# Chapter 9
# Pressure Buildup Test

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The most frequently used pressure transient test

The test is conducted by

(1) $q = const. \Rightarrow p_{wf} \downarrow$

(2) $q = 0 \Rightarrow p_{ws} \uparrow$

(3) $p_{ws} = f(t)$

$\Downarrow$

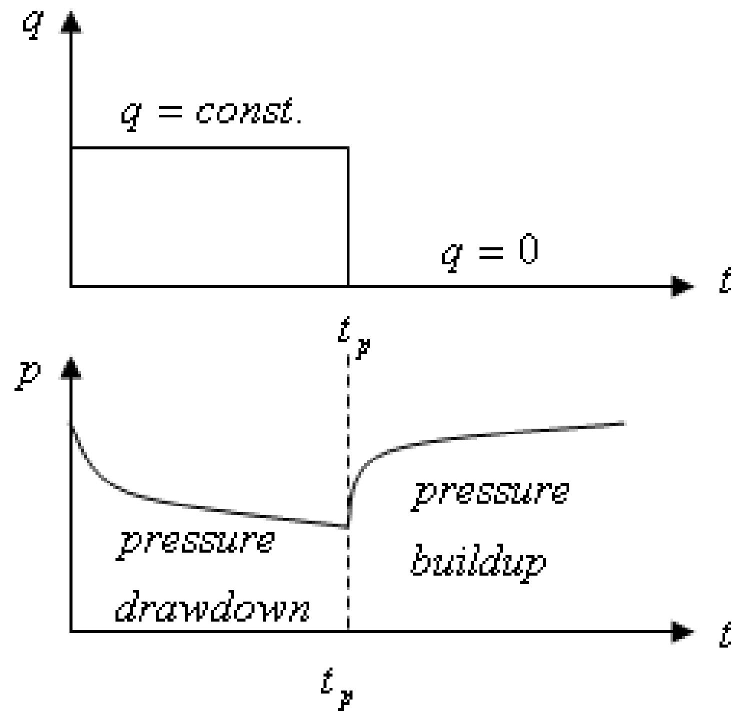
*To estimate*

(1) $k$

(2) $\bar{p}, p^*$

(3) $s$

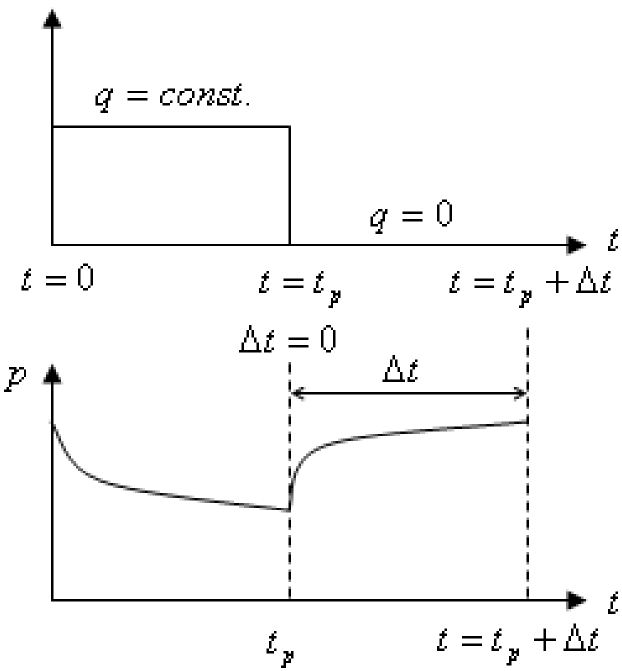
(4) *heterogeneities or boundary*



$$
\text{Pressure analysis}
\begin{cases}
\textit{Analytical solution} \begin{cases} \textit{plotting procedure (This Chapter)} \\ \textit{history match} \end{cases} \\
\textit{Numerical solution} \begin{cases} \textit{type curve (Chapter 4)} \\ \textit{history match} \end{cases}
\end{cases}
$$

# The Ideal Buildup Test (I)
# (Determination of permeability)

- Assumptions:
  - Reservoir is homogeneous, slightly compressible, single phase.
  - Infinite acting
  - Ei or log approximation is applied
  - Horner's pseudo-producing time application is applicable



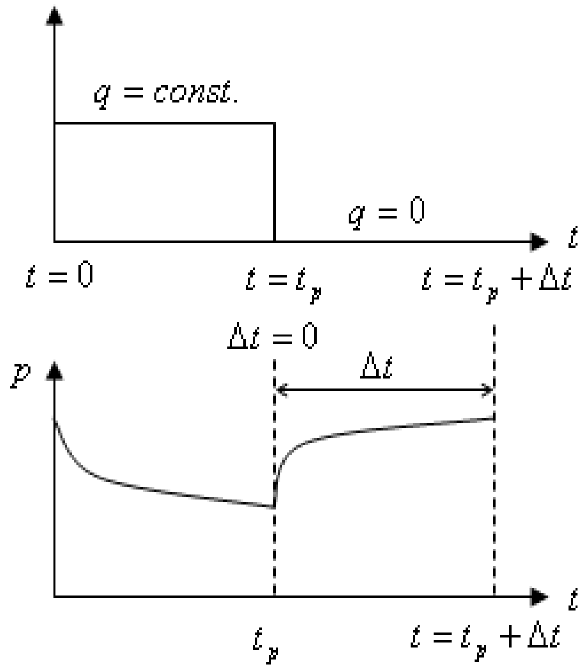
$$At \quad t = t_p + \Delta t$$

$$p_i - p_{ws} = -70.6\frac{q\mu B}{kh}\left\{\ln\left(\frac{1688\mu c\phi r_w^{\ 2}}{k\left(t_p + \Delta t\right)}\right) - 2s\right\}$$

$$-70.6\frac{(-q)\mu B}{kh}\left\{\ln\left(\frac{1688\mu c\phi r_w^{\ 2}}{k\left(\Delta t\right)}\right) - 2s\right\}$$

$$= -70.6\frac{q\mu B}{kh}\left\{\ln \Delta t - \ln\left(t_p + \Delta t\right)\right\}$$

$$= 70.6\frac{q\mu B}{kh}\ln\left(\frac{t_p + \Delta t}{\Delta t}\right)$$

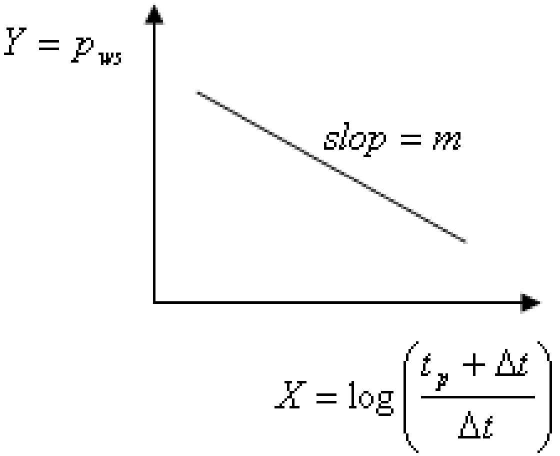



$$\Rightarrow p_{ws} = p_i - 162.6\frac{q\mu B}{kh}\log\left[\frac{t_p + \Delta t}{\Delta t}\right]$$

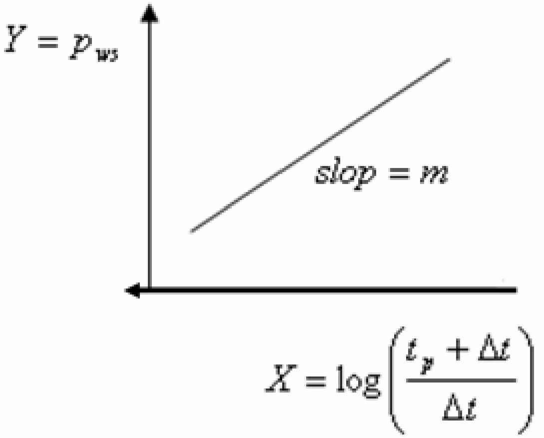
$$p_{ws} = p_i - 162.6\frac{q\mu B}{kh}\log\left[\frac{t_p + \Delta t}{\Delta t}\right]$$

$$let \qquad Y = p_{ws} \quad , \quad X = \log\left(\frac{t_p + \Delta t}{\Delta t}\right) \quad , \quad C = p_i \quad ,$$

$$m = 162.6\frac{q\mu B}{kh}$$




$$\Rightarrow Y = -mX + C$$




$$k = \frac{162.6 q\mu B}{mh}$$

# The Ideal Buildup Test (II) (Determination of skin factor)

$$At \quad t = t_p \quad the \quad ins\tan t \quad a \quad well \quad is \quad shut \quad in, \quad the \quad following \quad BHP, \quad p_{wf}, \quad is$$

$$p_{wf} = p_i + 70.6\frac{q\mu B}{kh}\left[\ln\left(\frac{1688\mu c\phi r_w^{\ 2}}{kt_p}\right) - 2s\right]$$

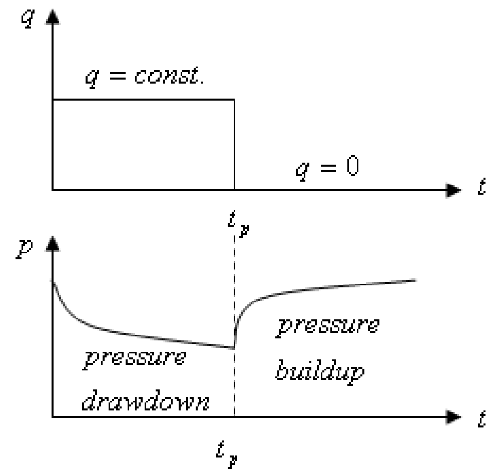
*or*

$$p_{wf} = p_i + 162.6\frac{q\mu B}{kh}\left[\log\left(\frac{1688\mu c\phi r_w^{\ 2}}{kt_p}\right) - 0.869s\right] ------(a)$$

$$where \quad m = 162.6\frac{q\mu B}{kh}$$

$$At \quad t = t_p + \Delta t$$

$$p_{ws} = p_i - m\log\left(\frac{t_p + \Delta t}{\Delta t}\right) ------(b)$$



$$Eq.(b) - Eq.(a)$$

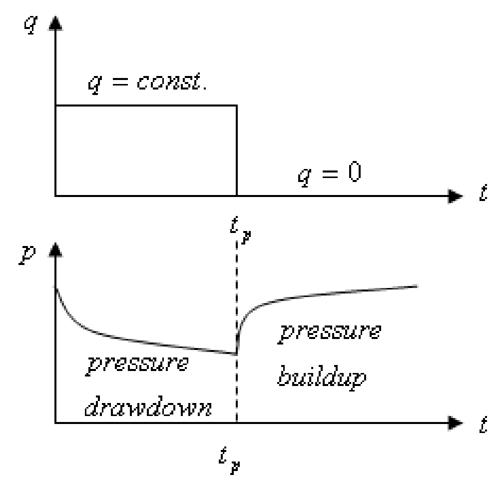
$$p_{ws} - p_{wf} = -m\log\left(\frac{t_p + \Delta t}{\Delta t}\right) - m\log\left(\frac{1688\mu c\phi r_w^{\ 2}}{kt_p}\right) + 0.869ms$$

$$\Rightarrow s = 1.151\left(\frac{p_{ws} - p_{wf}}{m}\right) + 1.151\log\left(\frac{t_p + \Delta t}{t_p}\right) + 1.151\log\left(\frac{1688\mu c\phi r_w^{\ 2}}{k\Delta t}\right)$$

$$For \qquad \Delta t = 1$$

$$\frac{t_p + \Delta t}{t_p} = \frac{t_p + 1}{t_p} = 1 + \frac{1}{t_p} \approx 1 \qquad for \qquad t_p >> 1$$

$$\Rightarrow s = 1.151\left\{\frac{\left(p_{ws@1hr} - p_{wf}\right)}{m} - \log\left(\frac{k}{\mu c\phi r_w^{\ 2}}\right) + 3.23\right.$$



Example 2.1 Analysis of ideal pressurebuildup test

GivenG

Oil reservoir

h = 22 ft

Bo = 1.3 RB/STB

$\phi = 0.2$

$c_t = 20 \times 10^{-6}\, psi^{-1}$

$\mu_o = 1.0 cp$

$r_w = 0.3 ft$

$q_o = 500 \quad STB/D \quad for \quad 3 days$

$(t_p = 3 days = 72 hrs)$

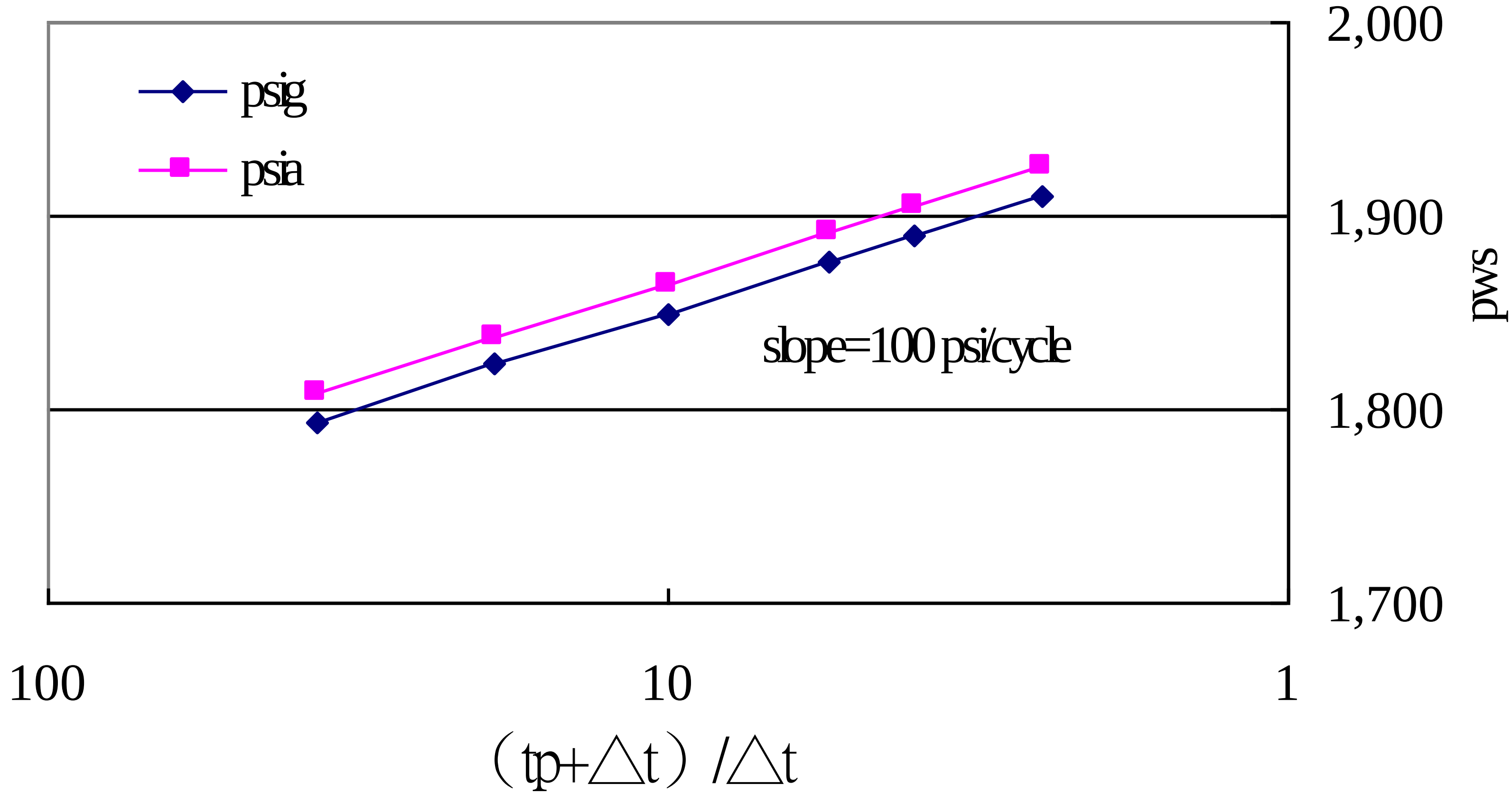
$then \quad shut - in$

| time after shut − in $\Delta t (hrs)$ | $p_{ws}$ (psig) |
|---|---|
| 0 | 1,150 |
| 2 | 1,794 |
| 4 | 1,823 |
| 8 | 1,850 |
| 16 | 1,876 |
| 24 | 1,890 |
| 48 | 1,910 |

