT.

Figure 2a displays the temporal distribution of the head from Equation 11 observed at a radial distance of $r = 10$ m, while varying the T values between 1×10^{-6} and 1×10^{-5} m²/min with intervals of 1×10^{-6} m²/min. The figure expectedly shows that as the aquifer permeability increases (as indicated by higher values of T), the head oscillations also increase. Specifically, an increase in T causes an evident upper-right shift in the positive peaks of the curves.

The effect of S on the head change is opposite to that of T . Figure 2b shows the temporal head distribution observed at $r = 10$ m while changing the S values from 1×10^{-4} to 2×10^{-4} in uniform intervals of 1×10^{-5} . An increase in S will damp the head amplitude and hinder head signal propagation to the observation well.

3.1.2. Influence of Skin Effects on the Aquifer Hydraulic Response

Figure 3a depicts the head fluctuations for different values of the skin factor S_f ranging from 1 to 10 with intervals of 1 and a constant value of $D = 0$ observed at $r = 10$ m. Increasing S_f has the effect of decreasing the hydraulic head amplitude, because a higher value of S_f results in a higher formation head loss. Importantly, the hydraulic

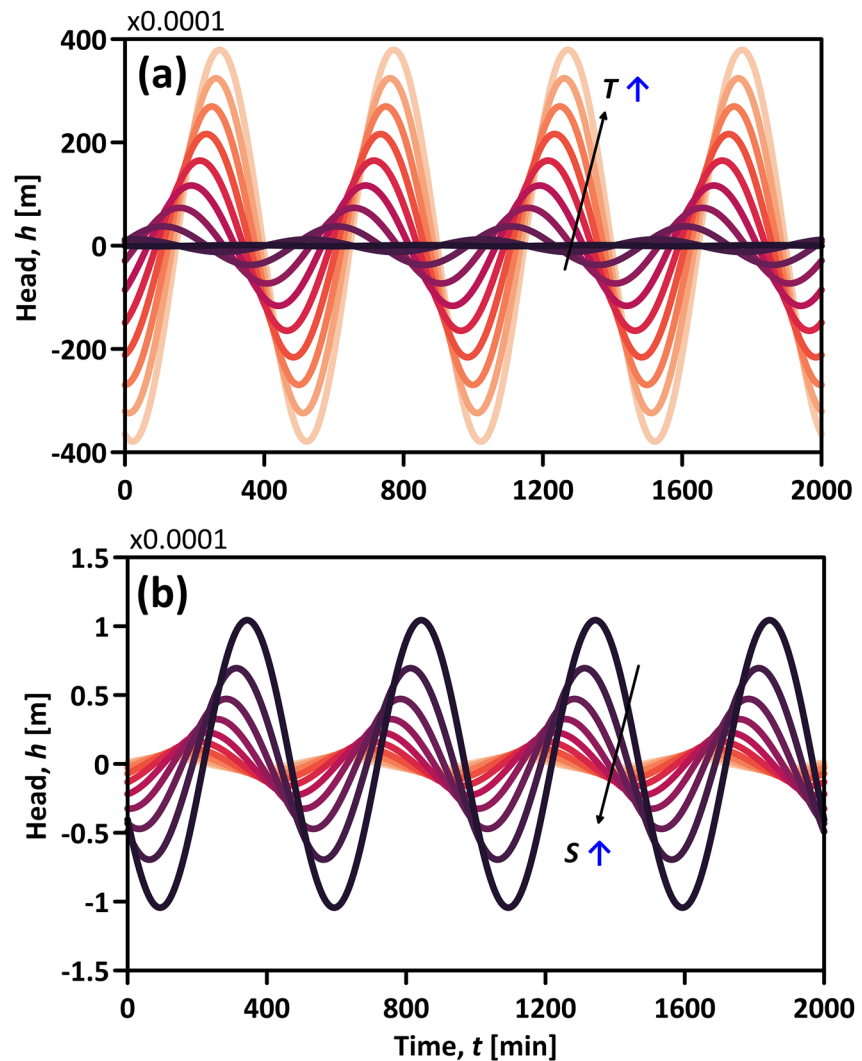


Figure 2. The hydraulic response to the OPT for different values of (a) T ranging from 1×10^{-6} to $1 \times 10^{-5} \text{ m}^2/\text{min}$ and of (b) S ranging from 1×10^{-4} to 2×10^{-4} observed at $r = 10 \text{ m}$ with $S_f = 10$.

