Groundwater hydraulics

2

FLUID MECHANICS – aimed at solving of technical tasks of balance and motion of fluids and mutual effect of fluid and solids

In civil (environmental) engineering – fluid is „**WATER**“

HYDRAULICS AND GROUNDWATER HYDRAULICS solves:

- under what external conditions
- with what losses
- under which discharge
- under what level and pressure
- in what form
- with what force effect

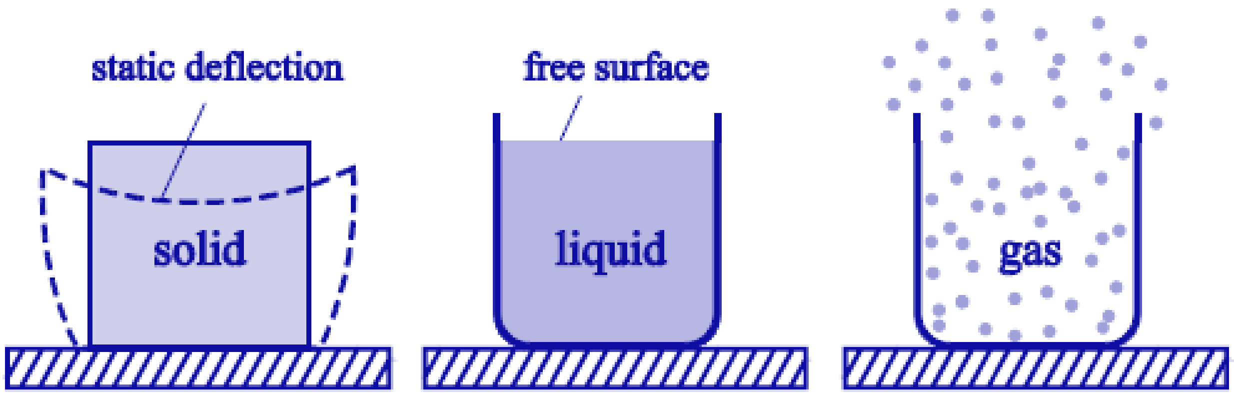
water moves through pipes, river channels, hydraulic structures or earth environment (**porous media**)

MECHANICS: The oldest physical science that deals with both stationary and moving bodies under the influence of forces.

FLUID MECHANICS: The science that deals with the behavior of fluids at rest (*fluid statics*) or in motion (*fluid dynamics*), and the interaction of fluids with solids or other fluids at the boundaries.

STATICS (HYDROSTATICS): The branch of mechanics that deals with water at rest.

DYNAMICS (HYDRODYNAMICS): The branch that deals with fluid (water) in motion.



Gases expand to fill the available volume

- Liquids:
  - water, oil, mercury, gasoline, alcohol
- Gasses:
  - air, helium, hydrogen, steam

Comparison Solids, Liquids and Gases

- **FLUID**- is a subset of the **phases of matter** and includes **liquids, gases, plasmas** and, to some extent, **plastic solids.**

Fluids → Liquids, Gases

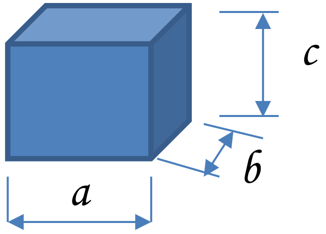
fluid is any substance that flows and takes the shape of its container.

LIQUID

-continuosly fills the open tank, doesn´t change spontaneously its volume

- with changes of presssure and tempetature – change of volume is very small

- forms free water level

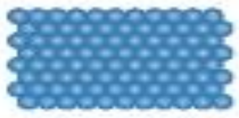


a=b=c= 1 mm

Contains $3x10^{10}$ molecules - air

Contains $3x10^{16}$ molecules - water

DISTINCTION BETWEEN SOLID, LIQUID AND GAS



solid

liquid

gas

Solid

- *Hold their shape; no need for container.*
- *Attractive forces* between the molecules are large enough to retain its shape.
- Effect of shear stress:

  produces deformation
- *Difficult to compress*
- *Molecular spacing: small-molecules are close together*
- *Typical density (Fe)*

  7800 kg/m³

Liquid

- *Take the shape of the container and will stay in open container.*
- *Attractive forces* between the molecules are smaller
- *Flow easily even though there are strong intermolecular forces between molecules.*
- Effect of shear stress .. flow
- *Difficult to compress*
- *Molecular spacing: small-molecules are held close together by intermolecular forces*
- Typical density(Water):

  1000 kg/m³

Gas

- *Expand to fill a closed container*
- *Move around freely with little interaction exept during collisions*
- *Attractive forces* between the molecules are very small
- Effect of shear stress ... flow
- Easy to compresss
- *Molecular spacing: large-molecules are far appart*
- Typical density (Air):

  1.2 kg/m³

- A **DIMENSION** is the measure by which a physical variable is expressed qualitatively

		International	SI-units
➢ Basic dimensions: (or primary quantities)	Length	L	m
	Time	T	s
	Mass	M	kg

➢ We can derive any **SECONDARY QUANTITY** from the primary quantities

i.e. Force = (mass) x (acceleration) : $F = M\,L\,T^{-2}$

➢ $F = kg\ m\ s^{-2}$

- A unit is a particular way of attaching a number to the qualitative dimension:

MULTIPLES OF UNITS

Name	Symbol	Factor	Number
giga	G	10^9	1 000 000 000
mega	M	10^6	1 000 000
kilo	K	10^3	1 000
milli	m	10^{-3}	0. 001
micro	µ	10^{-6}	0. 000 001

DERIVED UNITS WITH SPECIAL NAMES

Quantity	Unit	Symbol	Derivation
Force [F]	Newton	**N**	kg m s^{-2}
Work, Energy [E]	Joule	J	N m
Power [P]	Watt	W	J s^{-1}
Pressure [p]	Pascal	**Pa**	N m^{-2}

DIMENSIONS AND UNITS		
Quantity	**Symbol**	**Dimensions**
Velocity	v	LT^{-1}
Acceleration	a	LT^{-2}
Area	A	L^2
Volume	V	L^3
Discharge	Q	L^3T^{-1}
Force	F, G	$M\ L\ T^{-2}$
Pressure	p	$ML^{-1}T^{-2}$
Gravity acceleration	g	LT^{-2}
Temperature	T	Θ
Mass concentration	C	ML^{-3}

FORCES IN LIQUID

INTERNAL FORCES – molecular

electromagnetic phenomena, thermal motion of molecules

they are not taken into account (exception – surface tension and capilarity)

EXTERNAL FORCES – consequence of force field

A. Body (mass, volume) forces - inertia force, gravity force

From Newton ´s law:

$$\mathbf{F = m \,.\, A}$$

m- mass

a – acceleration

B. Surface forces– pressure force, tension force

$$\mathbf{F_\sigma = \sigma\, A}$$

σ -tension

A - area

FLUID PROPERTIES

- **DENSITY,** ρ (kg/m^3) (H_2O approx.1000) The density of a fluid is defined as mass per unit volume
  - SALINITY
  - TEMPERATURE

$$\rho = \frac{dm}{dV} \text{ .... } \rho = \frac{m}{V}$$

- **SPECIFIC WEIGHT**, γ (N/m^3)
- $\gamma = \rho \cdot g$ (H_2O approx. 9810)

The specific weight of fluid is its weight per unit volume.

Water

Temperature (°C)	Density (kg/m^3)
0	999.87
+4	1000
+10	999.73
+20	998.23
+100	958.4

- **SPECIFIC VOLUME:** $\upsilon = \frac{1}{\rho}$

Volume occupied by unit mass of fluid.