$Estimate: \quad k \quad , \quad p_i \quad , \quad and \quad s$

Solution：

| $\Delta t$ $(hrs)$ | $p_{ws}$ $(psig)$ | $p_{ws}$ $(psia)$ | $t_p + \Delta t$ | $\frac{t_p + \Delta t}{\Delta t}$ |
|---|---|---|---|---|
| 0 | 1,150 | 1165 | 72 | -- |
| 2 | 1,794 | 1809 | 74 | 37 |
| 4 | 1,823 | 1838 | 76 | 19 |
| 8 | 1,850 | 1865 | 80 | 10 |
| 16 | 1,876 | 1891 | 88 | 5.5 |
| 24 | 1,890 | 1905 | 96 | 4 |
| 48 | 1,910 | 1925 | 120 | 2.5 |


psig
psia
slope=100 psi/cycle
2,000
1,900
1,800
1,700
pws
100
10
1
（tp+△t）/△t

$$\mathrm{m = slope = 100 \quad psi/cycle\,(from\ figure)}$$

$$k = \frac{162.6 q\mu B}{mh} = \frac{162.6\times 500\times 1.0\times 1.3}{100\times 22} = 48 \quad md$$

$$p_i = 1950 \quad psig = 1965 \quad psia \quad at \quad \frac{t_p + \Delta t}{\Delta t} = 1 \quad \text{(from figure)}$$

$$\mathrm{s} = 1.151\left\{\frac{p_{1hr} - p_{wf}}{m} - \log\left(\frac{k}{\mu c\phi r_w{}^2}\right) + 3.23\right\}$$

$$At \quad \Delta t = 1hr \ , \quad \frac{t_p + \Delta t}{\Delta t} = \frac{72+1}{1} = 73 \quad \Rightarrow \quad p_{ws} = 1764 \quad psig$$

$$s = 1.151\left\{\frac{1764 - 1150}{100} - \log\left(\frac{48}{1.0\times 20\times 10^{-6}\times 0.2\times (0.3)^2}\right) + 3.23\right\}$$

$$= 1.43$$

# Actual Buildup Test — Flow regions

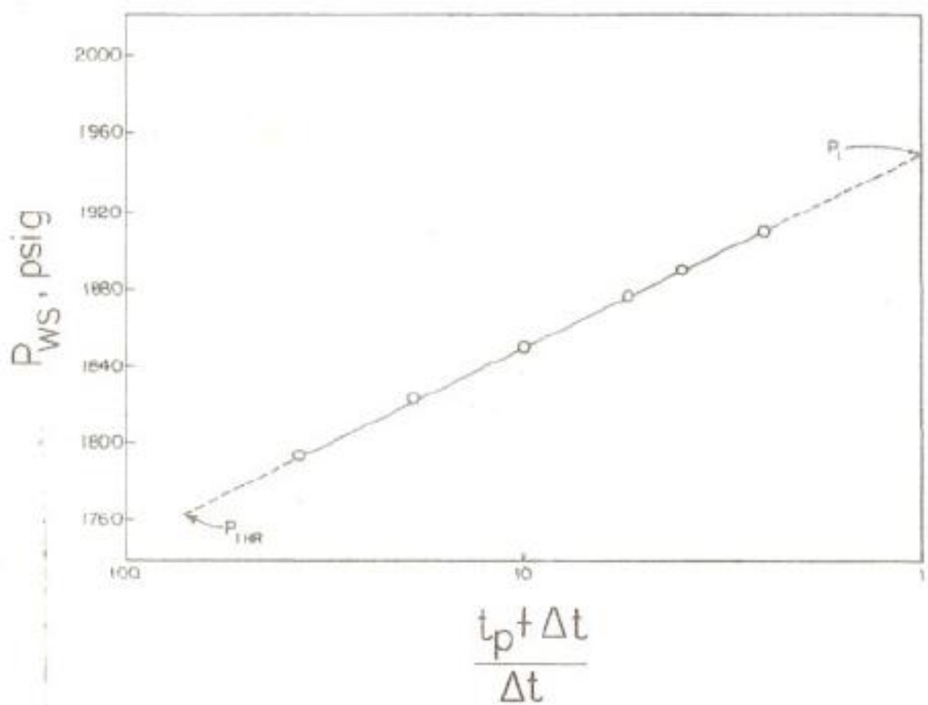



Fig. 2.3 – Ideal pressure buildup test graph.

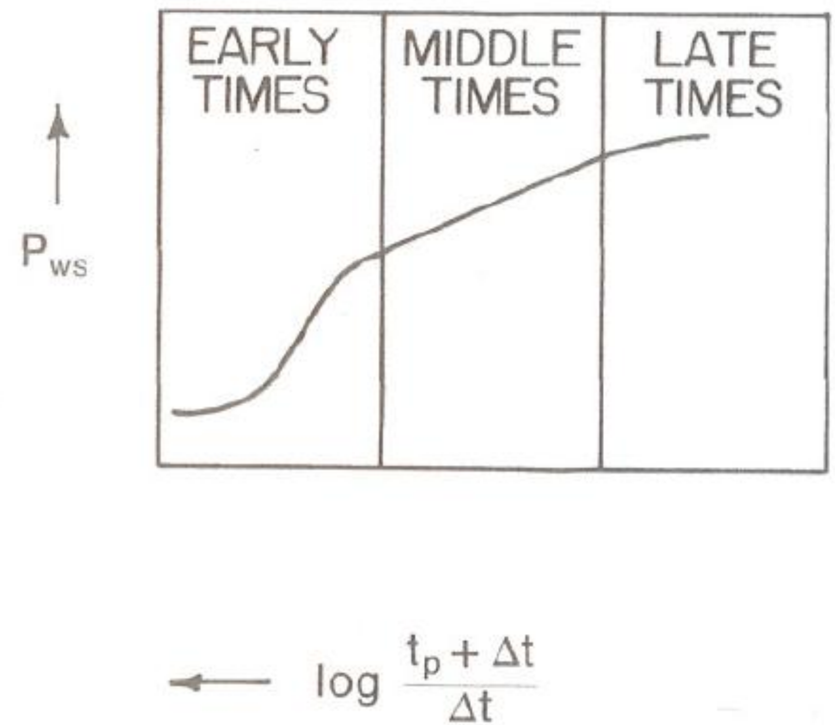
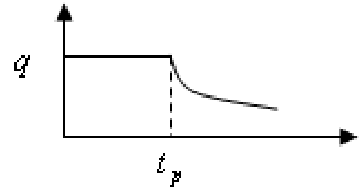



Fig. 2.4 – Actual buildup test graph.

Early-time region (pressure transient is moving through the formation nearest the wellbore)

It is caused by $\begin{cases} s \\ V_{starage} \end{cases} \Leftrightarrow$




Middle-time region (pressure transient has moved away from the wellbore and into the bulk formation)

$$\begin{array}{c} s\ \&\ V_{storage} \\ \textit{effect zone} \end{array} < \begin{array}{c} \textit{Radius of} \\ \textit{investigation} \\ (r_i) \end{array} < \begin{array}{c} \text{Re}\textit{servoir boundary}\ ; \\ \textit{Massive heterogeneities} \\ \textit{fluid / fluid contact} \end{array}$$

The data in the region can be used to determine $k, s$ and $p_i$ etc.

Late-time region (pressure behavior is influenced by boundary configuration,
interference from nearby wells
significant reservoir heterogeneities, and
fluid / fluid contacts.

# Actual Buildup Tests —
## Deviations from assumptions in ideal test theory

Infinite reservoir assumption

In principle, the Horner plot is incorrect when the reservoir is not infinite acting during the flow period preceding the buildup test. However, the Horner plot is used for all tests (even when the reservoir has reached pseudo steady-state for the following reasons:

(1) correct for infinite acting reservoir;

(2) to find p for t approaching to infinite;

(3) to find k for the data before affecting by boundary

Single - phase liquid assumption

$$c_t = c_o s_o + c_w s_w + c_g s_g + c_f$$

*where* $$c_o = -\frac{1}{B_o}\frac{dB_o}{dp} + \frac{B_g}{B_o}\frac{dR_s}{dp}$$

$$c_w = -\frac{1}{B_w}\frac{dB_w}{dp} + \frac{B_g}{B_w}\frac{dR_{sw}}{dp}$$

$$c_f = \frac{1}{V_p}\left(\frac{\partial V_p}{\partial p}\right)_T$$

$$c_g = \frac{1}{p} - \frac{1}{z}\left(\frac{\partial z}{\partial p}\right)_T$$

Oil is mobile phase

Water & gas are immobile

- -Homogeneous reservoir assumption

# Actual Buildup Tests- Qualitative behavior of field tests

- Early time region (ETR)
  - Effect of wellbore damage (with no after flow)
    - (1) without wellbore damage
    - (2) with wellbore damage

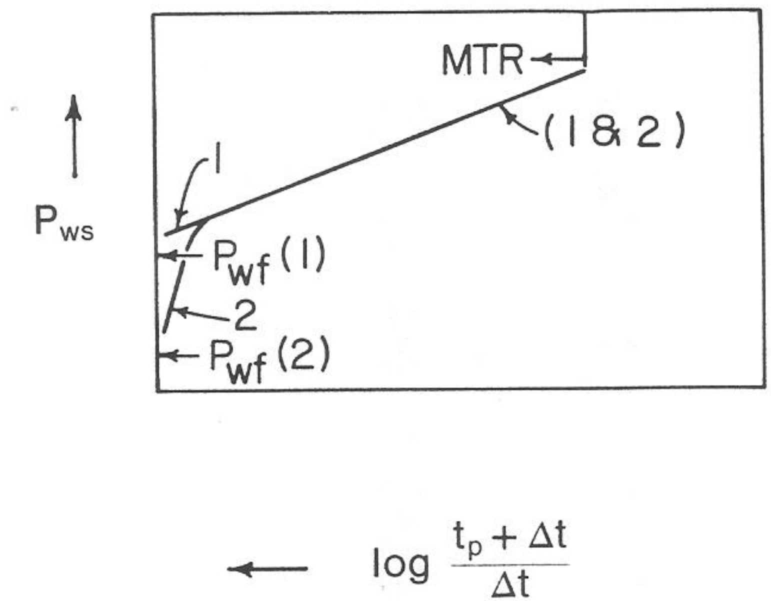



**Fig. 2.6** – Buildup test with no afterflow: (1) without wellbore damage and (2) with wellbore damage.

- Effect of afterflow (with formation damage)

(1) without afterflow

(2) with afterflow

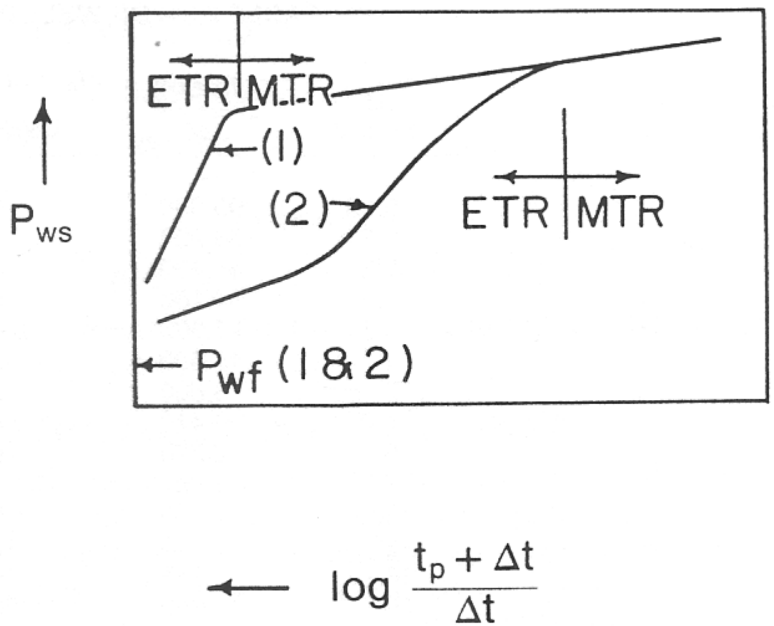



Fig. 2.7 – Buildup test with formation damage: (1) without afterflow and (2) with afterflow.

- Effect of hydraulically fractured well

(1) Non-fractured well

(2) Fractured well

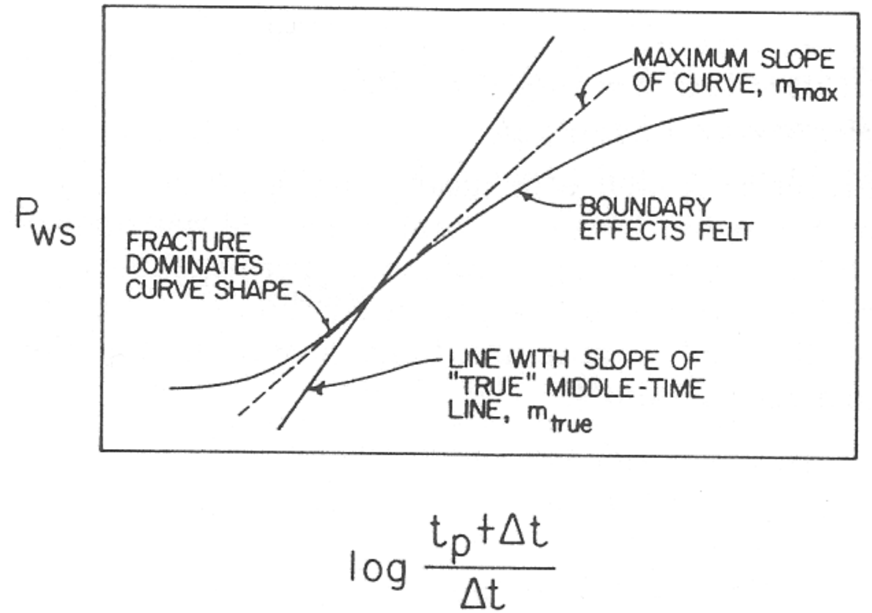



Fig. 2.14 – Buildup curve for hydraulically fractured well, bounded reservoir.

Late - time region (LTR)

--- boundary effect of a production well location

(1) well centered in drainage area

(2) well off - center in drainage area

# Effect and Duration of Afterflow

- Several problems that afterflow affects the buildup test analysis

(1) delay in the beginning of the MTR, ……

(2) total lack of development of the MTR in some cases, ……

(3) development of several false straight lines, ……

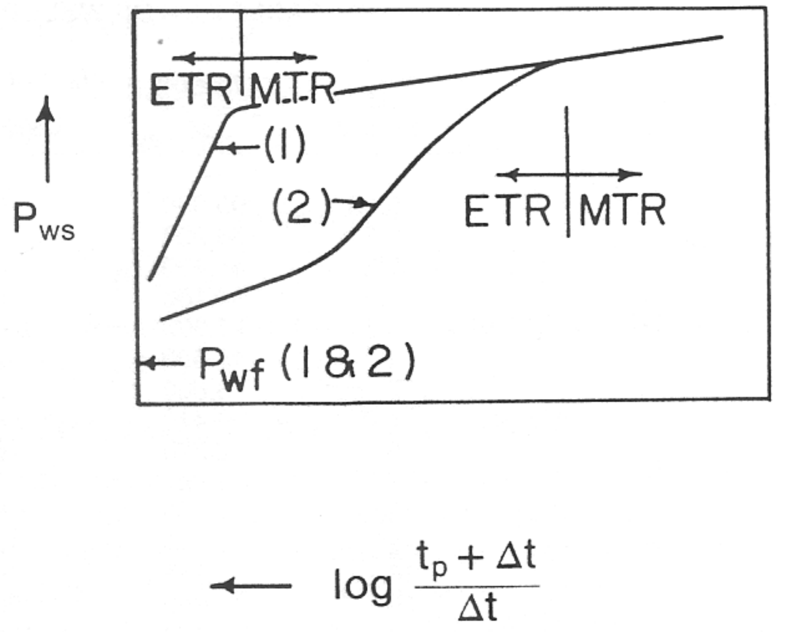



Fig. 2.7 – Buildup test with formation damage: (1) without afterflow and (2) with afterflow.

