Groundwater hydraulics

3

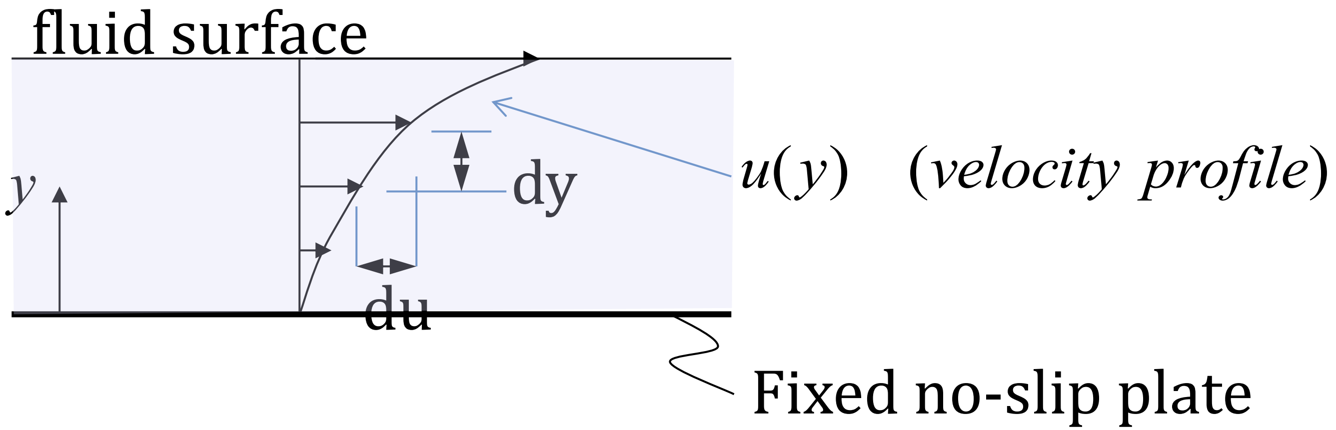
VISCOSITY – SHEAR FORCE

Viscosity is a **measure** of the resistance of a fluid to **deform under shear stress.**

SHEAR STRESS due to viscosity between layers: $\tau = \mu \dfrac{du}{dy}$

μ - **dynamic viscosity** (coeff. of viscosity)

$\nu = \dfrac{\mu}{\rho}$ - **kinematic viscosity**



Use definition of
SHEAR FORCE:

$$F = \tau A = \mu A \frac{du}{dy}$$

Dynamic viscosities of some fluids at 1 atm and 20°C (unless otherwise stated)

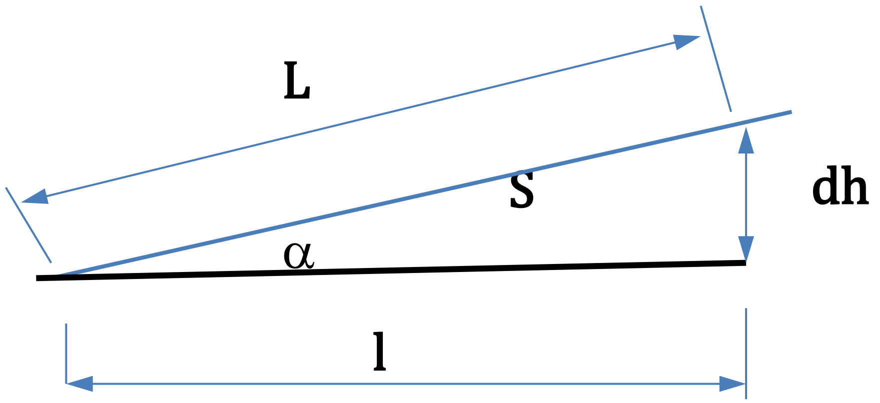
Fluid	Dynamic Viscosity μ, kg/m · s
Glycerin:	
−20°C	134.0
0°C	10.5
20°C	1.52
40°C	0.31
Engine oil:	
SAE 10W	0.10
SAE 10W30	0.17
SAE 30	0.29
SAE 50	0.86
Mercury	0.0015
Ethyl alcohol	0.0012
Water:	
0°C	0.0018
20°C	0.0010
100°C (liquid)	0.00028
100°C (vapor)	0.000012
Blood, 37°C	0.00040
Gasoline	0.00029
Ammonia	0.00015
Air	0.000018
Hydrogen, 0°C	0.0000088

Cengel_Cimbala, 2006

CHARACTERISTICS OF HYDRODYNAMICS

flow area, CROSS SECTIONAL AREA (perpendicular to velocity, v) A(m^2)