head is uniformly affected by the wellbore skin over time. The temporal head fluctuation predicted by the solutions for different rate-dependent factor D values observed at $r = 10 \text{ m}$ is shown in Figure 3b. The values of D range from 0 to $1 \times 10^4 \text{ min}/\text{m}^3$, while $S_f = 0$. The results show that as D increases, the absolute value of the hydraulic head decreases, especially in the later stages of the wave (or when the flow changes its direction through the wellbore screen), as indicated by the blue shaded areas in the figure. The effect is more pronounced for higher values of D and more strongly influences the middle of the half cycle.

As D decreases, the waveforms become more asymmetrical and exhibit a sawtooth pattern. Figure 4 presents three statistical indices (i.e., mean, standard deviation, and skewness) that quantify this behavior, corresponding to the positive half cycle of the head oscillations shown in Figure 3. Both the mean and the standard deviation of the head oscillations show a decreasing trend as D decreases. Obviously, a larger effect of the rate-dependent skin can result in higher skewness with lower mean and standard deviation. The negative skewness values indicate that the curves are skewed to the right, while skewness values associated with $D > 1 \times 10^4 \text{ min}/\text{m}^3$ indicate that the curves are skewed to the left. This finding implies that the effect of rate-dependent skin becomes significant when its value exceeds $1 \times 10^4 \text{ min}/\text{m}^3$ for the given reasonable range of parameters ($Q_{\text{peak}} = 1 \times 10^{-3} \text{ m}^3/\text{min}$, $r_w = 0.05 \text{ m}$, $r = 10 \text{ m}$, $T = 1 \times 10^{-6} \text{ m}^2/\text{min}$, $S = 1 \times 10^{-4}$, $\omega = 2\pi/500 \text{ min}^{-1}$, and $S_f = 10$).

These sawtooth patterns are evident in field data from OPTs. For example, Cardiff et al. (2020) presented raw data from an OPT in an unconfined aquifer at Boise Hydrogeophysical Research Site, Boise, Idaho, USA. In