Specific volume is the **reciprocal of density.**

Densities of Some Common Substances

Material	Density (kg/m^3)*	Material	Density (kg/m^3)*
Air (1 atm, 20°C)	1.20	Iron, steel	7.8×10^3
Ethanol	0.81×10^3	Brass	8.6×10^3
Benzene	0.90×10^3	Copper	8.9×10^3
Ice	0.92×10^3	Silver	10.5×10^3
Water	1.00×10^3	Lead	11.3×10^3
Seawater	1.03×10^3	Mercury	13.6×10^3
Blood	1.06×10^3	Gold	19.3×10^3
Glycerine	1.26×10^3	Platinum	21.4×10^3
Concrete	2×10^3	White dwarf star	10^{10}
Aluminum	2.7×10^3	Neutron star	10^{18}

- **VOLUME COMPRESSIBILITY** (p)
- It is defined as:

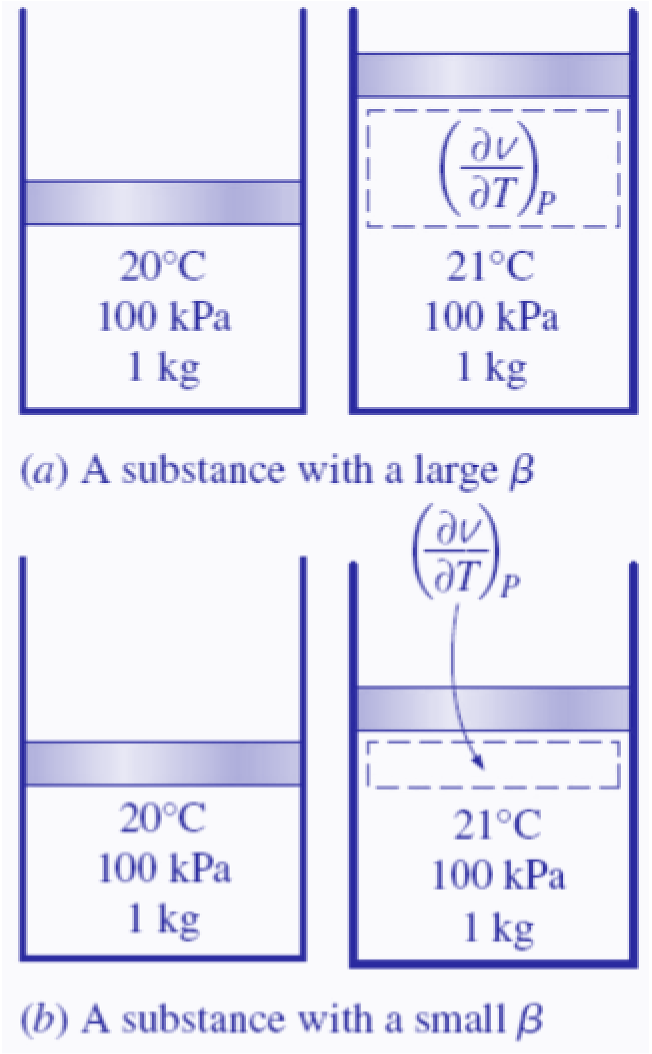
  Change in volume due to change in pressure."

$$\frac{\Delta V}{V_0} = -\beta_p \cdot \Delta p \qquad \beta_p = \frac{\Delta V}{V_0 \cdot \Delta p} \qquad [Pa^{-1}]$$

- **VOLUME EXPANSIVITY** (T)

  Change in volume due to change in temperature."

$$V = V_0\left(1 + \beta \Delta T\right) \qquad \beta = \frac{\Delta V}{V_0 \cdot \Delta T} \qquad [K^{-1}]$$



(a) A substance with a large β

(b) A substance with a small β

- **SURFACE TENSION - CAPILLARITY (GROUNDWATER)**

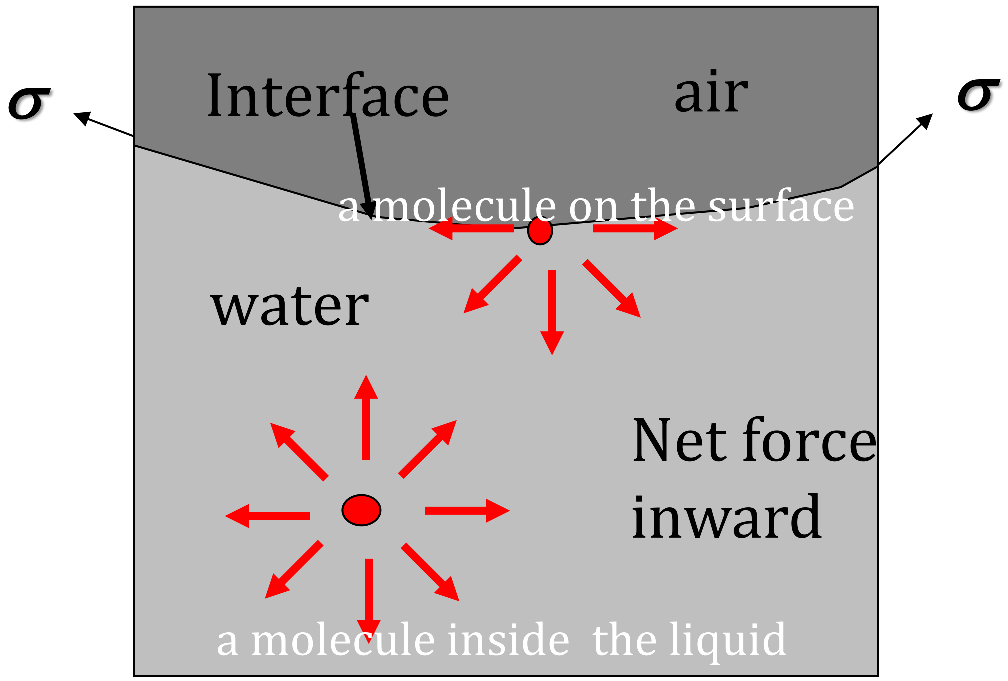
Below surface, forces act equally in all directions

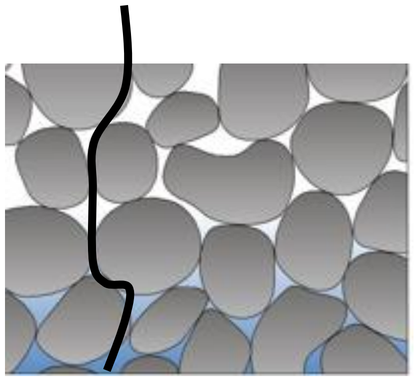
At surface, some forces are missing, pulls molecules down and together, like membrane exerting *tension* on the *surface*

If interface is curved, higher pressure will exist on concave side

Pressure increase is balanced by **surface tension,** σ

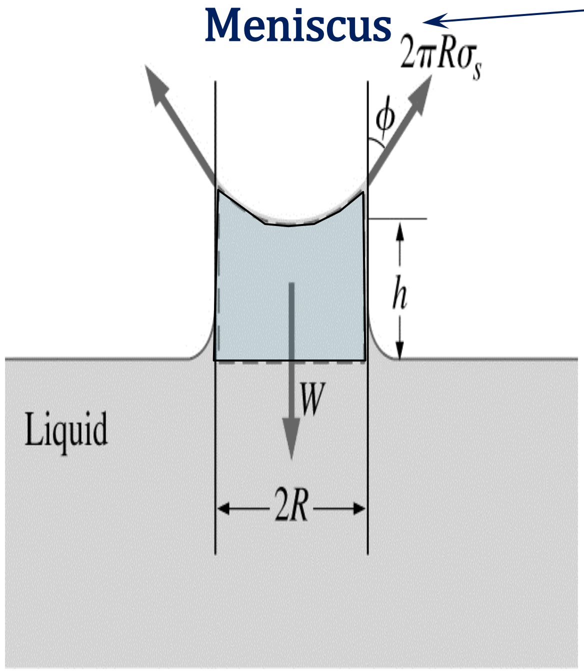
$\sigma = 0.073$ N/m (@ 20°C)





POROUS MEDIUM

- **CAPILLARY EFFECT** is the rise or fall of a liquid in a small-diameter tube.
- The curved free surface in the tube is call the **meniscus**.
- **Water** meniscus curves up because water is a ***wetting fluid.***



Equilibrium of surface tension force and gravitional pull on the water cylinder of height produces:

$$2\,\pi\,R\,\sigma\cos\phi = \pi\,R^2 h\,\gamma \quad \Rightarrow \quad h = \frac{2\,\sigma\cos\phi}{\gamma\,R}$$

σ	surface tension
ϕ	angle - liguid x solid
γ	specific weight of liquid
R	radius of tube

- **Viscosity**

Newton's equation of viscosity

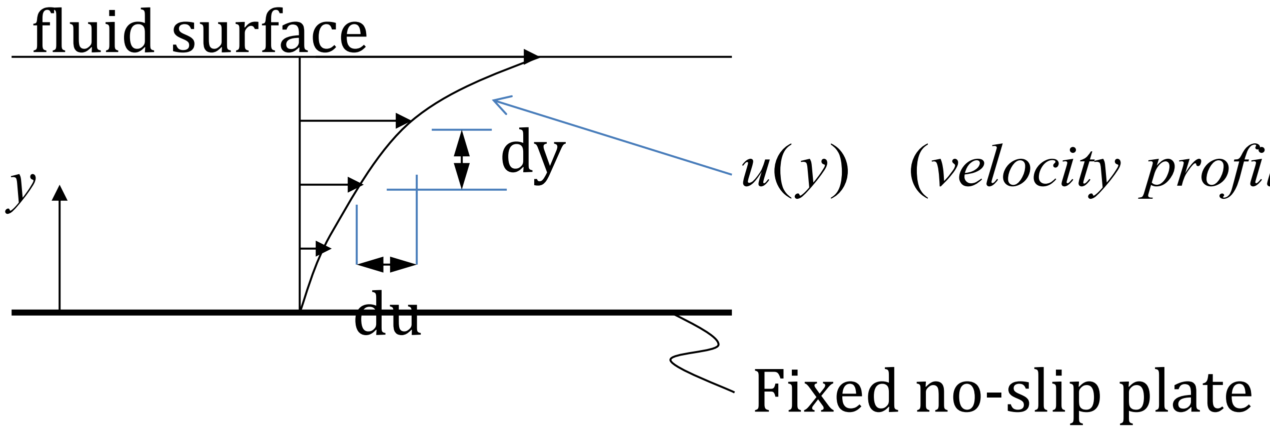
Viscosity is a measure of the resistance of a fluid to deform under **shear stress.**