- The characteristics of afterflow on a pressure buildup test plot

-- In Horner plot

(1) a laze S-shape at early time

(2) In some tests, parts of the S-shape may be missing in the time range during which data have been recorded

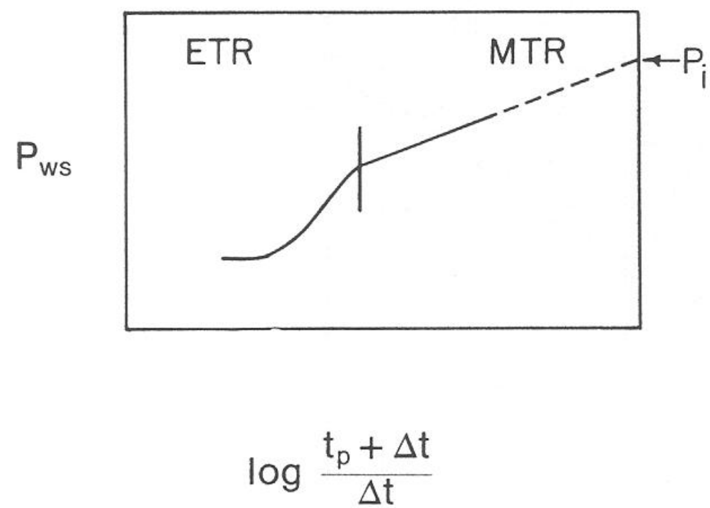



Fig. 2.15 – Buildup test graph for infinite-acting reservoir.

- In Rameys typecurve unit slope; End of afterflow ($1\frac{1}{2}$ cycle)

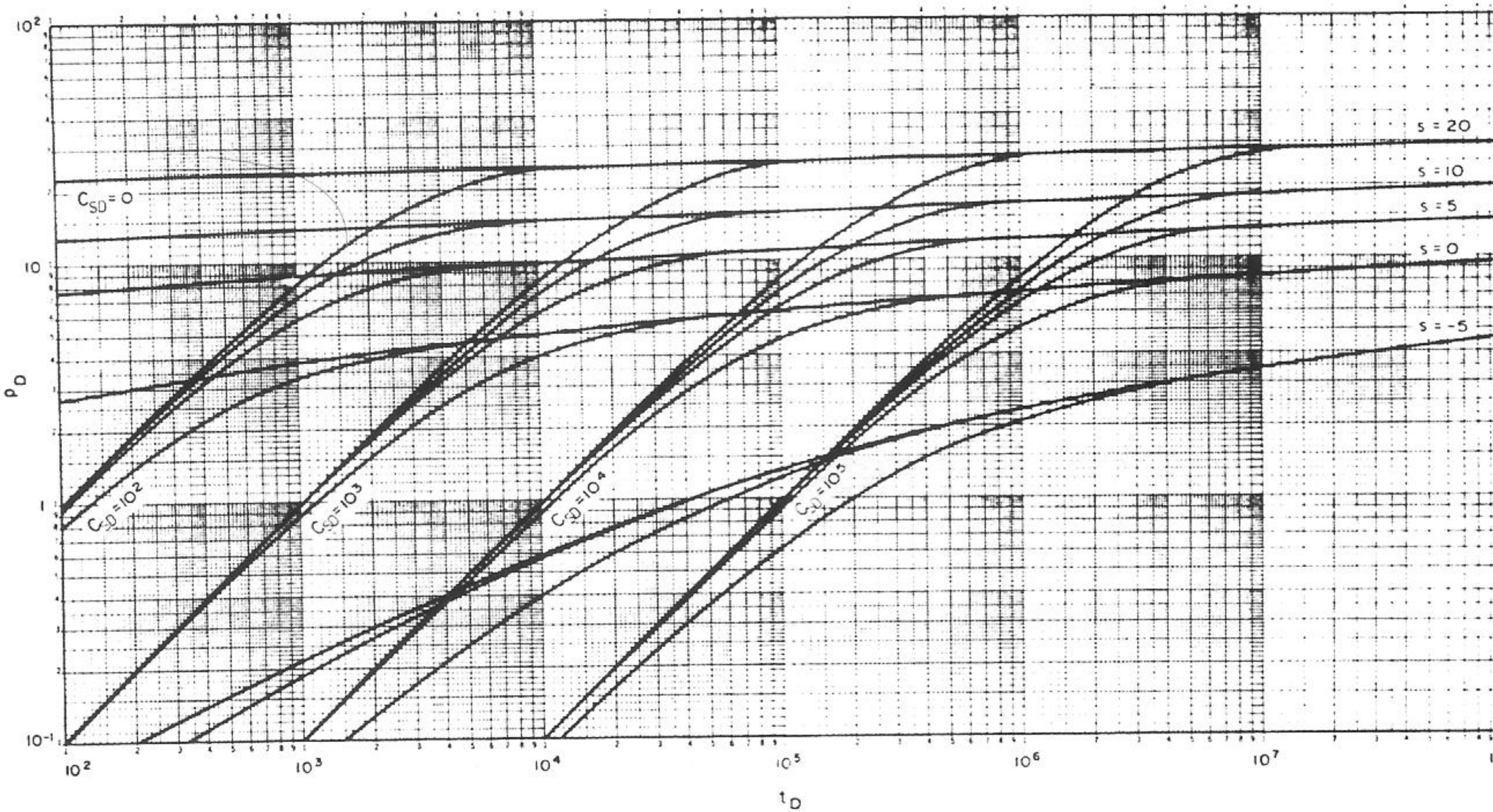



Fig. 4.1 – Type curves for constant production rate, infinite-acting reservoir (Ramey).

$\text{Pressure} \quad \textit{Drawdown}$

$$p_D = \frac{kh(p_i - p_w)}{141.2 q\mu B} \qquad t_D = \frac{2.637\times 10^{-4} kt}{\mu c_t \phi r_w^2}$$

$\text{Pressure} \quad \textit{Buildup}$

$$p_D = \frac{kh(p_{ws} - p_{wf})}{141.2 q\mu B} \qquad t_D = \frac{2.637\times 10^{-4} k\Delta t_e}{\mu c_t \phi r_w^2} \qquad \Delta t_e = \frac{\Delta t}{1 + \dfrac{\Delta t}{t_p}}$$

$$C_{sD} = \frac{0.894 C_s}{\phi c_t h r_w^2} \quad ,$$

$$\textit{where} \qquad C_s = 25.65 \frac{A_{wb}}{\rho} \quad , \quad \textit{or} \quad C_s = C_{wb} V_{wb}$$

$$Unit \quad Slope \quad line$$

$$1 - C_{sD}\frac{dp_D}{dt_D} = 0 \quad \Rightarrow C_{sD}\,p_D = t_D$$

$$\Rightarrow C_s = \frac{qB}{24}\frac{\Delta t_e}{\Delta p}$$

$$where \quad \Delta p = p_{ws} - p_{wf}$$

$$End \quad of \quad wellbore \quad sterage \quad effect$$

$$t_D \approx 50 C_{sD} e^{0.14s}$$

$$or \quad t_{wbs} \approx \frac{17{,}000 C_s e^{0.14s}}{(kh/\mu)}$$

# Example 2.2 – Finding the end of wellbore storage distortion

$t_p = 13{,}630 \quad hrs \quad (\text{an effective time at the final rate})$

$q_o = 250 \quad STB/D$

$\mu_o = 0.8 \quad cp$

$\phi = 0.039$

$B = 1.136 \quad RB/STB$

$c_t = 17 \times 10^{-6} \quad psi^{-1}$

$r_w = 0.198 \quad ft$

$r_e = 1489 \quad ft$

$\rho_o = 53 \quad lb_m/ft^3$

$A_{wb} = 0.0218 \quad ft^2$

$h = 69 \quad ft \;,\; and$

rising liquid level in well during shut-in

TABLE 2.3 – OILWELL PRESSURE BUILDUP TEST DATA

| $\Delta t$ (hours) | $\frac{t_p + \Delta t}{\Delta t}$ | $\Delta t_e = \Delta t / \left(1 + \frac{\Delta t}{t_p}\right)$ (hours) | $p_{ws}$ (psia) | $p_{ws} - p_{wf}$ (psia) |
|---|---|---|---|---|
| 0 | – | – | 3,534 | 0 |
| 0.15 | 90,900 | 0.15 | 3,680 | 146 |
| 0.2 | 68,200 | 0.2 | 3,723 | 189 |
| 0.3 | 45,400 | 0.3 | 3,800 | 266 |
| 0.4 | 34,100 | 0.4 | 3,866 | 332 |
| 0.5 | 27,300 | 0.5 | 3,920 | 386 |
| 1 | 13,600 | 1 | 4,103 | 569 |
| 2 | 6,860 | 2 | 4,250 | 716 |
| 4 | 3,410 | 4 | 4,320 | 786 |
| 6 | 2,270 | 6 | 4,340 | 806 |
| 7 | 1,950 | 7 | 4,344 | 810 |
| 8 | 1,710 | 8 | 4,350 | 816 |
| 12 | 1,140 | 12 | 4,364 | 830 |
| 16 | 853 | 16 | 4,373 | 839 |
| 20 | 683 | 20 | 4,379 | 845 |
| 24 | 569 | 24 | 4,384 | 850 |
| 30 | 455 | 29.9 | 4,393 | 859 |
| 40 | 342 | 39.9 | 4,398 | 864 |
| 50 | 274 | 49.8 | 4,402 | 868 |
| 60 | 228 | 59.7 | 4,405 | 871 |
| 72 | 190 | 71.6 | 4,407 | 873 |

Find :

(1) At what shut-in ($\Delta$t) time does afterflow cease distorting the pressure buildup test data ?

(2) At what shut-in ($\Delta$t) time does boundary effects appear ?

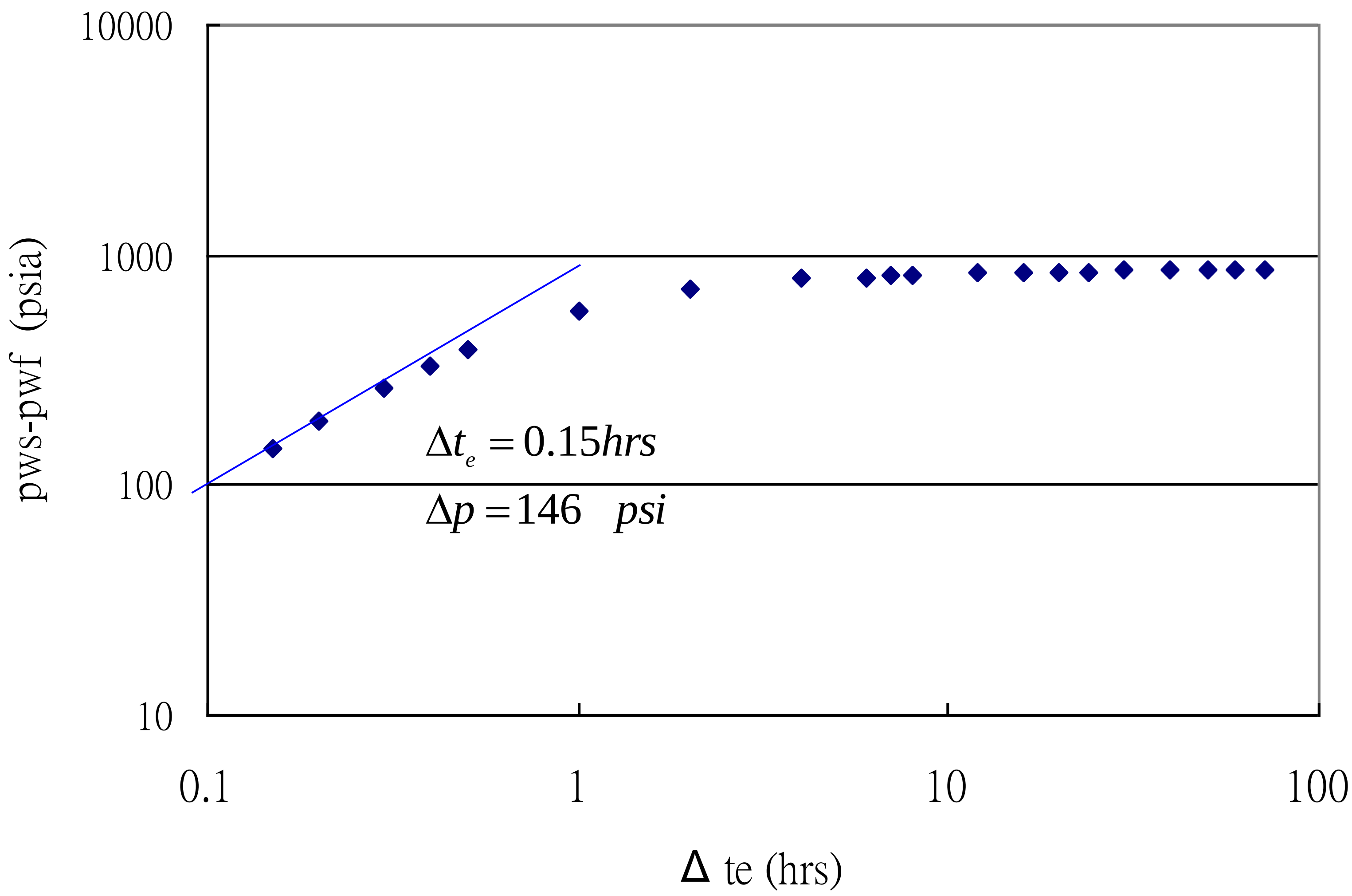
*Solution*:

(1)

(a) calculate the plotting variables $\dfrac{t_p + \Delta t}{\Delta t}$ , $\Delta t_e = \dfrac{\Delta t}{1 + \dfrac{\Delta t}{t_p}}$ , *and* $\left(p_{ws} - p_{wf}\right)$

$$qt_p + q\Delta t = q_e \Delta t \quad q_e \Delta t = q\left(t_p + \Delta t\right) \quad q_e\left(\frac{\Delta t}{t_p + \Delta t}\right) = q \quad q_e\left(\frac{\Delta t}{\dfrac{t_p + \Delta t}{t_p}}\right) = qt_p$$

$(b)$ plot $\log\left(p_{ws} - p_{wf}\right)$ v.s. $\log \Delta t_e$


10000
1000
100
10
pws-pwf (psia)
$\Delta t_e = 0.15 hrs$
$\Delta p = 146 \quad psi$
0.1
1
10
100
Δ te (hrs)

(c) Calculate wellbore storage constant from unit slope line

$$C_s = \frac{qB}{24}\frac{\Delta t_e}{\Delta p}$$

$\Delta t_e = 0.15 hrs$ & $\Delta p = 146$ *psi is on the unit slope line*

$$\Rightarrow C_s = \frac{250 \times 1.136}{24}\frac{0.15}{146} = 0.0121 \;\; bbl/psi$$

(c)" Calculate wellbore storage constant from well completion data (less accuracy)

$$C_s = \frac{25.65 A_{wb}}{\rho_0} = \frac{25.65 \times 0.0218}{53} = 0.0106 \;\; bbl/psia$$

(d) Calculate dimensionless wellbore storage constant

$$C_{sD} = \frac{0.894 C_s}{\phi c_t h r_w^2} = \frac{0.894 \times 0.0121}{(0.039) \times (17 \times 10^{-6}) \times (69) \times (0.198)^2} = 6031.5$$