Slope - S



α	sin(α)	tan(α)
0°	0	0
5°	0.087	0.087
10°	0.174	0.176
20°	0.342	0.346
30°	0.500	0.577
40°	0.643	0.839
50°	0.766	1.192

$$S = \frac{dh}{L} \Rightarrow \frac{dh}{l}$$

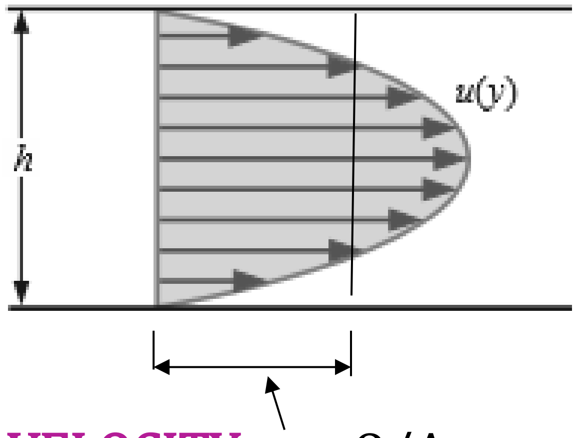
For small α (cca 8-10°)

$$\sin\alpha \approx tg\,\alpha$$

CHARACTERISTIC OF HYDRODYNAMICS

POINT VELOCITY $u = \frac{ds}{dt}$

THE AVERAGE (MEAN) VELOCITY - v - is defined as the average speed through a cross section.



elementary volume **discharge** $dQ = u\,dA$

$$v = \frac{1}{A}\int_S u.dA = \frac{Q}{A}$$

MEAN VELOCITY v=Q/A

DISCHARGE (mass) $= \rho \,.\, v \,.\, A$

MASS RATE PAST A CROSS-SECTION: Q_m (kg/s)

DISCHARGE (volume) $= v \,.\, A = Q$

VOLUME FLOW RATE PAST A CROSS- SECTION: Q (m^3/s)

KINDS AND FORMS OF FLOW

A. - **UNSTEADY FLOW** ….. Q = Q(x,y,z,t), v= v(x,y,z,t) $\frac{\partial Q}{\partial t} \neq 0$ $\frac{\partial Q}{\partial x_i} \neq 0$ $\frac{\partial v}{\partial t} \neq 0$ $\frac{\partial v}{\partial x_i} \neq 0$

- **STEADY FLOW** ….. Q = const. $\frac{\partial Q}{\partial t} = 0$ $\frac{\partial Q}{\partial x_i} = 0$

  a) **UNIFORM** flow … $\frac{\partial v}{\partial t} = 0$ $\frac{\partial v}{\partial x_i} = 0$ $\boldsymbol{A = const. \quad v = const.}$

  b) **NON – UNIFORM** flow $\frac{\partial v}{\partial t} = 0$ $\frac{\partial v}{\partial x_i} \neq 0$ $\boldsymbol{A \neq const. \quad v \neq const.}$

B. - **WITH FREE LEVEL** – flow limited by solid walls, free level on surface, motion caused by own weight of liquid

- **PRESSURE** – flow limited by solid walls from all sides, motion caused by difference of pressures

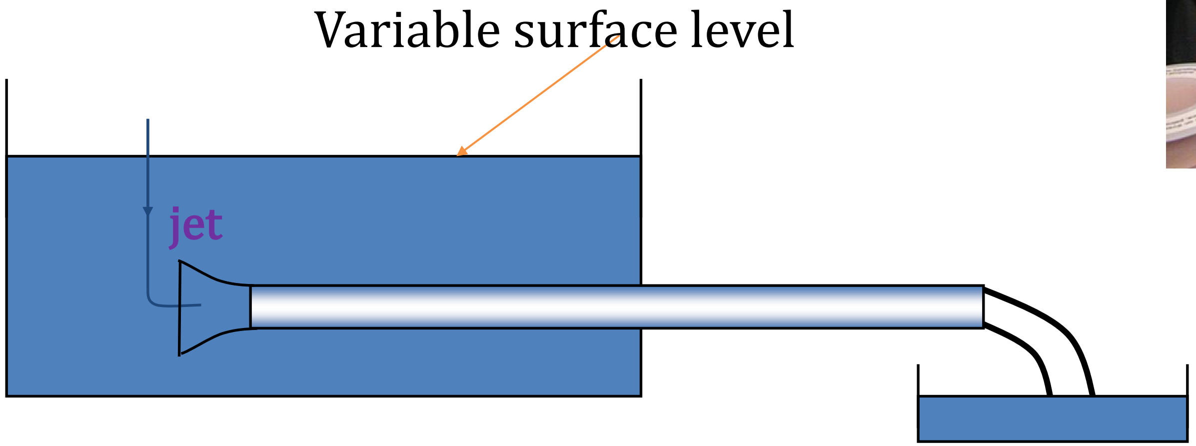
C. - **LAMINAR** flow
- **TURBULENT** flow