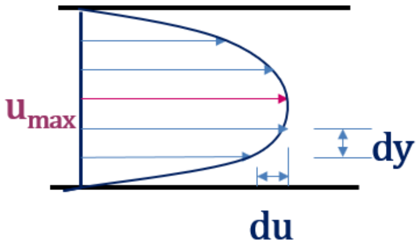
shear stress due to viscosity between layers:

$$\tau = \mu \frac{du}{dy}$$

μ - dynamic viscosity (coeff. of viscosity)

$$\nu = \frac{\mu}{\rho}$$ **- kinematic viscosity**





Use definition of **shear force:**

$$F = \tau A = \mu A \frac{du}{dy}$$

STANDARDS IN HYDRAULICS

Acceleration of gravity $g = 9.81\ m\ s^{-1}$

Atmospheric pressure (p_{at}) – $1.013 . 10^{5}$ Pa

Properties of water (T = 15 °C (39 °F) and p = 1 atm)

Density of water $\rho = 1000\ kg\ m^{-3}$

Density of air at 4° C : 1.20 kg/m^3

Specific weight $\gamma = 9810\ N\ m^{-3}$

Surface tension $\sigma = 0.073\ N\ m^{-1}$

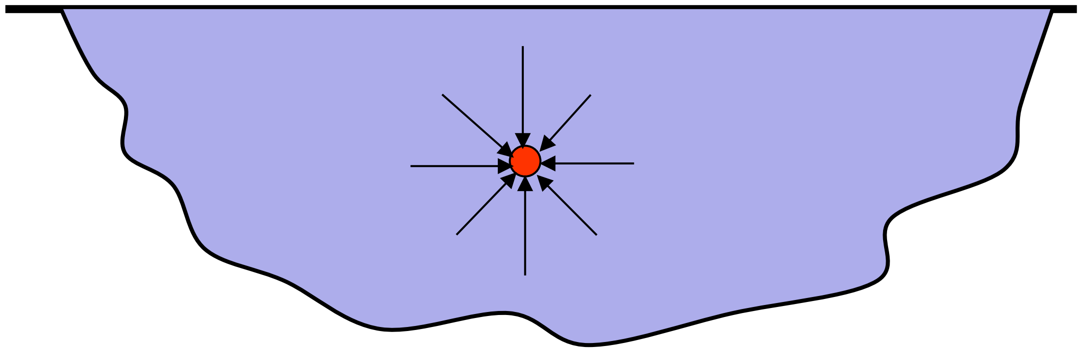
Viscosity $\mu = 1.14 . 10^{-3}\ N\ s\ m^{-2}$ (Pa.s)

Kinematic viscosity $\nu = 1.14 . 10^{-6}\ m^{2}\ s^{-1}$

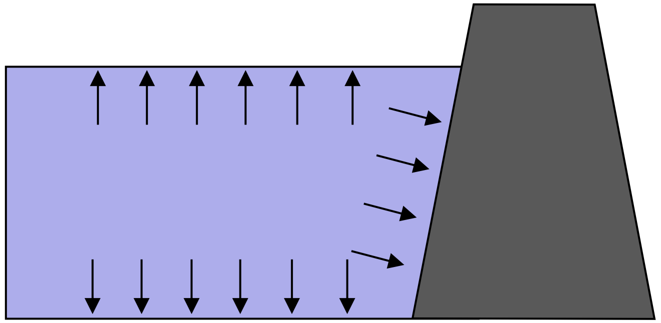
DEFINITION OF PRESSURE

Pressure is defined as the amount of force exerted on a unit area of a substance:

$$P = F / A$$



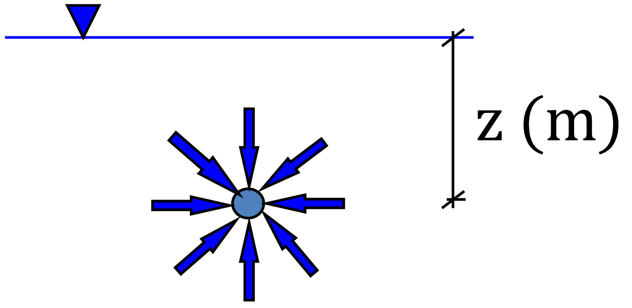
Pressure is a *Normal Force*
(It acts perpendicular to
the surface)
It is also called a *Surface Force*



Dam

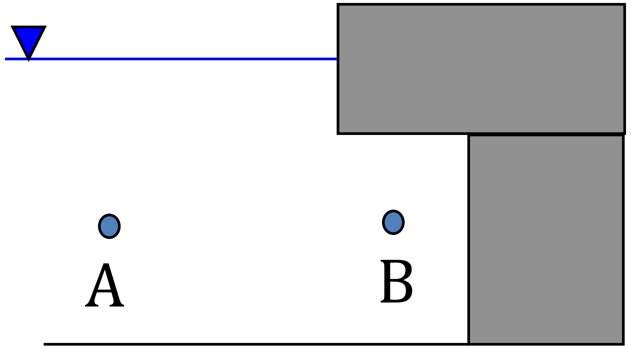
PROPERTIES OF PRESSURE

1.



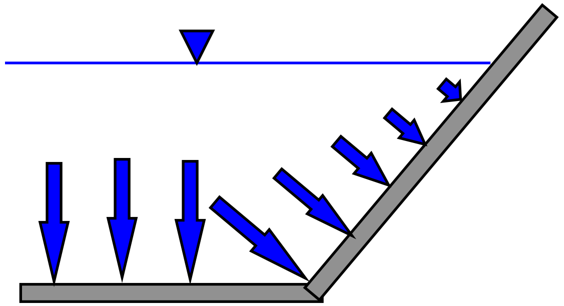
Pressure at any point in a fluid is the same in all directions $p=\rho gz$

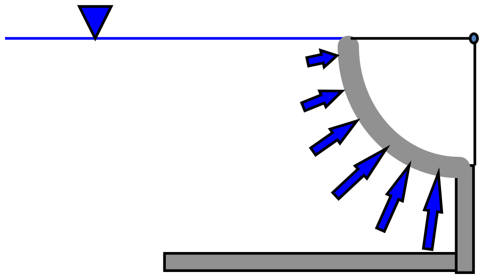
2.



Pressure the same at A and B.

3.

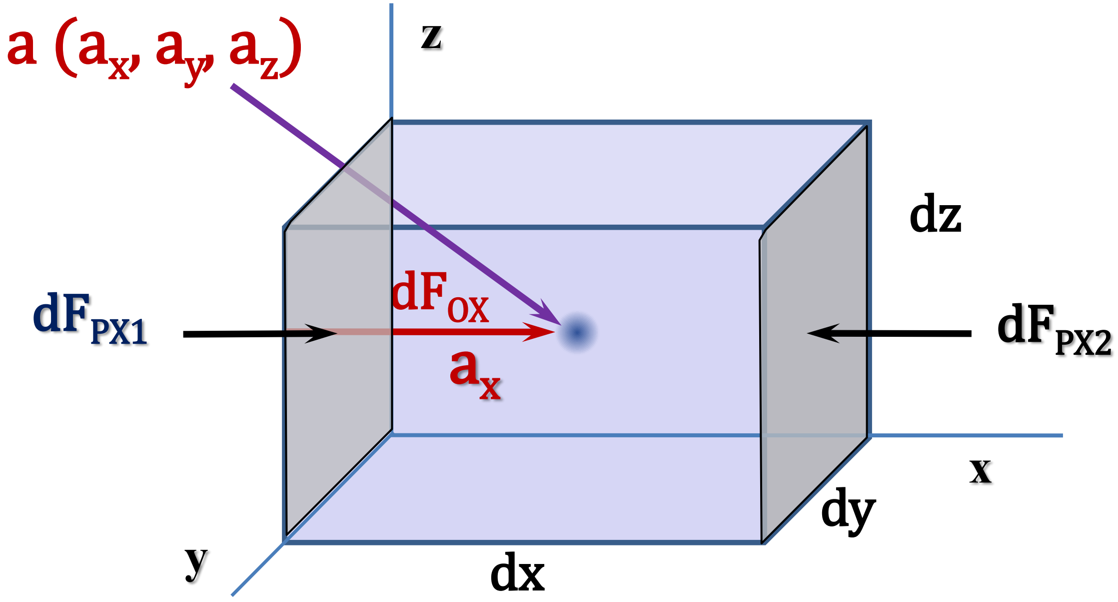




Pressure is always perpendicular to a surface.