(e) Calculate end of wellbore storage effect

- from the plot of $\log\left(p_{ws} - p_{wf}\right)$ $v.s.$ $\log \Delta t_e$

$1\frac{1}{2}$ *cycle from deviation of unit slope line* $\Delta t_e \approx 8$

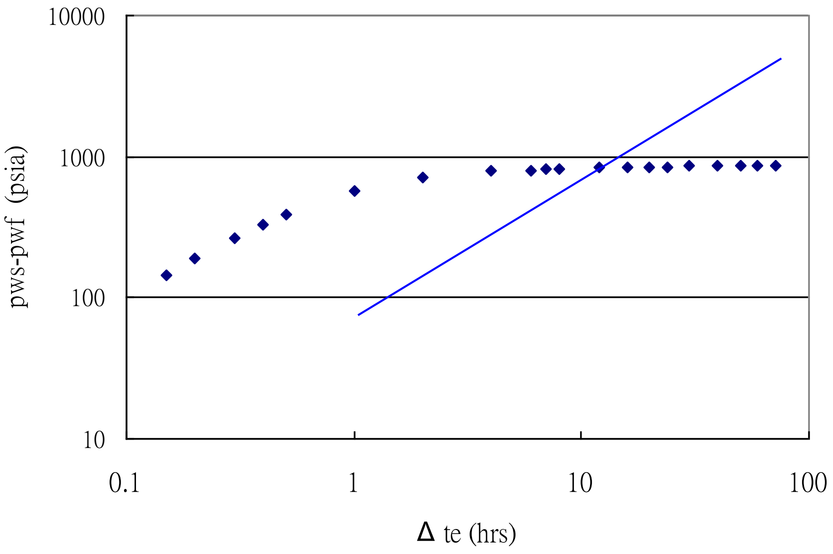
$$\Rightarrow \frac{\Delta t}{1+\frac{\Delta t}{t_p}} = 8 \quad \Rightarrow \frac{\Delta t}{1+\frac{\Delta t}{13630}} = 8$$

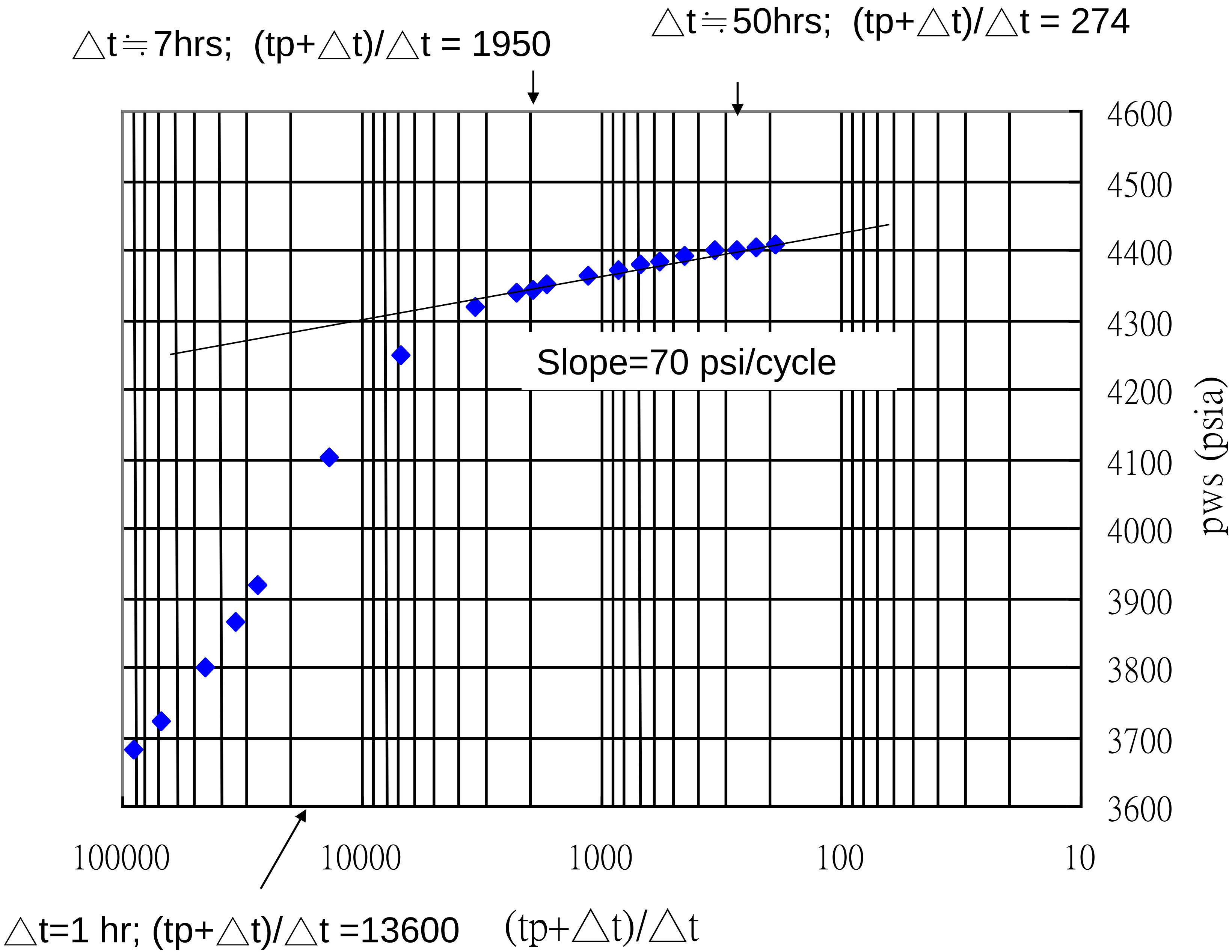
$$\Rightarrow \Delta t = 8 + 8\frac{\Delta t}{13630}$$

$$\Rightarrow \left(1 - \frac{1}{1363}\right)\Delta t = 8$$

$$\Rightarrow \Delta t = 8 \quad hrs \quad (The \quad result \quad is \quad very \quad rough)$$

# Plot of $\log\left(p_{ws} - p_{wf}\right)$ v.s. $\log \Delta t_e$




△t≒7hrs; (tp+△t)/△t = 1950
△t≒50hrs; (tp+△t)/△t = 274
4600
4500
4400
4300
4200
4100
4000
3900
3800
3700
3600
pws (psia)
Slope=70 psi/cycle
100000
10000
1000
100
10
△t=1 hr; (tp+△t)/△t =13600
(tp+△t)/△t

- from the equation such as

$$t_D \cong 50 C_{sD} e^{0.14s} \quad and \quad t_D \approx (60 + 3.5s) C_{sD}$$

$$\Rightarrow t_{wbs} = \frac{169{,}318 C_s e^{0.14}}{(kh/\mu)} \quad and \quad t_{wbs} = \frac{3386.3(60 + 3.5s) C_s}{(kh/\mu)}$$

In the above equations. The permeability must be known.

$$\underline{Deter\min ation \quad of \quad permeadility}$$

$$Form \quad the \quad Horner \quad plot \quad \left\{ \begin{matrix} m = 70\,psi/cycle \\ p_{1hr} = 4295\,psi \end{matrix} \right\}$$

$$k = 162.6\frac{q\mu B}{mh} = 162.6\frac{250 \times 1.136 \times 0.8}{70 \times 69} = 7.648 \quad md$$

$$s = 1.151\left[ \frac{p_{1hr} - p_{wf}}{m} - \log\left( \frac{k}{\mu c\phi r_w^2} \right) + 3.23 \right]$$

$$= 1.151\left\{ \frac{4290 - 3534}{70} - \log\left[ \frac{7.648}{0.039 \times 0.8 \times 17 \times 10^{-6} \times (0.198)^2} \right] + 3.23 \right\}$$

$$== 1.151[10.87 - 8.565 - 323] = 6.37$$

$$\Rightarrow t_{wbs} = \frac{169{,}318 \times 0.0118 \times e^{-0.14 \times 6.37}}{\left(\frac{7.648 \times 69}{0.8}\right)}$$

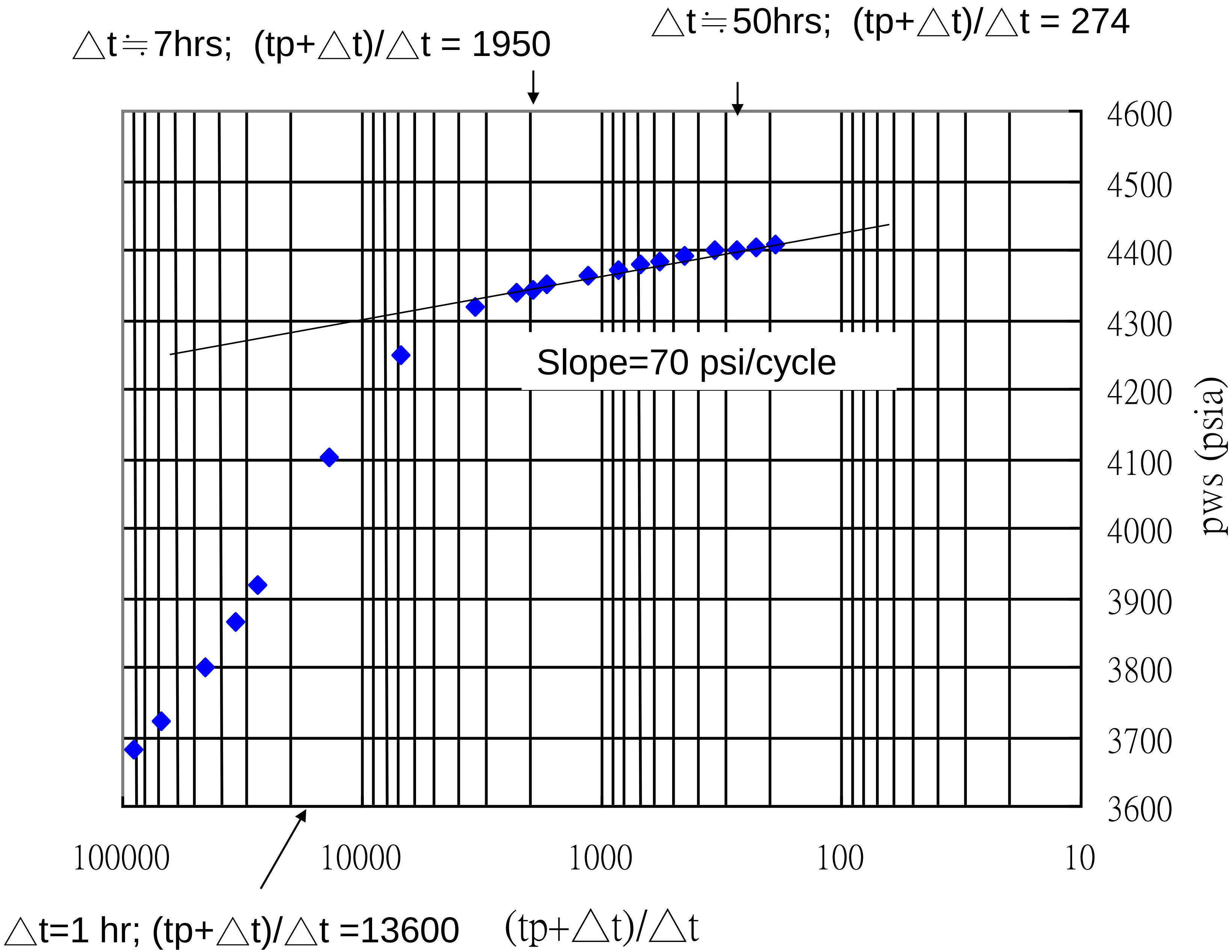
$$or \quad t_{wbs} = \frac{3386.3\left(60 + 3.5 \times 6.57\right) \times 0.0118}{\left(\frac{7.648 \times 69}{0.8}\right)} = 4.98 hrs$$

$$-\ from \quad Horner \quad plot \quad \frac{t_p + \Delta t}{\Delta t} \approx 2.27 \times 10^3 \Rightarrow \Delta t = 6 hrs$$

(2) To find end of MTR

From Horner plot, the data begin to deviate from the semi log straight line

$$at \quad \frac{t_p + \Delta t}{\Delta t} \approx 274 \Rightarrow \Delta t \approx 50 \quad hrs$$


△t≒7hrs; (tp+△t)/△t = 1950
△t≒50hrs; (tp+△t)/△t = 274
Slope=70 psi/cycle
4600
4500
4400
4300
4200
4100
4000
3900
3800
3700
3600
pws (psia)
100000
10000
1000
100
10
△t=1 hr; (tp+△t)/△t =13600
(tp+△t)/△t

# Determination of Permeability (procedure)

(1) Find the probable beginning of the MTR

(Cessation of afterflow effects)

(2) Find the probable end of the MTR

-- The Horner plot becomes nonlinear

-- Type curve matching technique

-- Equation $\Delta t_{lt} \approx \dfrac{38\phi\mu c_t A}{k}$

for a well centered in a square or circular drainage area

(3) Calclate the slope of the MTR, and estimate k from

$$\mathrm{k} = 162.6 \frac{\mathrm{q}\mu \mathrm{B}}{\mathrm{mh}} \quad \text{where } \mathrm{m} = \text{slope of the MTR}$$

(4) Radius of investigation at beginning and end of MTR

$$\mathrm{r_i} = \left( \frac{kt}{948 \mu c_t \phi} \right)^{1/2} = \left( 4 t_D \right)^{1/2}$$

(5) If there is no clear-cut MTR or if it is so short that it slope cannot be determined with confidence

$\Rightarrow$ using type-curve analysis.

Average permeability from pseudo-steady state test data

$$\mathrm{k_J} = \frac{141.2 q \mu B \left[ \ln\left( \frac{r_e}{r_w} \right) - \frac{3}{4} \right]}{h\left( \overline{p} - p_{wf} \right)}$$

For a damage well $\mathrm{k_J} < k$

For a stimulate well $\mathrm{k_J} > k$

# Well Damage and Stimulation

--Well damage -- It is caused by…..