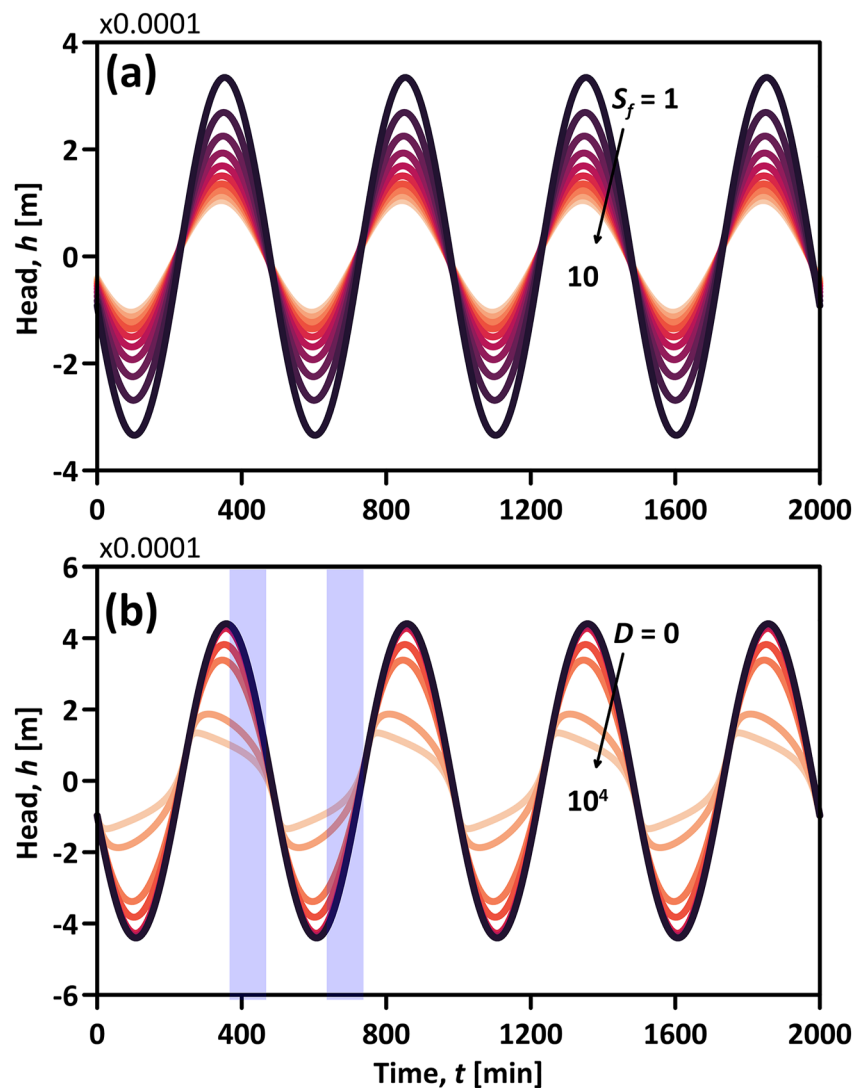


Figure 3. The hydraulic response of the OPT to different values of (a) S_f ranging from 1 to 10 and of (b) D ranging from 0 to 1×10^4 min/m³ (i.e., 0, 1, 5, 10, 50, 100, 500, 1,000, 5,000, and 10,000 min/m³) observed at $r = 10$ m. Note that the curves of head for $D = 0, 1, 5, 10$, and 50 overlap.

their Figure 3, the waveforms of the data in wells B1–6 and B4–5 form a sawtooth pattern as presented herein in Figure 5. Specifically, the head drops when the flow direction changes at the well screen, indicating the possible presence of rate-dependent skin in the field experiment.

3.1.3. Sensitivity Analysis

The sensitivity analysis of the DIM model examines the impact of five model parameters: T , S , ω , S_f , and D . Figure 6 displays the time histories of DIM for each parameter, along with close-up views. Figure 6a highlights that ω has the most significant sensitivity throughout the period, followed by T , S , S_f , and D . The curves of T and S behave similarly due to their high correlation. In contrast, S_f and D appear to have less sensitivity than other parameters, as their effects are mainly observed near the wellbore.

To provide a clearer view of the response pattern, Figure 6b illustrates the shock amplitudes for T , S , and ω , which increase with time, while the amplitude of S_f decreases. The oscillatory phenomenon is primarily due to the rate-dependent skin; as time passes, D starts to impact the DIM curves of T , S , ω , and S_f . The decreasing amplitude of the DIM for S_f can be attributed to the primary influence of the borehole skin on the aquifer system at early times. The pattern of D indicates that the rate-dependent skin has a consistent influence on the model output with a regular amplitude.