HYDROSTATIC DIFFERENTIAL EQUATION (EULER´S EQ.)

$$\sum F_{VOLUME} + \sum F_{PESSURE} = 0$$



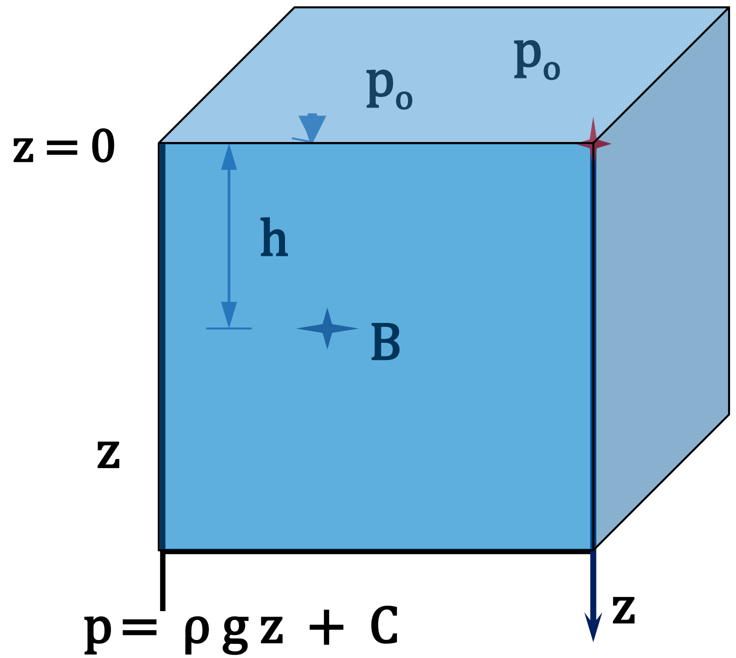
For x:

$$F_{OX} + F_{PX} = 0$$

$$F_{OX} + F_{PX1} - F_{PX2} = 0$$

$$\rho\, a_x = \frac{\partial p}{\partial x} \quad \rightarrow \quad a_x - \frac{1}{\rho}\frac{\partial p}{\partial x} = 0$$

THE FIRST TASK OF HYDROSTATICS : - DETERMINATION OF PRESSURE



$p = \rho g z + C$

Mass forces $\rho . a_x = \frac{\partial p}{\partial x}$

- For **Gravity force ....** $a_z = g$

$(a_x = a_y = 0)$

$dp = \rho g dz$

For $\rho = const.$ and $g = const.$

$$\int dp = \int \rho g \, dz$$

C - (integral constant) from condition at the free water level

Pressure at the point B $\quad p_B = p_0 + \rho g h \quad$ **$p = p_0 + \rho g h$** !!!

It is the pressure expressed in terms of height of fluid.

PRESSURE HEAD

$$\frac{p}{\rho g}$$

The term elevation (head) means the vertical distance from some reference level to a point of interest.

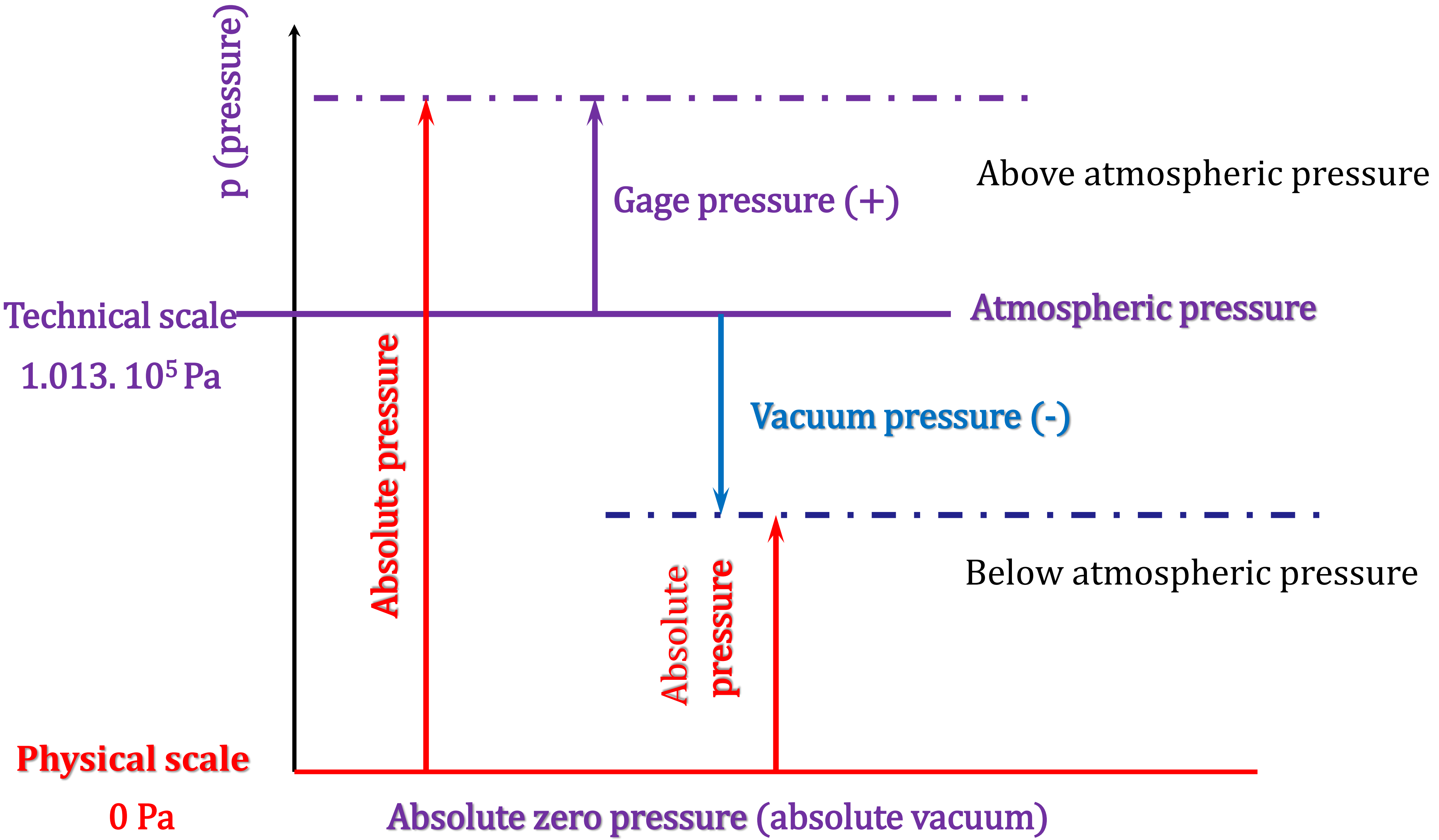
PIESOMETRIC HEAD

$$h = \frac{p}{\rho g} + h(z) = \frac{p}{\gamma} + h(z) \qquad \frac{p_B}{\rho g} = \frac{p_0}{\rho g} + h(z)$$