--Stimulation -- To improve a well's productivity

by acidization and hydraulic fracturing

Skin method

-- Wellbore damage $\Rightarrow \quad s > 0 \Leftarrow$ *In*completely perforated interval

-- Well stimulation $\Rightarrow \quad s < 0$

$$s \Rightarrow r_{wa} = r_w e^{-s} \qquad (for \quad s > 0 \quad or \quad s < 0)$$

$$L_f\left(= x_f\right) \cong 2r_{wa} \quad (for \quad s < 0)$$

$$(\Delta p)_s = 141.2 \frac{q\mu B}{kh} s = 0.869ms \qquad where \quad m = \frac{162.6q\mu B}{kh}$$

Flow efficiency, $E$

$$E=\frac{J_{actual}}{J_{ideal}}=\frac{\overline{p}-p_{wf}-(\Delta p)_s}{\overline{p}-p_{wf}}$$

$$where \quad J=\frac{q}{\overline{p}-p_{wf}}=\frac{k_J h}{141.2 q\mu B\left[\ln\left(\frac{r_e}{r_w}\right)-\frac{3}{4}\right]}$$

$$E=\frac{\overline{p}-p_{wf}-(\Delta p)_s}{\overline{p}-p_{wf}}\cong\frac{p^*-p_{wf}-(\Delta p)_s}{p^*-p_{wf}}$$

$$E=1 \quad (no \quad damage \quad ; \quad no \quad stimulation)$$

$$E<1 \quad damaged \quad well$$

$$E>1 \quad stimulated \quad well$$

Effect of incompletly perforated interval

$$s = \frac{h_t}{h_p} s_d + s_p$$

$$\text{where} \qquad s_p = \left( \frac{h_t}{h_p} - 1 \right) \left[ \ln \left( \frac{h_t}{r_w} \sqrt{\frac{k_h}{k_v}} \right) - 2 \right]$$

$s = total \quad skin \quad factor$

$s_d = true \quad skin \quad factor, \quad caused \quad by \quad formation \quad damage$

$s_p = apparent \quad skin \quad factor, \quad caused \quad by \quad an \quad incompletely \quad perforated \quad interval$

$h_t = total \quad interval \quad height, \quad ft$

$h_p = the \quad perforated \quad interval, \quad ft$

$--Example \quad 2.5--$

# Analysis of Hydraulically Fractured Wells (Vertical fractures)

There are three basic analytical solutions for hydraulically fractured wells

(1) Uniform-flux fracture system (Nature fracture)

(2) Infinite conductivity fracture system (Hydraulic fracture)

(3) Finite conductivity fracture system (Hydraulic fracture)

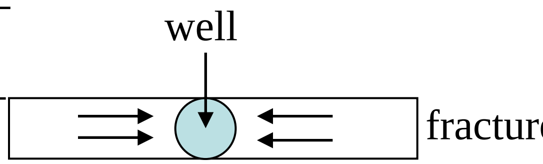
(Low or internendiate conductivity fracture system)

Finite conductivity fracture system

Four flow period :

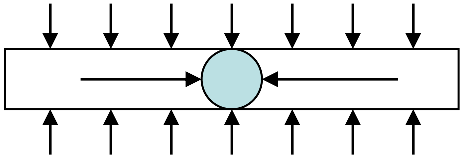
(1) Fracture linear flow period

$$p_i - p_{wf} = \frac{282.4 q\mu B x_f}{k_f w h}\sqrt{0.0008293\frac{k_f t}{\phi_f c_{ft}\mu x_f^2}}$$



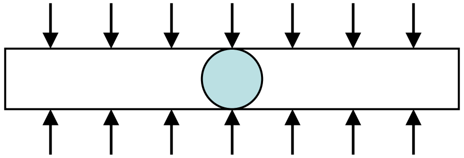

(2) Bilinear flow period

$$p_i - p_{wf} = \frac{44.1 q\mu B}{h_f\left(k_f w\right)^{1/2}\left(\phi\mu c_t k\right)^{1/4}}t^{1/4}$$



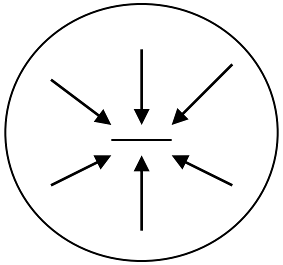
(3) Formation linear flow period

$$p_i - p_{wf} = \frac{4.064 qB}{h}\sqrt{\frac{\mu t}{k\phi c_t x_f}}$$



(4) Pseudo radial flow period

$$p_D = \frac{1}{2}\left(\ln t_D + 0.80907\right)$$



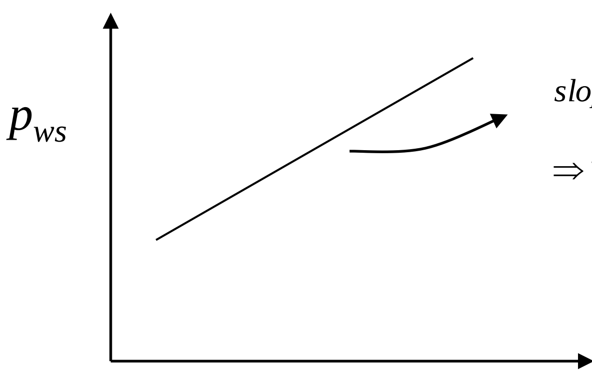
# Pressure Buildup for Hydraulically Fractured Wells --Formation linear flow

Pressure drawdown:

$$p_i - p_{wf} = \frac{4.064qB}{hx_f}\sqrt{\frac{\mu}{k\phi c_t}}\sqrt{t}$$

Pressure buildup :

$$p_i - p_{wf} = \frac{4.064qB}{hx_f}\sqrt{\frac{\mu}{k\phi c_t}}\left(\sqrt{t_p + \Delta t} - \sqrt{\Delta t}\right)$$




$$slope = \frac{4.064qB}{hx_f}\sqrt{\frac{\mu}{k\phi c_t}}$$

$\Rightarrow$ *To calculate* $x_f$ *by knowing* $k$

# Pressure Buildup for Hydraulically Fractured Wells --Pseudo-radial flow

Pressure dradown:

$$p_i = \frac{1}{2}\left(\ln t_D + 0.80907\right)$$

Pressure buildup :

$$p_{ws} = p_i - 162.6\frac{q\mu B}{kh}\log\left(\frac{t_p + \Delta t}{\Delta t}\right)$$

$$\Rightarrow slop = 162.6\frac{q\mu B}{kh}\left(to \quad find \quad k\right)$$

$$s = 1.151\left[\frac{p_{1hr} - p_{wf}}{m} - \log\left(\frac{k}{\mu c\phi r_w^2}\right) + 3.23\right]$$

$$\Rightarrow x_f = 2r_{wa} = 2r_w e^{-s}$$

**TABLE 2.4 – BUILDUP TEST SLOPES FOR HYDRAULICALLY FRACTURED WELLS**

| $L_f/r_e$ | $m_{max}/m_{true}$ |
|---|---|
| 0.1 | 0.87 |
| 0.2 | 0.70 |
| 0.4 | 0.46 |
| 0.6 | 0.32 |
| 1.0 | 0.28 |

# Pressure Level in Surrounding Formation

Formation pressure:

$$
\begin{cases}
\left.\begin{array}{l}
\textit{Original reservoir pressure } (p_i) \\
\textit{False pressure } (p^*)
\end{array}\right\} \textit{from pressure buildup test} \\
\left.\begin{array}{c}
\textit{Static drainage-area pressure } (\bar{p}) \\
(\textit{average pressure})
\end{array}\right\} \begin{array}{l}
\textit{from } (1) \textit{ MBH method, or } p^* \textit{ method} \\
\qquad (2) \textit{ Modified Muskat method}
\end{array}
\end{cases}
$$

# Original reservoir pressure

- For an infinite acting reservoir and for a well in a new reservoir (No pressure depletion)
- For a reservoir with one or more boundaries relatively near a test well (pressure depletion is negligible)

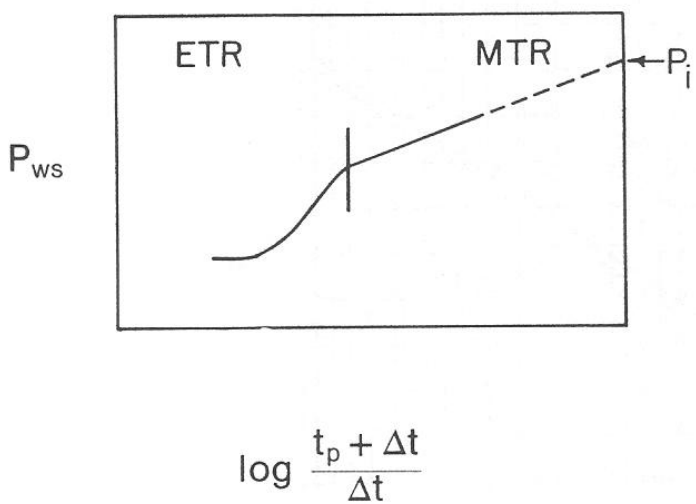



**Fig. 2.15** – Buildup test graph for infinite-acting reservoir.

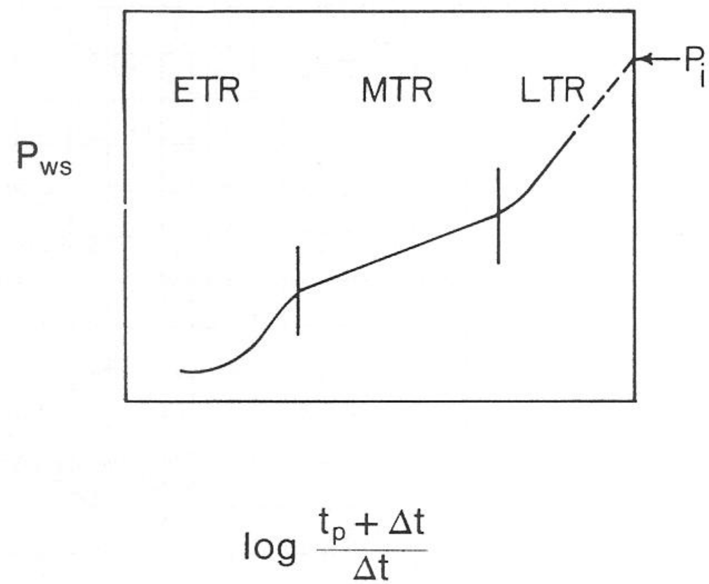
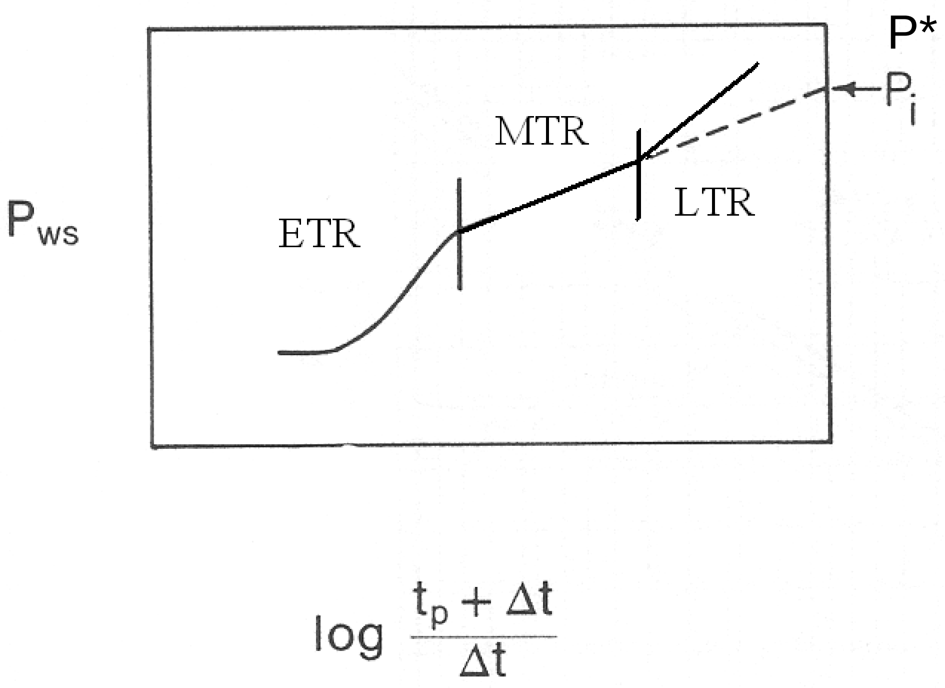



**Fig. 2.16** – Buildup test graph for well near reservoir limit(s).

# Original reservoir pressure (cont.)

- False pressure



# Static drainage-area pressure ($\bar{p}$)

$$\begin{cases} (1)\ \text{The Matthews-Brons-Hazebroek (MBH) method, or p* method} \\ (2)\ \text{The modified Muskat method} \end{cases}$$

P* method or MBH method

(1) Extrapolate the midde-time line to $\dfrac{t_p + \Delta t}{\Delta t} = 1$

$$\left( in \quad the \quad plot \quad of \quad p_{wf} \quad vs. \ \frac{t_p + \Delta t}{\Delta t} \right) \Rightarrow \quad p^*$$

(2) Estimate the drainage area shape:

(3) Choose the proper curve from figs 2.17A through 2.17G (p.36-39) for the drainage-area shape of the tested well

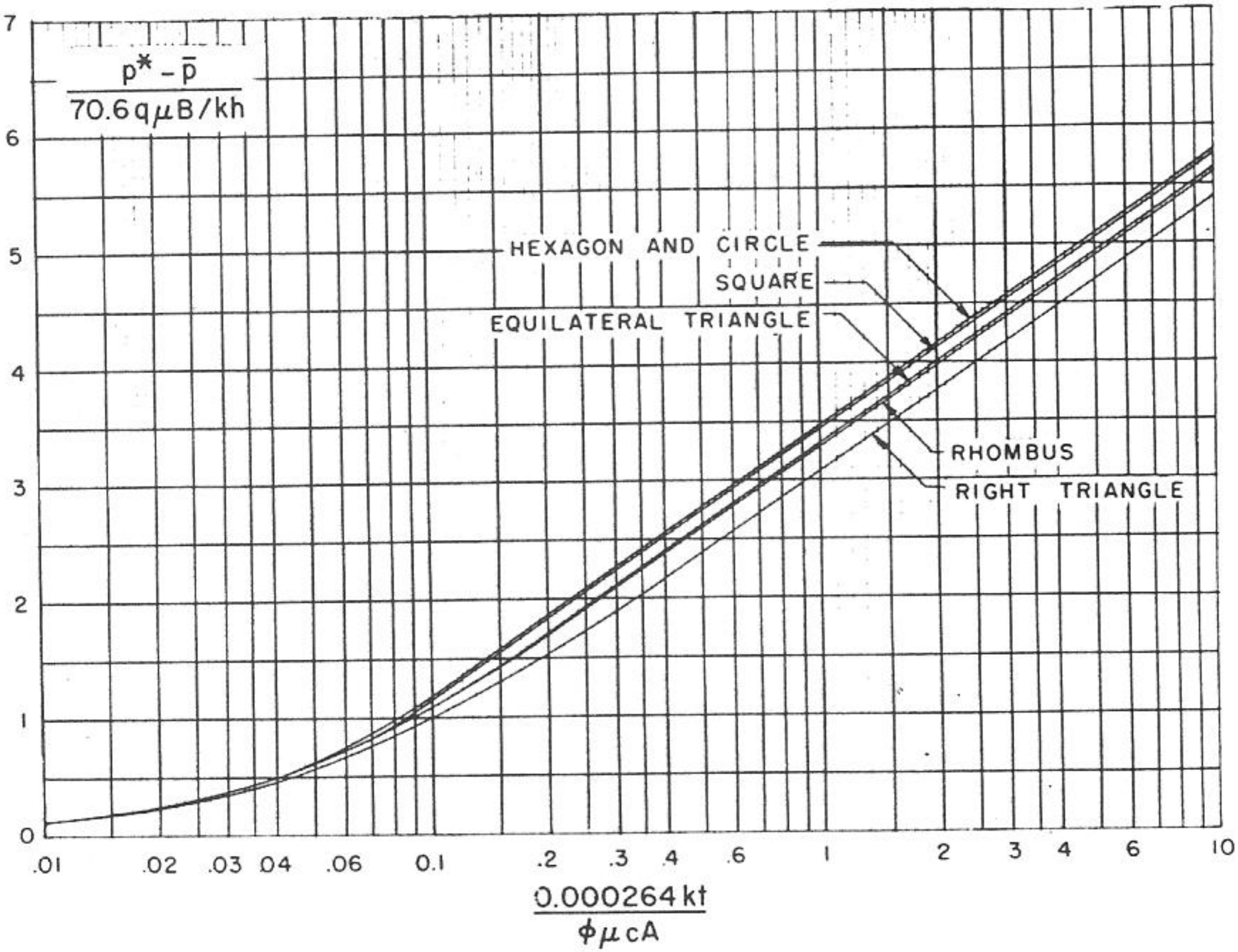



Fig. 2.17A – MBH pressure function for well in center of equilateral figures.