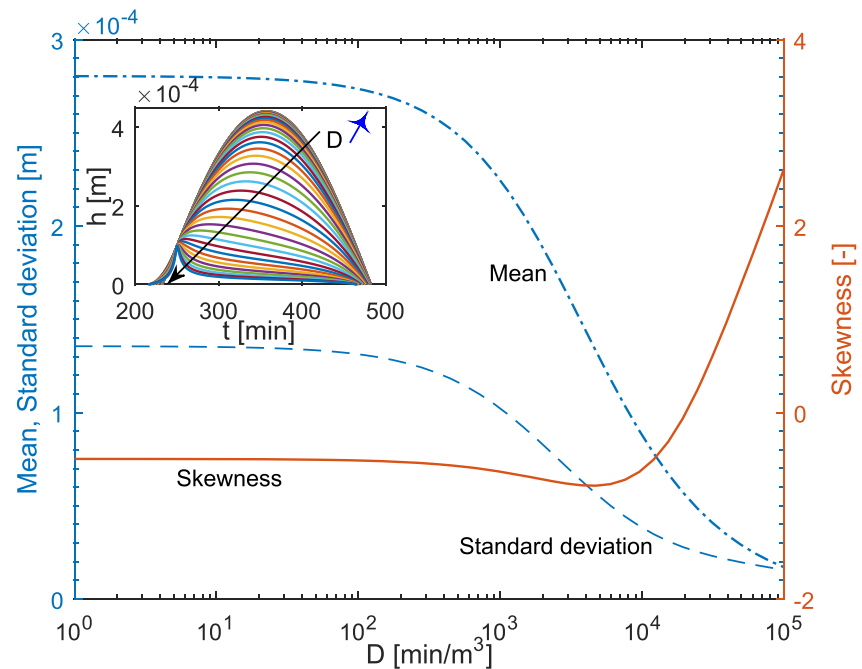


Figure 4. Statistical measures for the positive half cycle of head oscillations shown on the margin of the figure. Higher D results in higher skewness and lower mean and standard deviation of the half cycle of head.

Similar to studies using tidal signals in groundwater to estimate hydraulic properties, the frequency of the OPT is critical in the sensitivity analysis. For instance, if a low-frequency spring-neap tide signal and a semi-diurnal signal have the same amplitude, the former will penetrate further into an aquifer because the exponential damping factor depends on the tidal period (Jacob, 1950). Likewise, low-frequency OPT tests can penetrate further into aquifers than high-frequency ones, assuming both have the same amplitude.

3.2. Analysis of Field Data

Table 1 summarizes the results for the estimated parameters from the three distinct solutions utilized for analysis of the previously collected and described field data (Rabinovich et al., 2015). The application of inverse forms of these solutions to infer aquifer properties from the field data were evaluated based on three versions of Equation 11: the original (Solution 1), a variant with $D = 0$ (Solution 2), and another with both $D = 0$ and $S_f = 0$ (Solution 3). As Table 1 indicates, Solution 1 emerged as the superior model, yielding lower SEE and ME values than the other solutions. Although the SEE value yielded by Solution 2 is close to that from Solution 1, the ME value of Solution 1 is only approximately half of that for Solution 2. This implies that Solution 1, while maintaining a similar level of predictive accuracy (i.e., SEE), has a lower systematic bias (i.e., ME). A lower systematic bias generally implies that the model predictions are more reliable. In addition, the markedly different hydraulic and storage parameter values inferred from the different solutions (Table 1) underscore the importance of accounting for wellbore head loss in this particular field case. The four orders of magnitude difference in T between Solution 1 and Solution 2 is due to the omission of the effect of D , which results in a lower estimated T to more closely match the field data. This implies that neglecting inertial forces could result in a lower T if the solution only considers S_f . Conversely, if the effects of both S_f and D are ignored, the estimated solution may be biased toward a higher T and lower S .

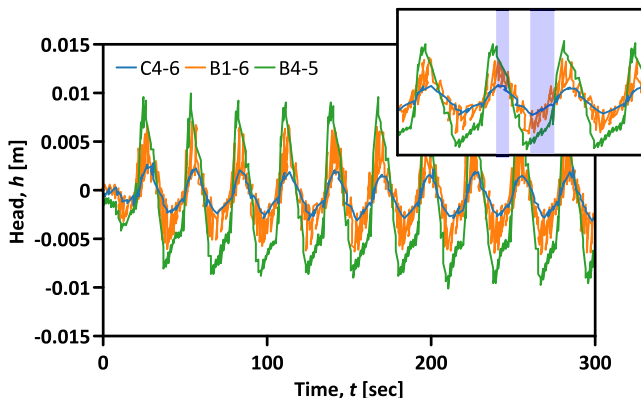


Figure 5. Example raw data (digitized from original study) recorded at different depths from the OPT conducted by Cardiff et al. (2020). The close-up view shows the similar pattern results from the rate-dependent skin (blue shaded area).