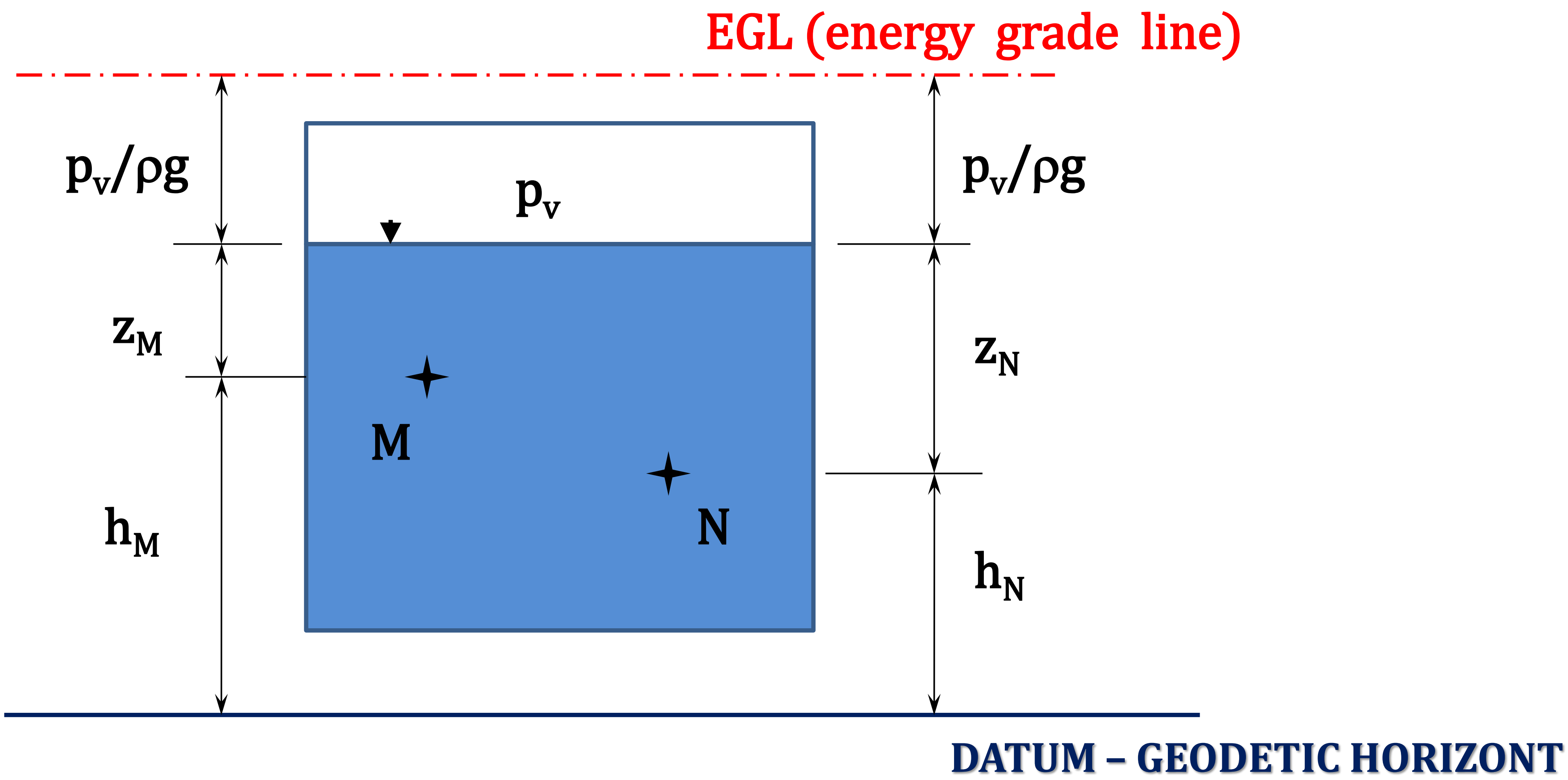
- **ATMOSPHERIC PRESSURE:** It is the force per unit area exerted by the weight of air above that surface in the atmosphere of Earth (or that of another planet). It is also called as barometric pressure.
- **GAGE PRESSURE:** It is the pressure, measured with the help of pressure measuring instrument in which the atmospheric pressure is taken as Datum (reference from which measurements are made).
- **ABSOLUTE PRESSURE:** It is the pressure equal to the sum of atmospheric and gauge pressures. Or
- If we measure pressure relative to absolute zero (perfect Vacuum) we call it **absolute pressure.**
- **VACUUM PRESSURE**: If the pressure is below the atmospheric pressure we call it as vacuum.

$$p_{abs} = p_{atm} + p_{gage}$$

ABSOLUTE AND RELATIVE PRESSURE



PRESSURE TANK WITH FLUIDS



$$h_M + z_M + \frac{p_v}{\rho g} = h_N + z_N + \frac{p_v}{\rho g} = konst.$$

PASCAL'S LAW

Blaise Pascal
(1623-1662)

Pressure at a Point: **Pascal's Law** $F_S \rangle\rangle F_{V(B)}$

F_S – surface force; F_V – volume force

Pressure is the **normal** force per unit area at a given point acting on a given plane within a fluid mass of interest.

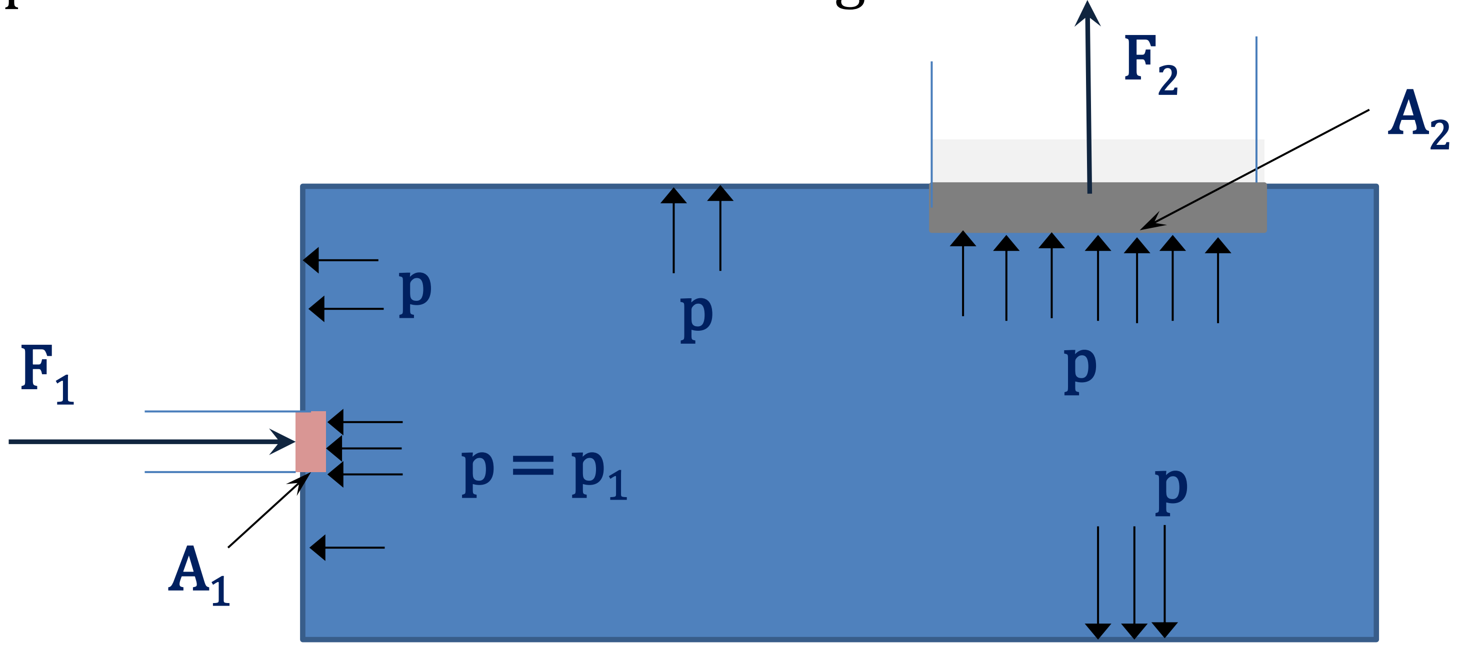
Pressure is independent of direction!

In a closed system, pressures transmitted to a fluid are identical to all parts of the container.

Gradual pressure change dp in small closed volume of liquid is the same in all directions and passes on all points of liquid without any change.

PASCAL' S LAW

Pascal's Law: the pressure at a point in a fluid at rest, or in motion, is independent of the direction as long as there are no shearing stresses present.



$$p_1 = \frac{F_1}{A_1} = p_2 = \frac{F_2}{A_2} = p \quad \Rightarrow \quad F_2 = p\,A_2 = F_1 \frac{A_2}{A_1}$$

$$F_2 = \eta\, F_1 \frac{A_2}{A_1}$$

η - **loss coefficient**

p-pressure; A-area; F-force;