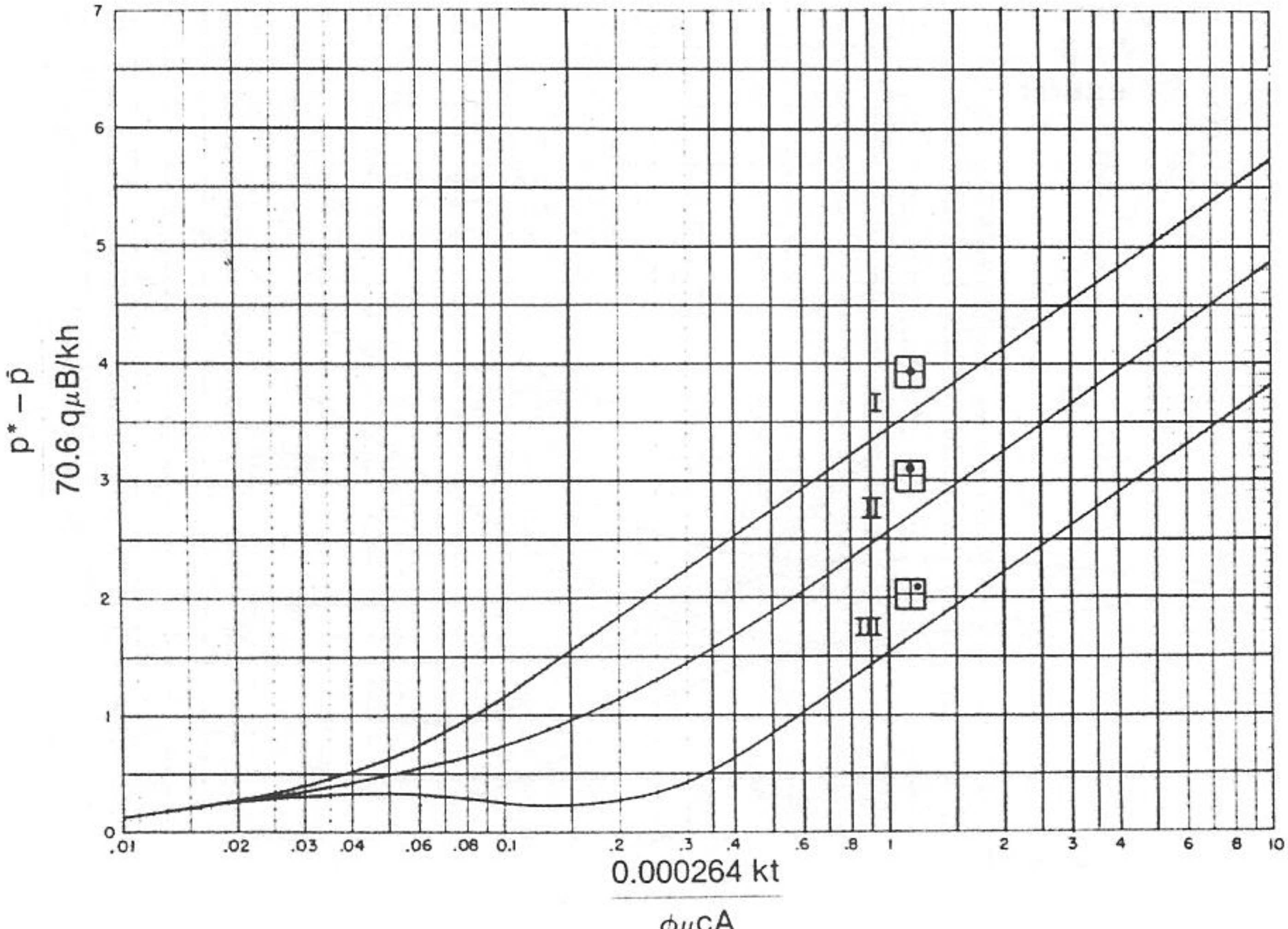


Fig. 2.17B – MBH pressure function for different well locations in a square boundary.

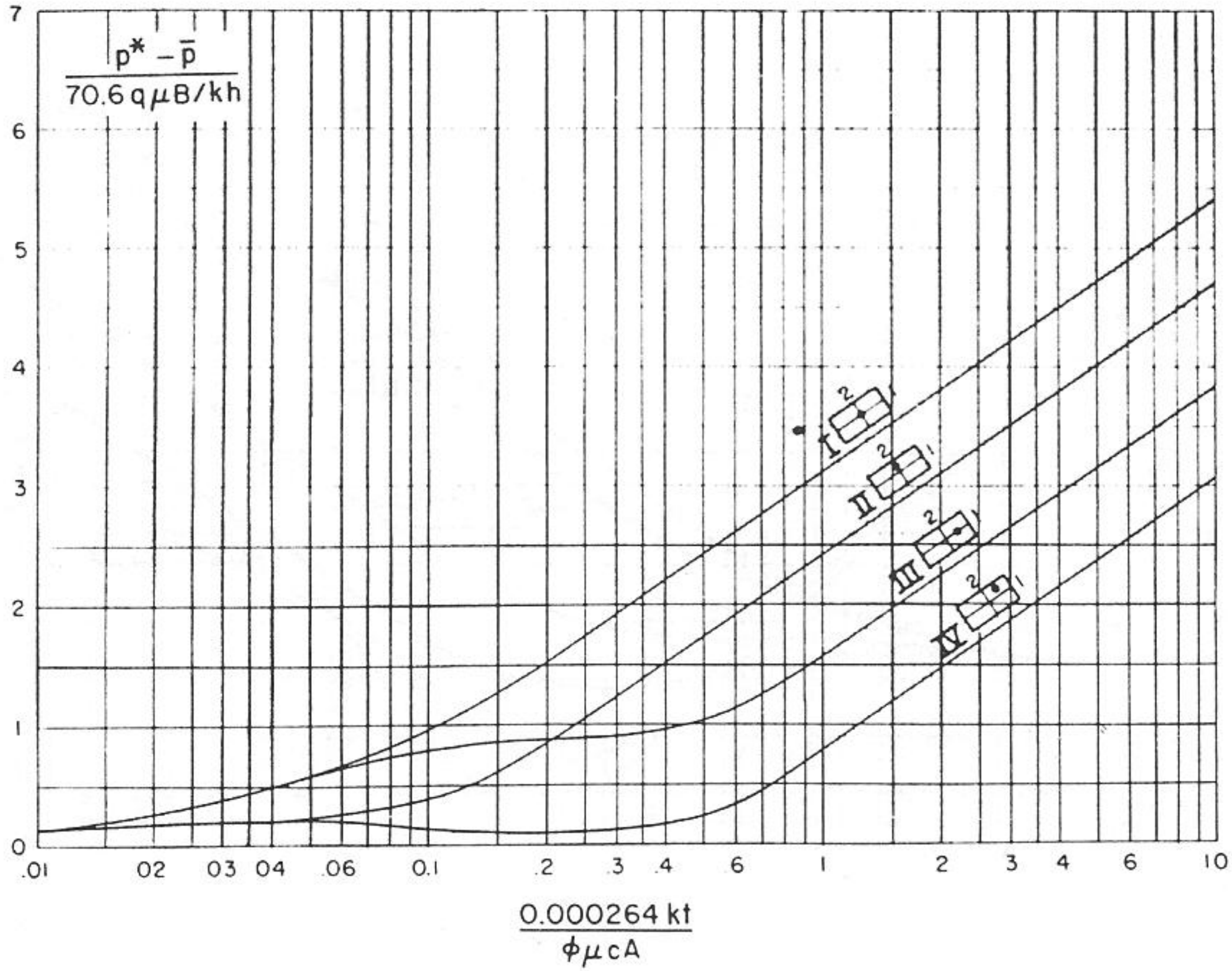



Fig. 2.17C – MBH pressure function for different well locations in a 2:1 rectangular boundary

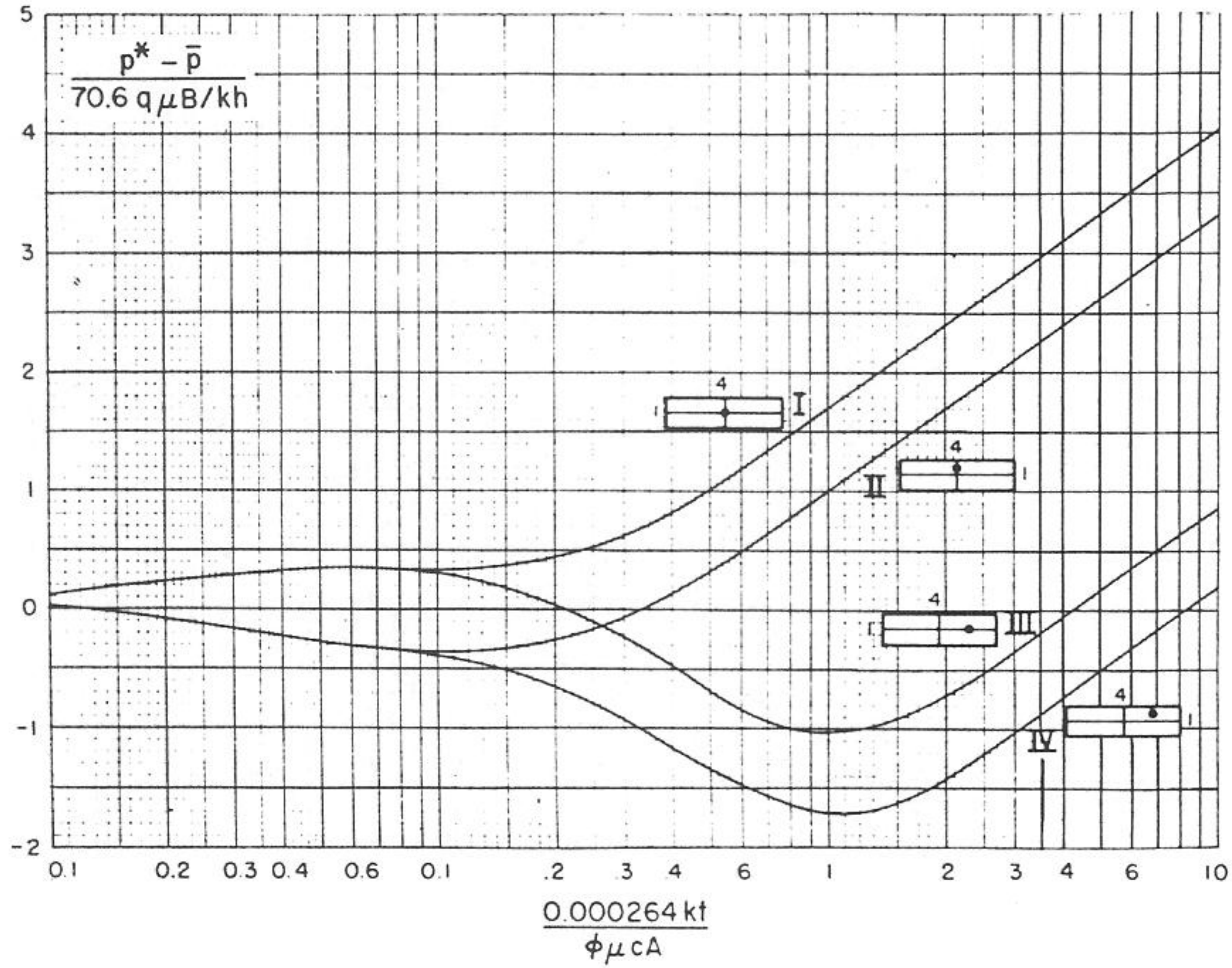



Fig. 2.17D – MBH pressure function for different well locations in a 4:1 rectangular boundary.

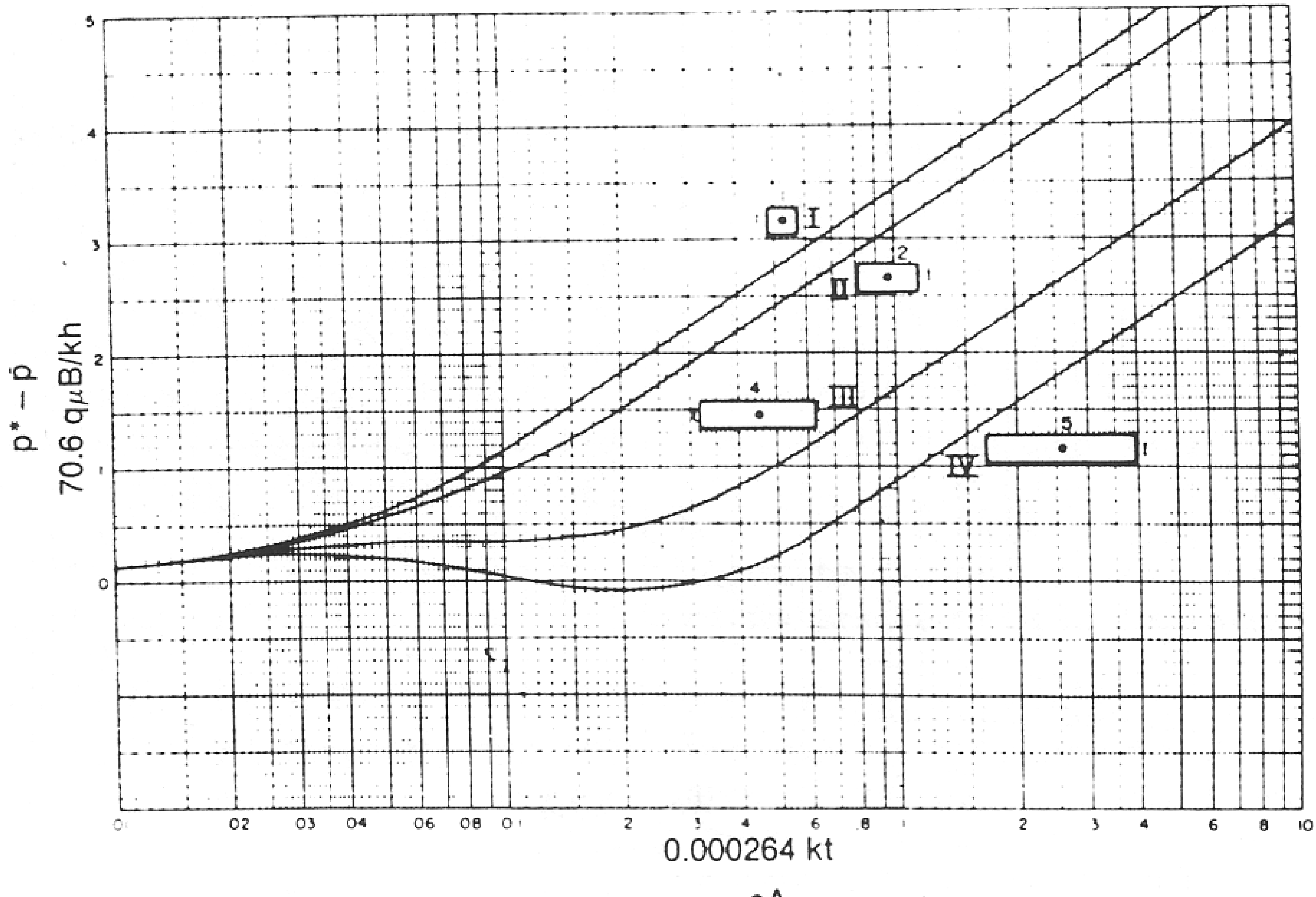



Fig. 2.17E – MBH pressure function for rectangles of various shapes.