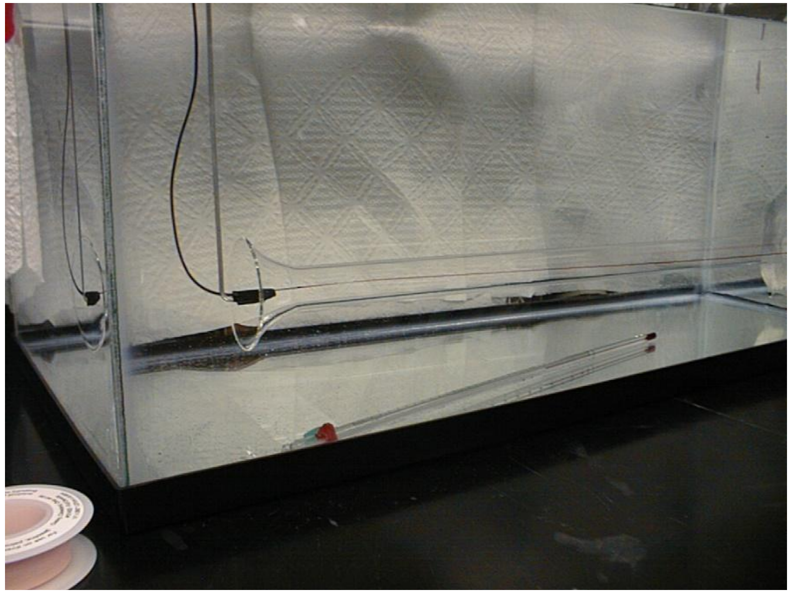
REAL FLUID

“A fluid in which there is ***friction*** *i.e* **viscosity**.”

LAMINAR AND TURBULENT FLOW

Reynolds experiment **1883:**





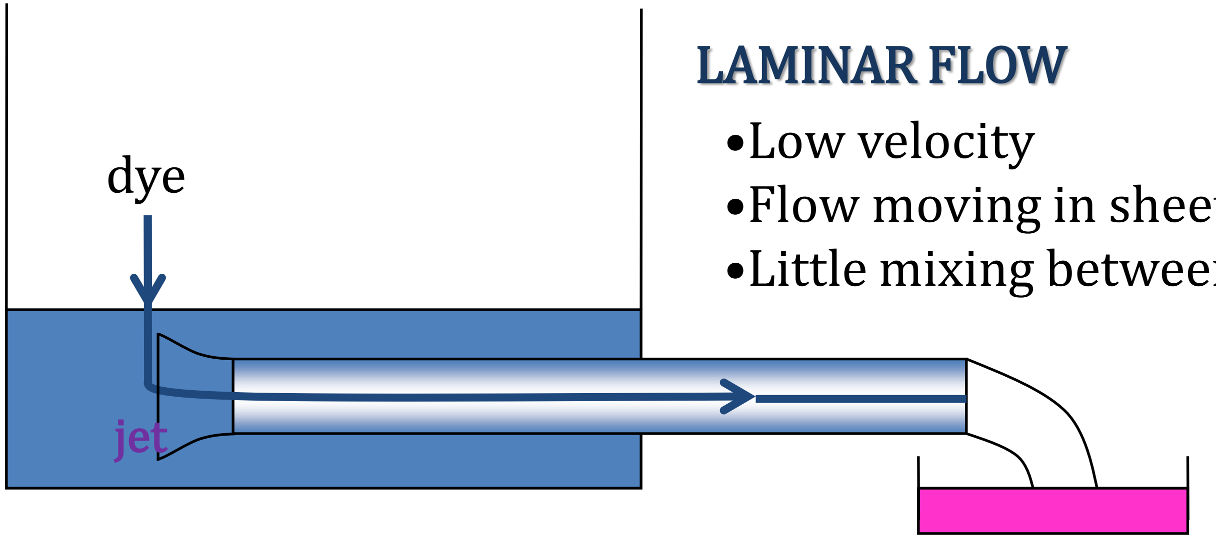
Two different, distinct **flow regimes**:

A) **LAMINAR FLOW**

B) **TURBULENT FLOW**

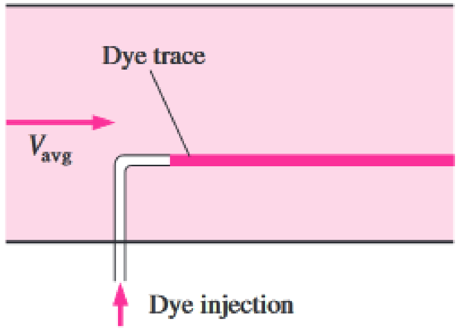
Osborne Reynolds (1842-1912)

REYNOLDS EXPERIMENT 1883

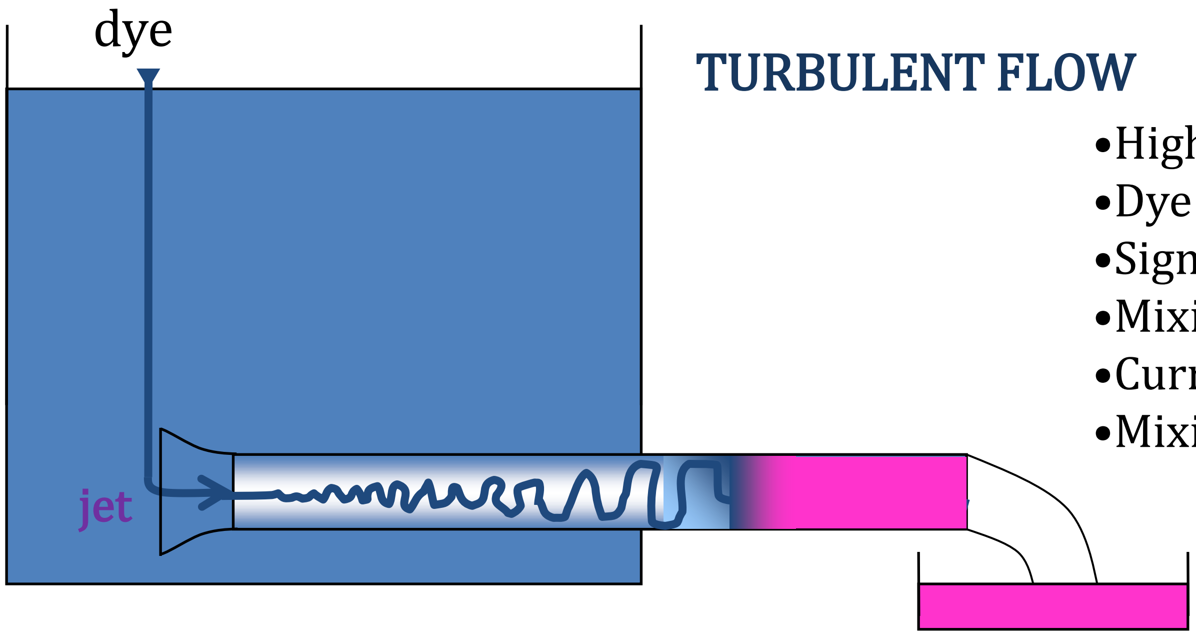


LAMINAR FLOW

- Low velocity
- Flow moving in sheets
- Little mixing between sheets

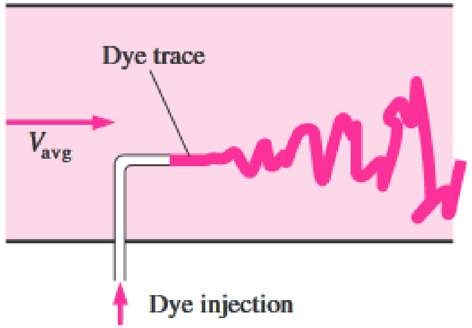


(*a*) Laminar flow



TURBULENT FLOW

- High velocity
- Dye breaks up in a diffuse cloud
- Significant mixing
- Mixing increases with velocity
- Currents perpendicular to pipe
- Mixing is initiated by roughness on the flow boundaries



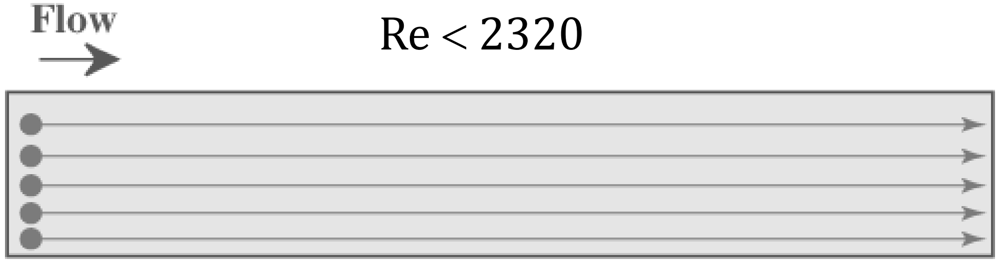
(*b*) Turbulent flow

REYNOLDS CLASSIFIED THE FLOW TYPE ACCORDING TO THE MOTION OF THE FLUID.