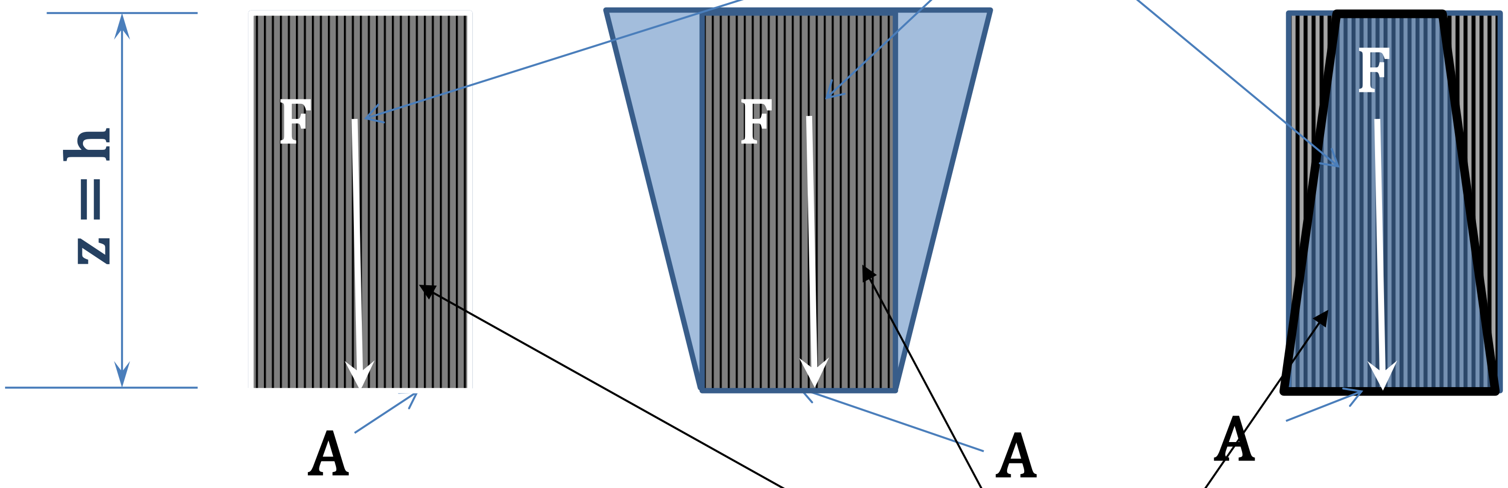
a) horizontal bottom

$$F = p.A = \rho.g.h.A$$

Volume of pressure body

Hydrostatic paradoxon

Pressure prism



$$dF = \rho.g.z.dA \rightarrow \quad F = \int_A dF = \int_A \rho.g.z.dA = \rho.g.z\int_A dA$$

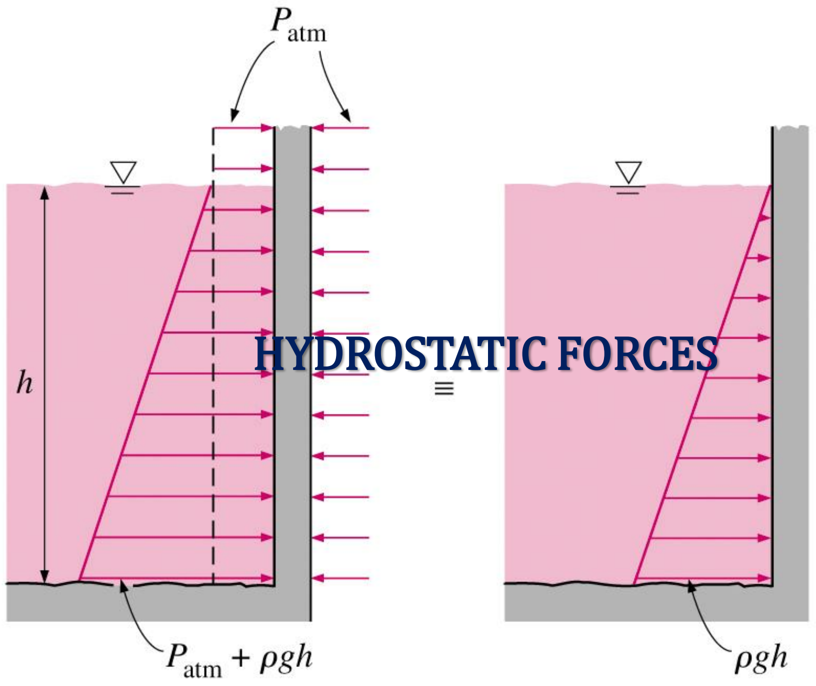
$$\rightarrow \quad F = p.A = \rho.g.h.A$$

V_{PB}=h. A – Volume of pressure body

A_{PD} - area of pressure diagram

Pressure prism is a geometric representation of *hydrostatic forces*

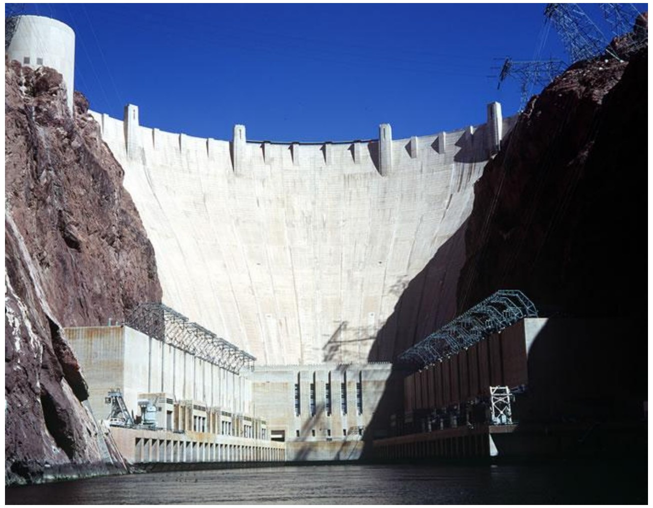
HYDROSTATIC FORCES ON PLANE SURFACES

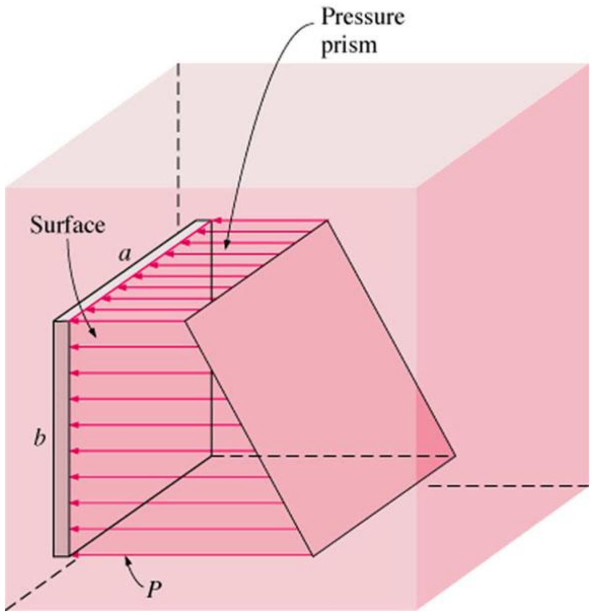


HYDROSTATIC FORCES

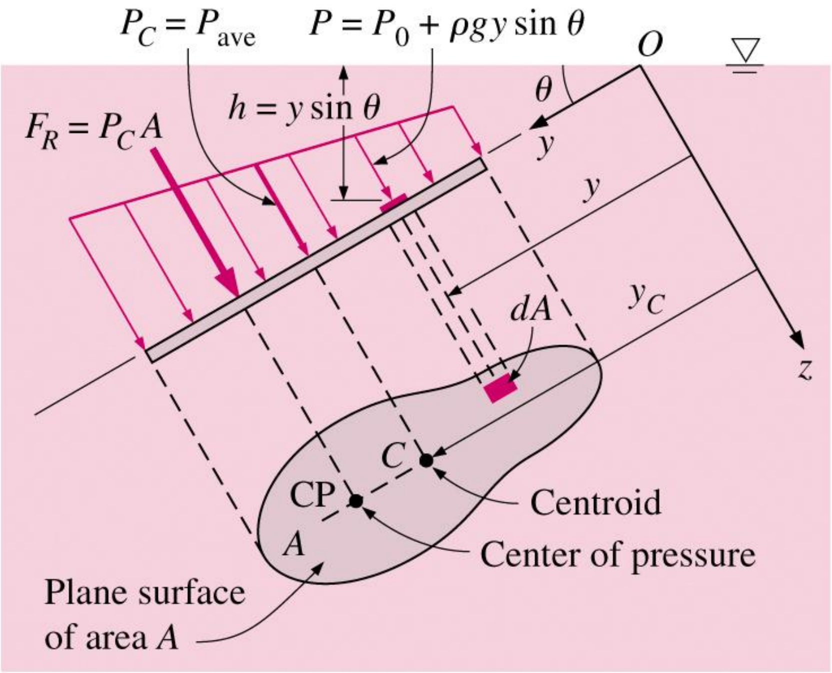
(a) P_{atm} considered   (b) P_{atm} subtracted

- On a *plane* surface, the hydrostatic forces form a system of parallel forces
- Atmospheric pressure P_{atm} can be neglected when it acts on both sides of the surface.