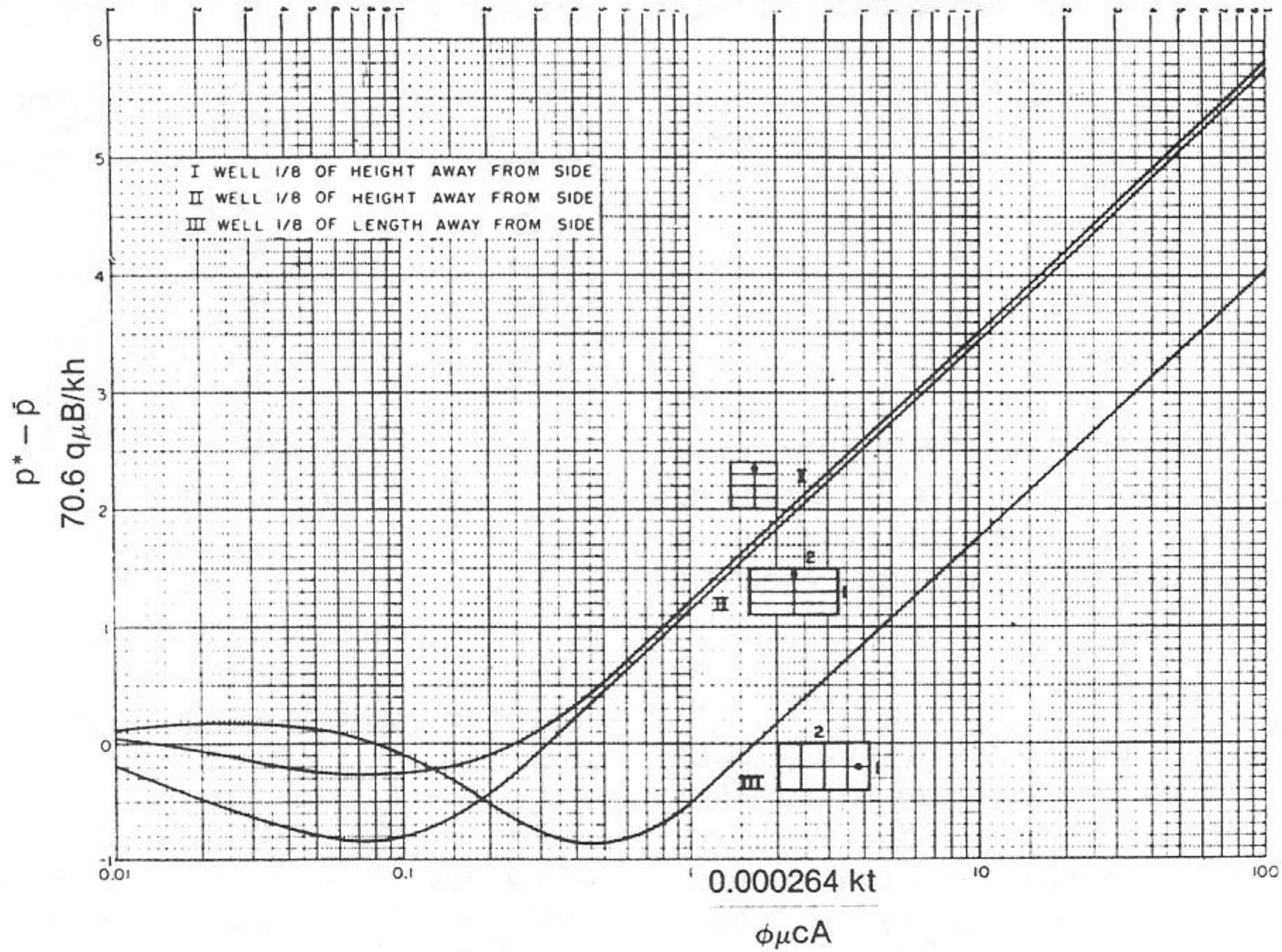



Fig. 2.17F – MBH pressure function in a square and in 2:1 rectangles.

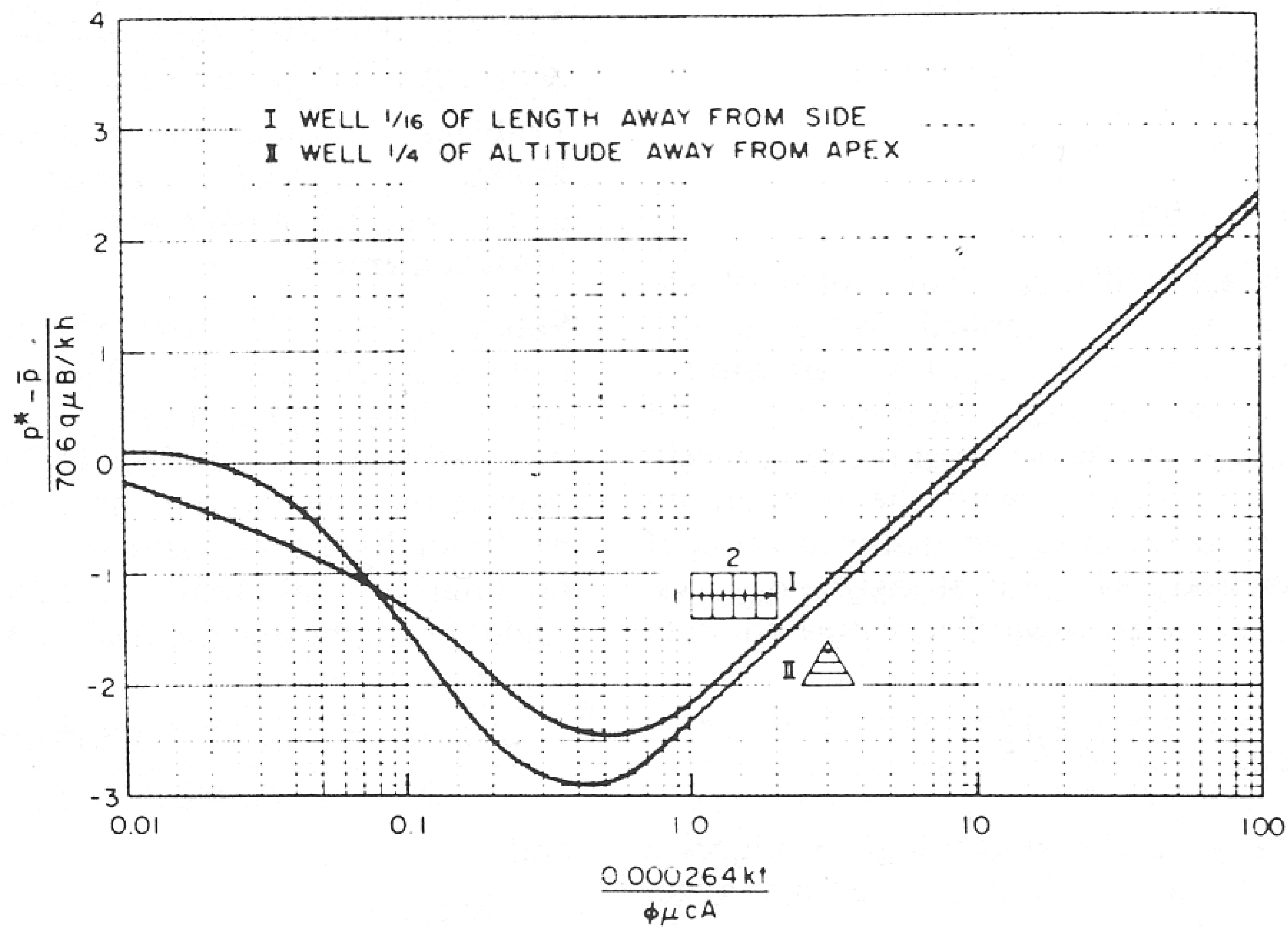



Fig. 2.17G – MBH pressure function on a 2:1 rectangle and equilateral triangle.

(4) Estimate $\dfrac{2.64\times10^{-4}kt_p}{\mu c_t \phi A}$ $\quad and \quad find \quad p_{DMBH} = \dfrac{\left(p^* - \overline{p}\right)}{70.6q\mu B/kh} = \dfrac{2.303\left(p^* - \overline{p}\right)}{m}$

$$where \quad m = 162.6\frac{q\mu B}{kh}; \quad t_p = producing \quad time \quad in \quad Horner \quad plot$$

(5) $\overline{p} = p^* - \dfrac{mp_{DMBH}}{2.303}$

Advantages : It does not reguire data beyond the MTR and it is applicable to a wide varity of drainage - area slops.

Disadvantages : It is requires knownledge of drainage - area size and shape, and estimates of reservoir and fluid properties such as $\phi$ and $c_t$ , which are not always known with great accuracy.

----- Example 2.6 -----

After pseudo - staady state flow has been achieved (figs 2.17A - 2.17G)

$P_{DMBH} = \ln\left(C_A\ t_{DA}\right)$

# Modified Muskat Method

$$\begin{cases} p_{wf} = p_i - 141.2 \dfrac{q\mu B}{kh} \left\{ \dfrac{2t_D}{r_{eD}^2} + \ln r_{eD} - \dfrac{3}{4} + 2\displaystyle\sum_{n=1}^{\infty} \dfrac{e^{-\alpha_n^2 t_D} J_1(\alpha_n r_{eD})}{\alpha_n^2 [J_1^2(\alpha_n r_{eD}) - J_1^2(\alpha_n)]} \right\} \\ p_i = \bar{p} + \dfrac{0.0744 qBt}{\phi c_t h r_e^2} \end{cases}$$

$$In \quad the \quad pressure \quad buildup$$

$$\bar{p} - p_{ws} = 118.6\frac{q\mu B}{kh} e^{-\frac{0.00388 k\Delta t}{\mu c_t \phi r_e^2}}$$

$$\Rightarrow \log(\bar{p} - p_{ws}) = \log(118.6\frac{q\mu B}{kh}) - \frac{0.00168 k\Delta t}{\mu\phi c_t r_e^2}$$

$$for \quad \frac{250\mu\phi c_t r_e^2}{k} \le \Delta t \le \frac{750\mu\phi c_t r_e^2}{k}$$

$$\Rightarrow \log(\bar{p} - p_{ws}) = A + B\Delta t$$

$$where \quad A = \log(118.6\frac{q\mu B}{kh})$$

$$B = -\frac{0.00168 k}{\mu\phi c_t r_e^2}$$

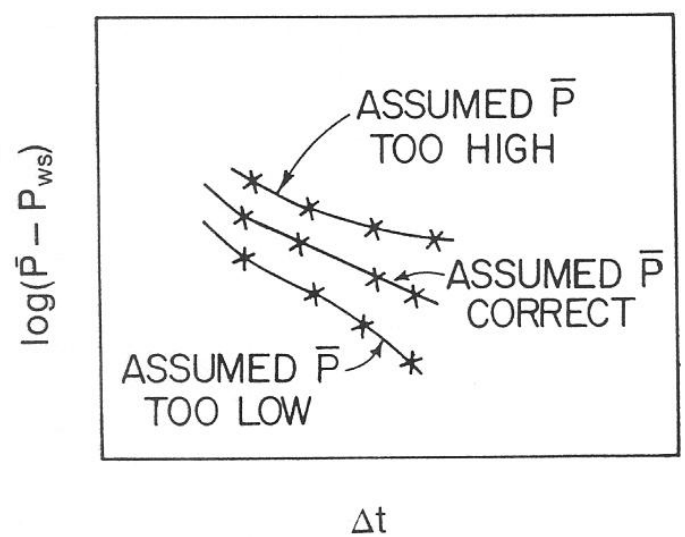



Fig. 2.18 – Schematic graph for modified Muskat method.

Advantages

(1) It requires no estimates of reservoir properties when it is used to establish $\bar{p}$

(2) It has been found to provide satisfactory $\bar{p}$ estimates for hydraulically fractured wells and for wells with layers of different permeability that communicate only at the wellbore.

In these case, the $p^*$ method fails.

Dis advantages

(1) It fails when the tested well is not reasonably centered in its drainage area.

(2) The required shut-in time of $\dfrac{250\mu c_t \phi r_e^2}{k}$ to $\dfrac{750\mu c_t \phi r_e^2}{k}$ are frequently impractically long, particular in low-permeability reservoir.

--- Example 2.7 ---

# Reservoir Limits Test (Ⅰ)

$$--To\quad estimate$$

$$\left.\begin{cases}(1)\quad reservoir\quad size\\(2)\quad dis\tan ce\quad to\quad boundaries\end{cases}\right\}\; based\quad on\quad pressure\;\; buildup$$

$$Superposition \quad principle \quad for \quad pressure \quad drawdown$$

$$(p_i - p_{wf})_{total\ at\ well\ A}$$

$$= (\Delta p)_{due\ to\ well\ A} + (\Delta p)_{due\ to\ well\ I}$$

$$= -70.6\frac{q\mu B}{kh}\left\{Ei\left(\frac{-948\mu c_t \phi r_w^2}{kt}\right) - 2s\right\}$$

$$-70.6\frac{q\mu B}{kh}\left\{Ei\left(\frac{-948\mu c_t \phi (2L)^2}{kt}\right)\right\}$$

$$\approx -70.6\frac{q\mu B}{kh}\left\{\ln\left(\frac{1688\mu c_t \phi r_w^2}{kt}\right) - 2s\right\}$$

$$-70.6\frac{q\mu B}{kh}\left\{\ln\left(\frac{1688\mu c_t \phi (2L)^2}{kt}\right)\right\}$$

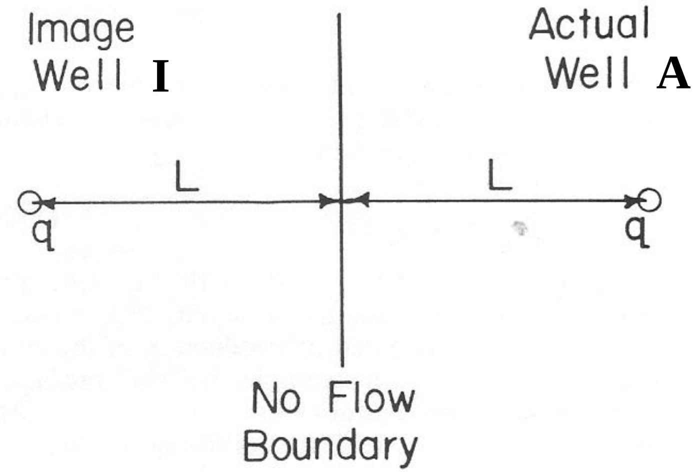



Fig. 1.9—Well near no-flow boundary illustrating use of imaging.

$$Superposition \quad principle \quad for \quad pressure \; buildup$$

$$(p_i - p_{ws})_{total \; at \; well \; A} = (\Delta p)_{sell \; 1} + (\Delta p)_{well \; 2}$$

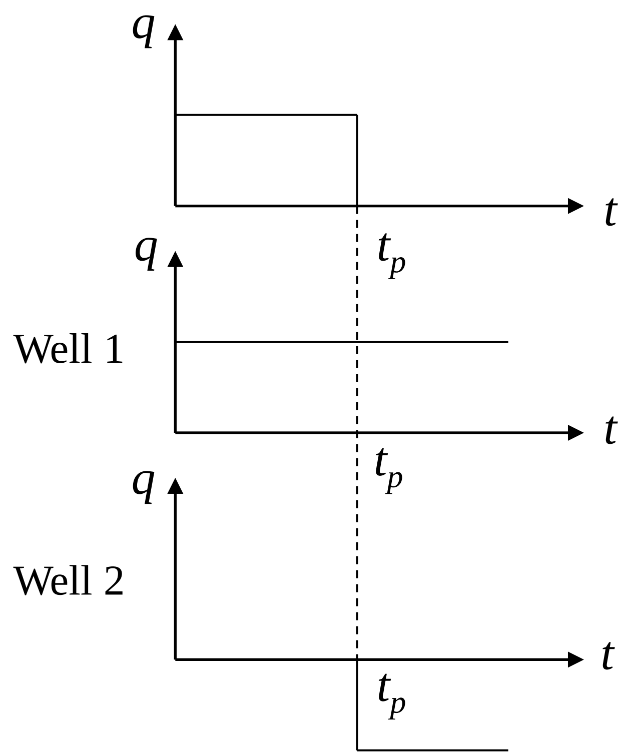
$$\Rightarrow p_i - p_{ws}$$

$$= -70.6\frac{q\mu B}{kh}\left\{\ln(\frac{1688\mu c_t \phi r_w^2}{k(t_p + \Delta t)}) - 2s\right\}$$

$$-70.6\frac{q\mu B}{kh}\left\{Ei(\frac{-3792\mu c_t \phi (L)^2}{k(t_p + \Delta t)})\right\}$$

$$-70.6\frac{(-q)\mu B}{kh}\left\{\ln(\frac{1688\mu c_t \phi r_w^2}{k\Delta t}) - 2s\right\}$$

$$-70.6\frac{(-q)\mu B}{kh}\left\{Ei(\frac{-3792\mu c_t \phi (L)^2}{k\Delta t})\right\} - - - - - - (1)$$



$$\approx -70.6\frac{q\mu B}{kh}\left\{\ln(\frac{1688\mu c_t\phi r_w^2}{k(t_p+\Delta t)})-2s\right\}-70.6\frac{q\mu B}{kh}\left\{\ln(\frac{6537\mu c_t\phi(L)^2}{k(t_p+\Delta t)})\right\}$$

$$+70.6\frac{q\mu B}{kh}\left\{\ln(\frac{1688\mu c_t\phi r_w^2}{k\Delta t})-2s\right\}+70.6\frac{q\mu B}{kh}\left\{\ln(\frac{6753\mu c_t\phi(L)^2}{k\Delta t})\right\}--(2)$$

$$\approx 2\times 70.6\frac{q\mu B}{kh}\ln\left(\frac{t_p+\Delta t}{\Delta t}\right)------(3)$$

$$\Rightarrow p_i-p_{ws}=2\times 162.6\frac{q\mu B}{kh}\log\left(\frac{t_p+\Delta t}{\Delta t}\right)=2\times m\log\left(\frac{t_p+\Delta t}{\Delta t}\right)$$

$$where \quad m=162.6\frac{q\mu B}{kh}$$

$note:(1)Double \quad slope=2m$

$(2)The \quad time \quad required \quad for \quad the \quad slope \quad to \quad double$

$$\frac{3792\mu c_t\phi(L)^2}{k\Delta t}<0.02 \quad or \quad \Delta t>1.9\times 10^5\frac{\mu c_t\phi L^2}{k}$$