The water levels predicted by the three solutions and the measured data are illustrated in Figure 7. Results show that the solution accounting for both

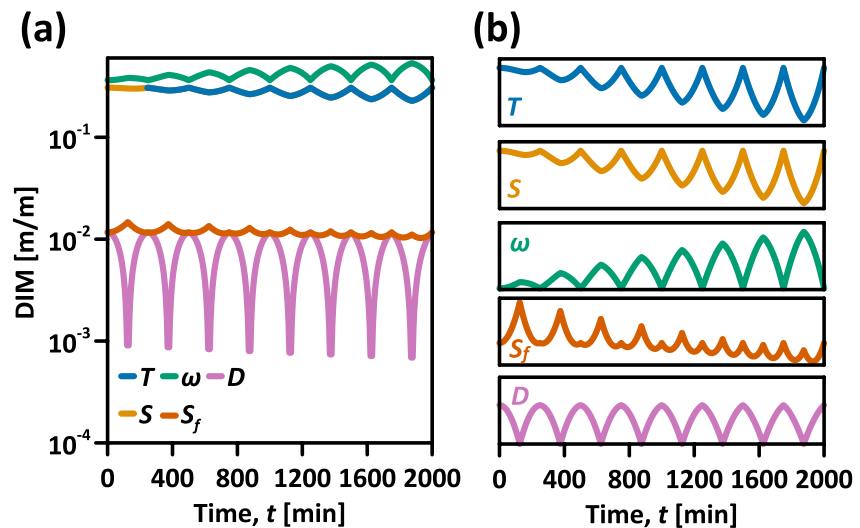


Figure 6. (a) Results of the DIM sensitivity for five parameters and (b) a DIM curve close-up view.

wellbore skin and rate-dependent skin is not only flexible in parameter estimation but also capable of capturing the unique sawtooth pattern (very high and sharp peaks) observed in the field-measured data, while the other two solutions underestimate the peak values. This explains the higher errors for Solutions 1 and 2 (see ME, Table 1). In addition to generating the primary peaks, the new solution generates additional peaks as a consequence of the inertial forces caused by the change in pumping direction. Although subtle, additional positive and negative peaks are also evident near the main peaks in the field data.

The present solution is based on assumptions of aquifer homogeneity. Therefore, the estimated parameters reflect the mean value within a radius of influence of the OPT, as detailed in Zech et al. (2016). Considering the behavior of log scale fields for both T and S under varying distributions, it is possible that inferred parameters may be overestimated or underestimated due to extreme values when using a generalized extreme value distribution with a long tail. Alternatively, a log-normal distribution might result in estimates for T and S that are closer to the mean of the study field. Furthermore, the sensitivity analysis indicates that the sensitivity curves for T and S exceed those of the skin factors, implying that aquifer heterogeneity might play a more substantial role in the estimates of T and S than in the skin factor estimates. In future work, numerical models could potentially be integrated to explore the combined impacts of aquifer heterogeneity and skin factors.

4. Conclusions

This study introduces a novel analytical solution for interpreting data from OPTs that considers the impacts of both wellbore skin and rate-dependent skin. The study demonstrates that rate-dependent skin has a notable effect on hydraulic head oscillations that occur during an OPT, particularly when flow changes direction at the well screen. Our qualitative and quantitative analysis of field data and modeled heads from analytical solutions shows that rate-dependent skin generates a sawtooth pattern in the resultant head oscillations. This effect is especially pronounced when the rate-dependent factor has a value of $D \geq 1 \times 10^4 \text{ min/m}^3$ for the parameter set used herein (defined in the beginning of Section 3). The oscillation frequency of the hydraulic head is the most sensitive parameter, followed by transmissivity, storage coefficient, skin factor, and rate-dependent skin.