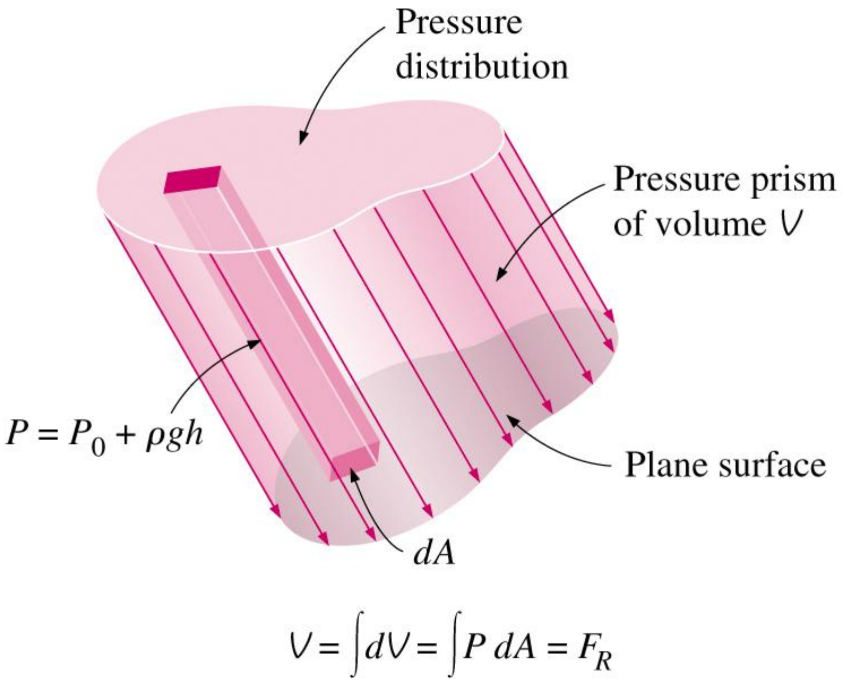




Pressure prism
Surface
a
b
P

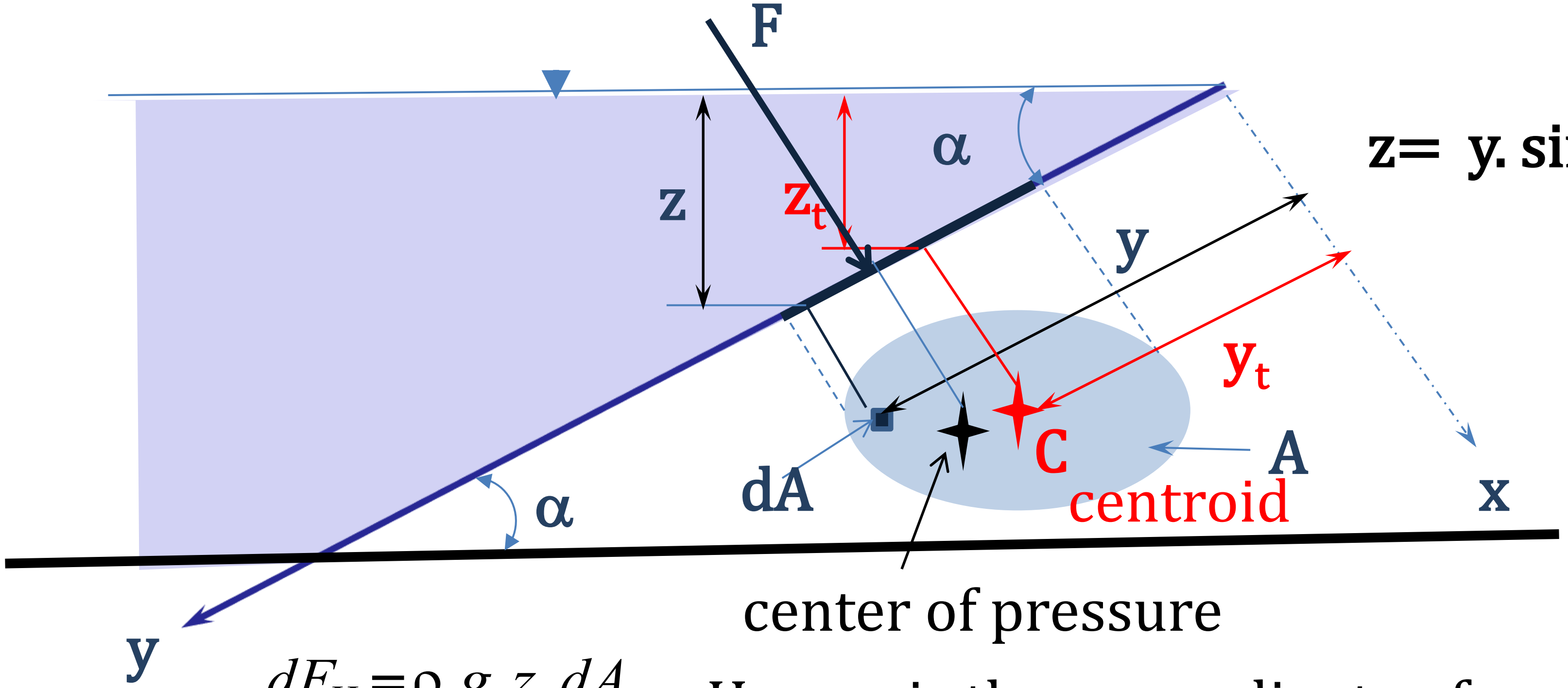


$P_C = P_{ave}$
$P = P_0 + \rho g y \sin\theta$
O
θ
$h = y \sin\theta$
$F_R = P_C A$
y
y
y_C
dA
z
C
CP
Centroid
Center of pressure
A
Plane surface of area A



Pressure distribution
Pressure prism of volume V
$P = P_0 + \rho g h$
Plane surface
dA
$V = \int dV = \int P\, dA = F_R$

METHOD FOR SIMPLE INCLINED PLANE SURFACES



z= y. sin α

$$dF_H = \rho . g . z . dA$$

Here y_t is the *y*-coordinate of centroid of area

$$F_H = \int_A \rho . g . z . dA = \int_A \rho . g . \ y . sin\alpha . dA = \rho . g . sin\alpha \int_A y . dA$$

$\int_A y . dA$ .....Moment of an area A about the x axis

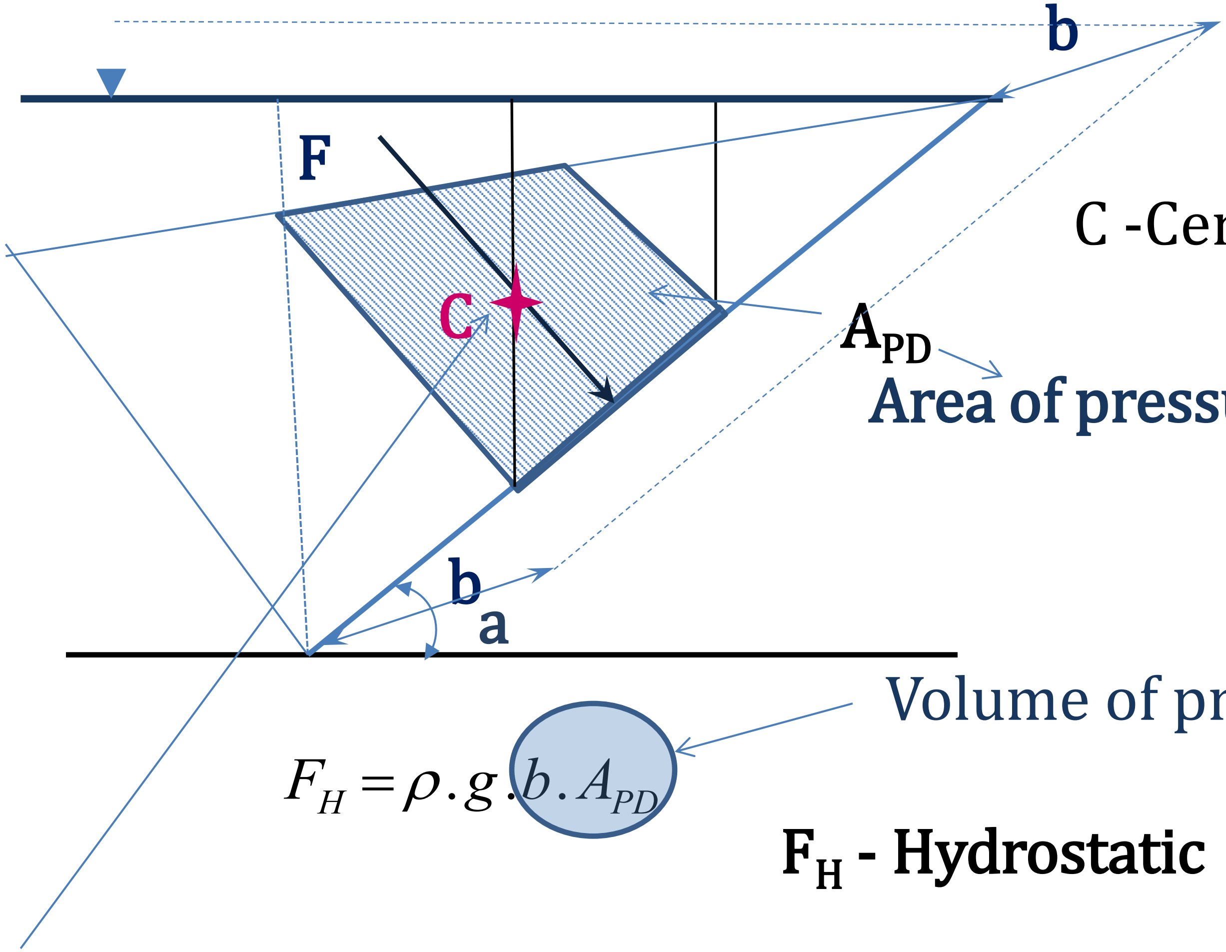
$$M_x = y_T . A$$

$$F_H = \rho . g . sin\alpha . y_T . A = \rho . g . z_T . A$$

Pressure prism ...