# Reservoir Limits Test ( Ⅱ )

$$From \quad Eq \quad (1)$$

$$\Rightarrow p_i - p_{ws} = 70.6\frac{q\mu B}{kh}\ln\left(\frac{t_p+\Delta t}{\Delta t}\right) - 70.6\frac{q\mu B}{kh}Ei\left\{(\frac{-3792\mu c_t\phi L^2}{k(t_p+\Delta t)})\right\}$$

$$-70.6\frac{(-q)\mu B}{kh}Ei\left[(\frac{-3792\mu c_t\phi L^2}{k\Delta t})\right]$$

$$For \quad t_p >> \Delta t \Rightarrow t_p + \Delta t \approx t_p$$

$$\Rightarrow p_i - p_{ws} = 70.6\frac{q\mu B}{kh}\ln\left(\frac{t_p+\Delta t}{\Delta t}\right) - 70.6\frac{q\mu B}{kh}Ei\left\{(\frac{-3792\mu c_t\phi L^2}{k(t_p)})\right\}$$

$$+70.6\frac{q\mu B}{kh}Ei\left[(\frac{-3792\mu c_t\phi L^2}{k\Delta t})\right]$$

$$\Rightarrow p_i - p_{ws} = \left\{162.6\frac{q\mu B}{kh}\log\left(\frac{t_p+\Delta t}{\Delta t}\right) - 70.6\frac{q\mu B}{kh}Ei\left[\frac{-3792\mu c_t\phi L^2}{kt_p}\right]\right\}$$

$$+70.6\frac{q\mu B}{kh}Ei\left[(\frac{-3792\mu c_t\phi L^2}{k\Delta t})\right]------(4)$$

$$\Rightarrow p_i - p_{ws} = \left\{162.6\frac{q\mu B}{kh}\log\left(\frac{t_p+\Delta t}{\Delta t}\right) - 70.6\frac{q\mu B}{kh}Ei\left[\frac{-3792\mu c_t\phi L^2}{kt_p}\right]\right\}$$

$$+70.6\frac{q\mu B}{kh}Ei\left[(\frac{-3792\mu c_t\phi L^2}{k\Delta t})\right] ------- (4)$$

$Note: (1) \quad In \qquad \{\cdots\cdots\},$

$$162.6\frac{q\mu B}{kh}\log(\frac{t_p+\Delta t}{\Delta t}) \quad determine \quad the \quad slope \quad of \quad MTR$$

$$Ei\left\{(\frac{-3792\mu c_t\phi L^2}{k(t_p)})\right\} \quad determine \quad the \quad position \quad of \quad MTR$$

$(2) \quad For \quad \Delta t \to small$

$$Ei\left\{(\frac{-3792\mu c_t\phi L^2}{k(\Delta t)})\right\} \quad is \quad negligible$$

$For \quad \Delta t \to large$

$$Ei\left\{(\frac{-3792\mu c_t\phi L^2}{k(\Delta t)})\right\} \quad can \quad not \quad be \quad negligible$$

# Reservoir Limits Test (Ⅲ)

Use Eq.(4) to analyze pressure buildup test data

(1) Plot $p_{ws}$ *v.s.* $\log \dfrac{t_p + \Delta t}{\Delta t}$

(2) Establish the MTR

(3) Extrapolate the MTR into the LTR

(4) Tabulate the differences, $\Delta p_{ws}^*$, between the buildup curve and extrapolated MTR for several points ($\Delta p_{ws}^* = p_{ws} - p_{MT}$)

(5) Estimate L from the relationship implied by the following equation :

$$\Delta p_{ws}^* = 70.6 \frac{q \mu B}{kh} \left[ - Ei \left( \frac{-3792 \mu c_t \phi L^2}{k \Delta t} \right) \right]$$

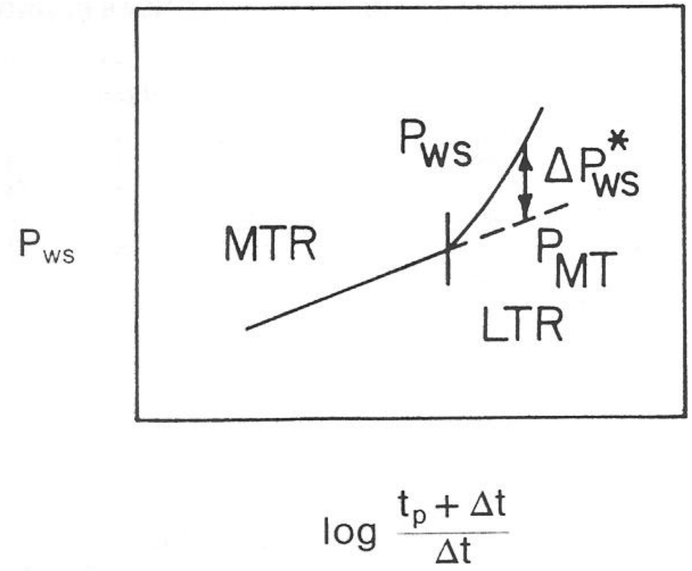



Fig. 2.20 – Buildup test graph for well near reservoir boundary.

Note:

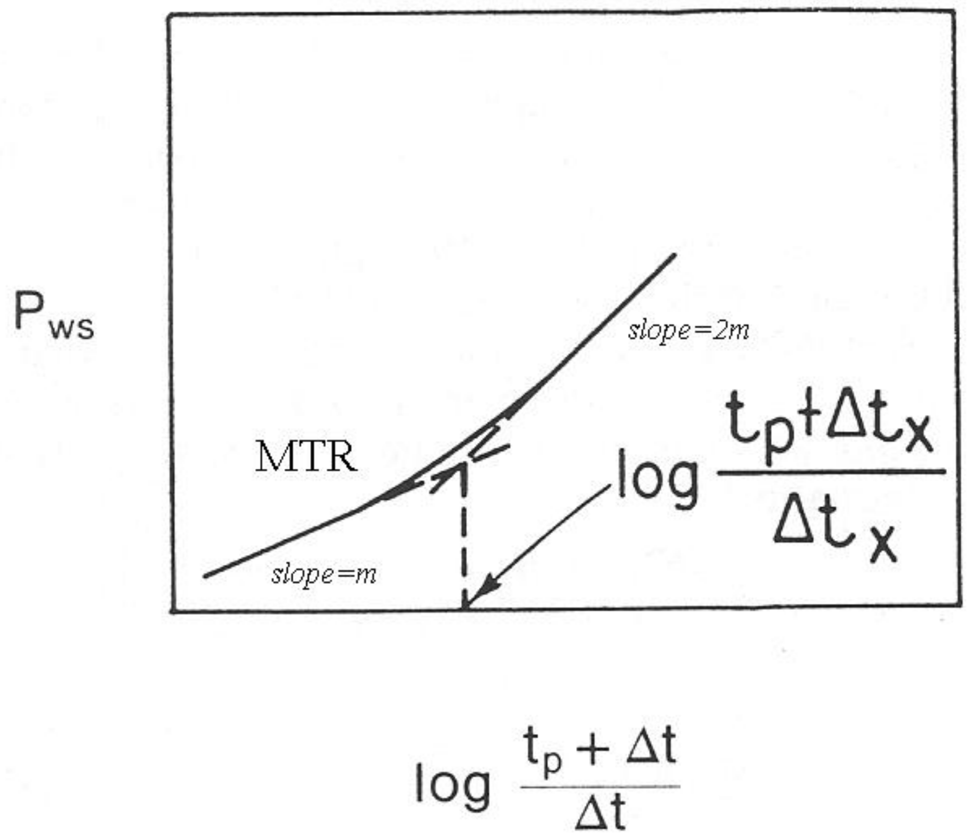
(1) the above equation is for $t_p >> \Delta t$

(2) if the apparent value of L tends to increase or to decrease systematically with time, there is a strong indication that the model does not describe the reservoir adequately.

--- Example 2.8 ---

•Other method to estimate distance from well to boundary (method suggested by Gary)

$$L = \sqrt{\frac{0.000148 k \Delta t_x}{\mu c_t \phi}}$$



# Estimation of Reservoir Size

- The basic idea is to compare average static reservoir pressure before and after production of a known quality of fluid from a closed volumetric reservoir.

$$\Delta N_p B_o = (\overline{p}_1 - \overline{p}_2) c_t V_R \phi$$

$$\Rightarrow V_R = \frac{(\Delta N_p)(B_o)}{(\overline{p}_1 - \overline{p}_2) c_t \phi}$$

$$V_R [=] bbls \qquad \Delta N_p [=] STB$$

$$\overline{p}_1 \;\&\; \overline{p}_2 [=] psi \qquad c_t [=] psi^{-1}$$

$$A \cdot h = V_R$$

$$A = \frac{V_R}{h}$$

Example 2.9 Estimating reservoir size

$Given:$

$\bar{p}_1 = 3000\,psi \qquad \bar{p}_2 = 2100\,psi \qquad q_{avg} = 150\,{}^{STB}\!/_{D} \quad for \quad one \quad year$

$B_o = 1.3\,{}^{RB}\!/_{STB} \qquad c_t = 10\times 10^{-6}\,psi^{-1} \qquad \phi = 0.22 \qquad h = 10\,ft$

$estimate : A_R = ?\,acres$

$solution:$

$$\Delta N_p = qt = 150(STB/D)\times 365(days) = 54750 \quad STB$$

$$V_R = \frac{(\Delta N_p)(B_o)}{(\bar{p}_1 - \bar{p}_2)c_t\phi} = \frac{54750\times 1.3}{(3000-2100)\times 10\times 10^{-6}\times 0.22} = 35.94\times 10^6 \quad bbls$$

$$A = \frac{V_R}{h} = \frac{35.94\times 10^6(bbls)\times 5.61458(ft^3/bbl)}{10} = 20.18\times 10^6 \quad ft^2$$

$$= 20.18\times 10^6(ft^2)/43560(ft^2/acres) = 463 \quad acres$$

# Modifications for Gases

Pressure drawdown

$$p_D = \frac{1}{2}[\ln(t_D) + 0.80907]$$

*oil reservoir*

$$p_i - p_{wf} = -162.6\frac{q\mu B}{kh}\left\{\log(\frac{1688\mu c\phi r_w^2}{kt}) - \frac{s}{1.151}\right\}$$

*where*

$q[=]STB/D \qquad B[=]RB/STB \qquad p[=]psi \qquad \mu[=]cp$

$k[=]md \qquad h[=]ft \qquad r_w[=]ft \qquad t[=]hrs$

$\phi, s[=]\text{dim}\,ensionless \qquad c_t[=]psi^{-1}$

$gas \quad reservoir$

$$(a) \qquad p_i - p_{wf} = -162.6\frac{q_g \mu_{gi} B_{gi}}{kh}\left\{\log\left(\frac{1688\mu_{gi}c_{gi}\phi r_w^2}{kt}\right) - \frac{(s+Dq)}{1.151}\right\}$$

$where$

$q_{gi}[=]Mcf\ /\ D \qquad B_{gi}[=]RB/Mcf \qquad c_{ti} \approx c_{gi}S_g[=]psi^{-1}$

$$B_{gi} = \frac{178.1 z_i T p_{sc}}{\left(\frac{p_i - p_{wf}}{2}\right)T_{sc}} = 10.069\frac{z_i T}{p_i + p_{wf}}$$

$\Rightarrow$

$$(b) \qquad p_i^2 - p_w^2 = -1637\frac{q_g \mu_{gi} z_{gi} T}{kh}\left\{\log\left(\frac{1688\mu_{gi}c_{gi}\phi r_w^2}{kt}\right) - \frac{s'}{1.151}\right\}$$

$where \quad s' = s + Dq$

$Eq.(a) \quad for \quad p > 3000\,psi$

$Eq.(b) \quad for \quad p < 2000\,psi$

Pressure Buildup

Use superposition to develop equation describing a buildup test for gas well

(1) $For \quad p > 3000\,psi \quad based \quad on \quad eq(a)$

$$p_{ws} = p_i - 162.6\frac{q_g B_{gi}\mu_i}{kh}\log\left[\frac{t_p + \Delta t}{\Delta t}\right]$$

$$In \quad p_{ws} \quad vs \quad \log\left[\frac{t_p + \Delta t}{\Delta t}\right] \quad plot$$

$$m = 162.6\frac{q_g B_{gi}\mu_i}{kh} \quad or \quad k = 162.6\frac{q_g B_{gi}\mu_i}{mh}$$

$$s^{'} = s + Dq_g = 1.151\left[\frac{(p_{1hr} - p_{wf})}{m} - \log\left(\frac{k}{\phi\mu_i c_{ti} r_w^2}\right) + 3.23\right]$$

$$(2) \quad For \quad p < 2000\,psi \quad based \quad on \quad eq(b)$$

$$p_{ws}^2 = p_i^2 - 1637\frac{q_g \mu_i z_i T}{kh}\log\left(\frac{t_p + \Delta t}{\Delta t}\right)$$

$$In \quad p_{ws}^2 \quad vs \quad \log\left(\frac{t_p + \Delta t}{\Delta t}\right) \quad plot$$

$$m^{"} = 1637\frac{q_g \mu_i z_i T}{kh} \quad or \quad k = 1637\frac{q_g \mu_i z_i T}{m^{"} h}$$

$$s^{'} = s + Dq_g = 1.151\left[\frac{p_{1hr}^2 - p_{wf}^2}{m^{"}} - \log\left(\frac{k}{\phi \mu_i c_{ti} r_w^2}\right) + 3.23\right]$$

# Modifications for Multiphase Flow

*pressure drawdown*

$$p_{ws} = p_i + 162.6 \frac{q_t}{\lambda_t h}\left[\log\left(\frac{1688 c_t \phi r_w^2}{\lambda_t t}\right) - \frac{s}{1.151}\right]$$

*where*

$$q_t = q_o B_o + (q_g - \frac{q_o R_s}{1000}) B_g + q_w B_w$$

$q_o, q_t [=] STB/D$ $\quad$ $q_g [=] Mcf/D$ $\quad$ $B_o, B_w [=] RB/STB$

$B_g [=] RB/Mcf$ $\quad$ $R_s [=] CF/STB$

$$\lambda = total \quad mobility = \frac{k_o}{\mu_o} + \frac{k_w}{\mu_w} + \frac{k_g}{\mu_g} \qquad c_t = S_o c_o + S_w c_w + S_g c_g + c_f$$

*pressure drawdown*

$$p_{ws} = p_i - 162.6\frac{q_t}{\lambda_t h}\log\left[\frac{t_p + \Delta t}{\Delta t}\right]$$

*In the plot of* $p_{ws}$ *vs.* $\log\left[\frac{t_p + \Delta t}{\Delta t}\right]$

$$m = 162.6\frac{q_t}{\lambda_t h} \quad \Rightarrow \quad \lambda_t = 162.6\frac{q_t}{mh}$$

$$k_o = 162.6\frac{q_o B_o \mu_o}{mh}$$

$$k_g = 162.6\frac{(q_g - \frac{q_o R_s}{1000})\mu_g B_g}{mh}$$

$$k_w = 162.6\frac{q_w \mu_w B_w}{mh}$$

$$s = 1.151\left[\frac{p_{1hr} - p_{wf}}{m} - \log\left(\frac{\lambda_t}{\phi c_t r_w^2}\right) + 3.23\right]$$

$\overline{p}$ *is calculated just as for a single-phase reservoir*