Table 1
Estimated Parameters for Field Data

	T (m^2/min)	S	S_f	D (min/m^3)	SEE (m)	ME (m)
Solution 1	0.184	0.0208	2.123	262.57	5.51×10^{-3}	1.22×10^{-3}
Solution 2	3.75×10^{-5}	0.32	1.08	—	5.52×10^{-3}	2.19×10^{-3}
Solution 3	243.6	3.79×10^{-300}	—	—	9.67×10^{-3}	3.95×10^{-3}

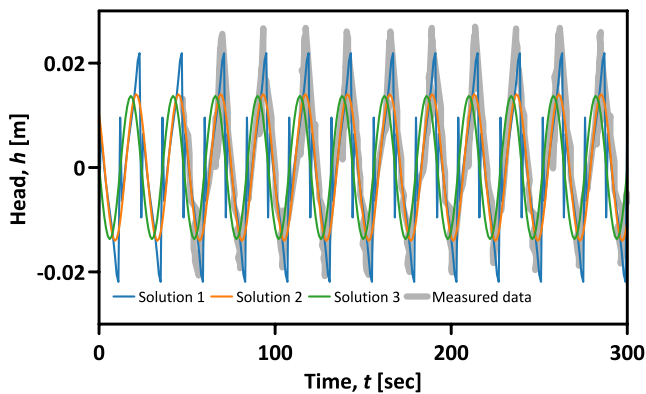


Figure 7. Field data and head predicted by the three solutions.

To assess out model and investigate the role of skin effects, we performed parameter estimation for the proposed new solution, a solution neglecting the rate-dependent skin, and a solution excluding both the rate-dependent skin effect and the wellbore skin. Our analysis demonstrates that neglecting the rate-dependent skin may result in highly inaccurate parameter estimation from OPT data. For example, the solution including rate-dependent skin is capable of capturing sharp peaks displayed in the field data, whereas previously existing solutions neglecting rate-dependent skin fail to do so and thus must compensate by adjusting other parameters when fitting the solution to field data. Moreover, if a model considers only wellbore skin and ignores rate-dependent skin, the inferred T will tend to be underestimated while the inferred S will be overestimated. Conversely, if the model neglects the effects of both skin types, the result will be an overestimated T and an anomalously low S . In conclusion, the new solution is a valuable tool for estimating aquifer parameters from OPT data.

Notation

D	Rate-dependent factor, [$L^{-3} T$]
e_j	j th prediction error
H	Hydraulic head in wellbore, [L]
h	Hydraulic head in aquifer, [L]
(K_0, K_1)	Modified Bessel functions of the second kind of zero-order and first-order
M	Total number of observations
P_k	k th input parameter
Q	Pumping rate, [$L^3 T^{-1}$]
Q_{peak}	Maximum pumping rate achieved during a pumping cycle, [$L^3 T^{-1}$]
r	Radial distance from the center of the pumping well, [L]
r_w	Radius of the pumping well, [L]
S	Storage coefficient, [–]
S_f	Wellbore skin factor, [–]
T	Transmissivity, [$L^2 T^{-1}$]
t	Time, [T]
v	Number of uncertain parameters
X_k	Normalized sensitivity analysis with k th input parameter
Φ	Wave phasor for the aquifer head, [L]
Φ^*	Wave phasor for the wellbore head, [L]
ω	Angular pumping frequency, [T^{-1}]

Acronyms

DIM	Differential importance measure
ME	Mean error
SEE	Standard error of estimation
OB	Objective function
OPT	Oscillatory pumping test

Data Availability Statement

Data used in this manuscript were from Cardiff et al. (2020) and Rabinovich et al. (2015). No new data were used in this manuscript.

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