HYDROSTATIC FORCES - PRESSURE DIAGRAM METHOD

Direction of force
is normal to the plane



C -Centroid of the area (PD)

A_{PD}

Area of pressure diagram (PD) [m^2]

Volume of pressure body [m^3]

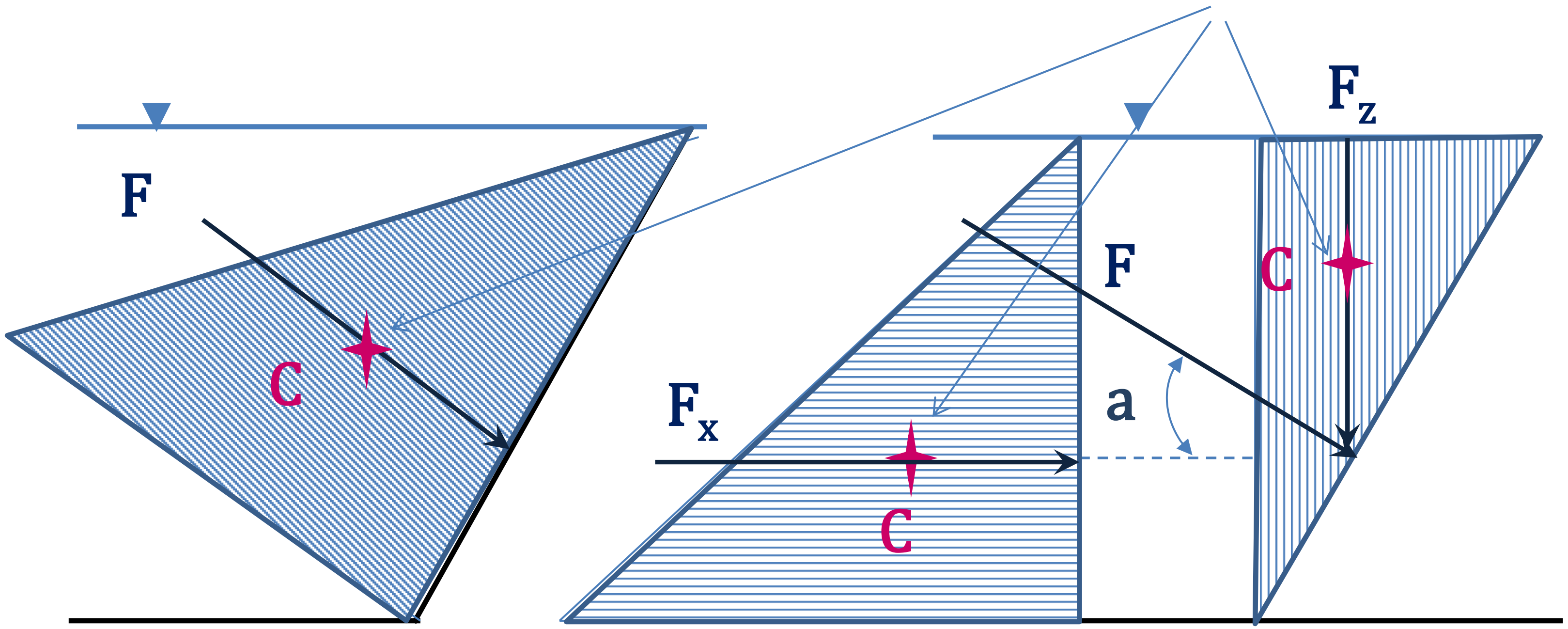
$$F_H = \rho . g . b . A_{PD}$$

F_H - Hydrostatic Force

F passes through the centroid of the area (PD)

APLICATION OF PRESSURE DIAGRAMS

F passes through the centroid of the area (PD).



Pressure diagram - complex

$$F_H = \rho . g . b . A_{PD}$$

Pressure diagram - components

$$tg\,\alpha = \frac{F_z}{F_x}$$

$$F_x = \rho . g . b . A_{PDx}$$

$$F_z = \rho . g . b . A_{PDz}$$

$$F_H = \sqrt{F_x^2 + F_z^2}$$

APLICATION OF PRESSURE DIAGRAMS

Pressure diagram

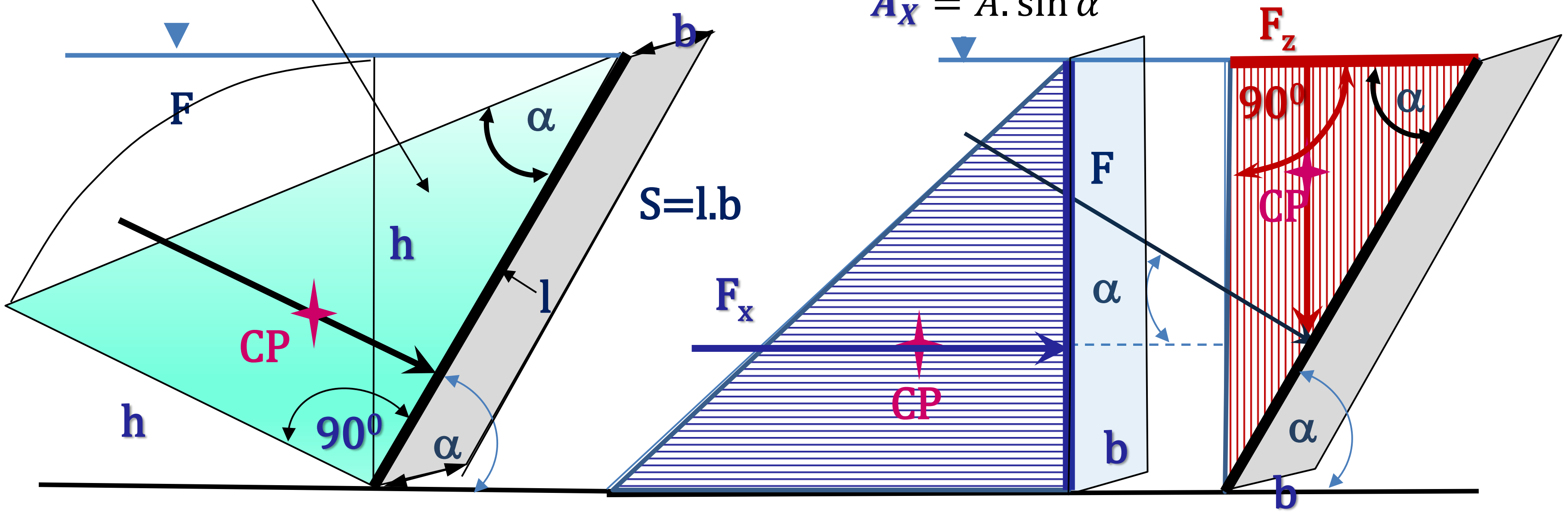
Pressure diagram – components – x,z

$$\cos\alpha = \frac{A_Z}{A}$$

$$A_Z = A.\cos\alpha$$

$$A_X = A.\sin\alpha$$

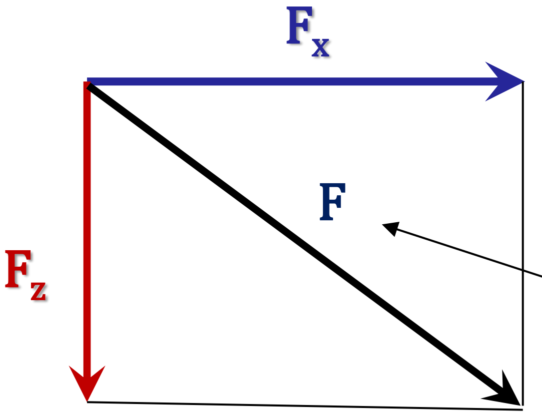
$$\sin\alpha = \frac{A_X}{A}$$



$$F = \rho\, g\, b\, A_{PD}$$

$$F_H = F_X = \rho g b \boldsymbol{A}_{(PD)X} \qquad F_V = F_Z = \rho g b \boldsymbol{A}_{(PD)Z}$$

$S_{(PD)X}; S_{(PD)Z}$- areas of pressure diagrams – x anz direction



Hydrostatic force– Pythagorean Theorem

$$F = \sqrt{F_X^2 + F_Z^2}$$ vector sum