$$Reynolds\ number\ for\ pipe\ \ \boldsymbol{Re} = \frac{\boldsymbol{v\,D}}{\upsilon}$$

$Re_{CR} = 2320$

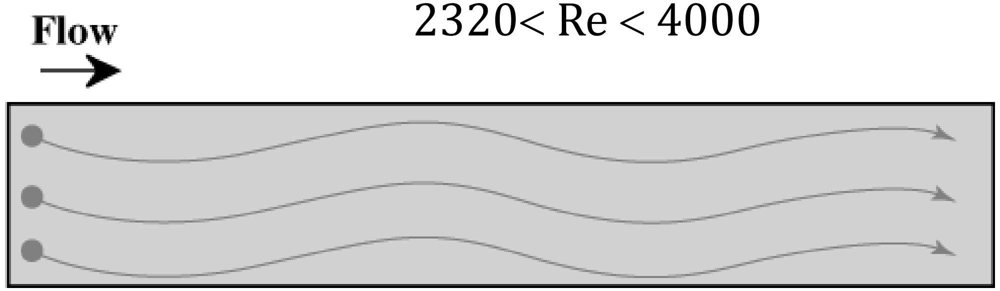
$Re < 2320$



Low discharge

LAMINAR FLOW: every fluid molecule followed a straight path that was parallel to the boundaries of the tube.

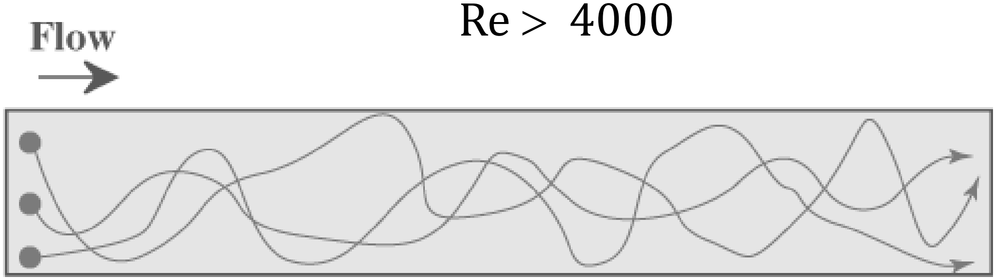
$2320 < Re < 4000$



Medium discharge

TRANSITIONAL FLOW: every fluid molecule followed wavy but parallel path that was not parallel to the boundaries of the tube.

$Re > 4000$



High discharge

TURBULENT FLOW: every fluid molecule followed very complex path that led to a mixing of the dye.

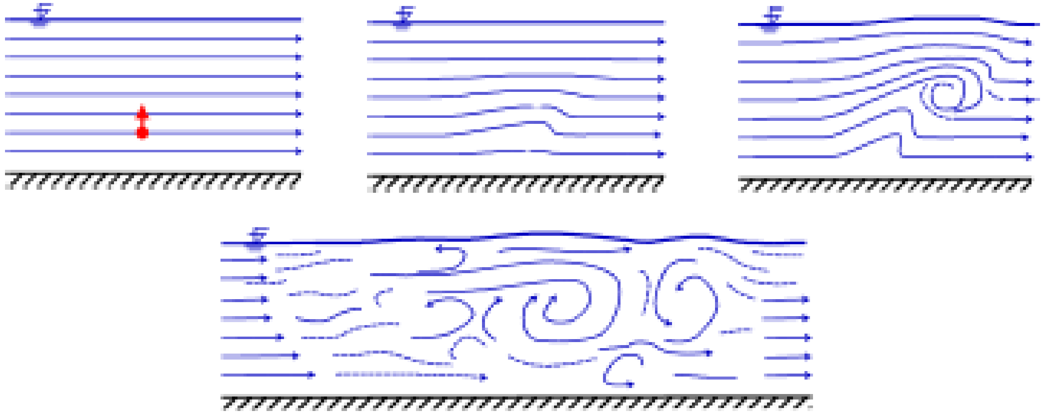
LAMINAR AND TURBULENT FLOW

- **laminar – particles of liquid move at parallel paths**
- **turbulent – motion of particles of liquid: irregular and**

inordinate, fluctuations of velocity vector in time and space, mixing inside flow

- Criterion – **Reynolds number**

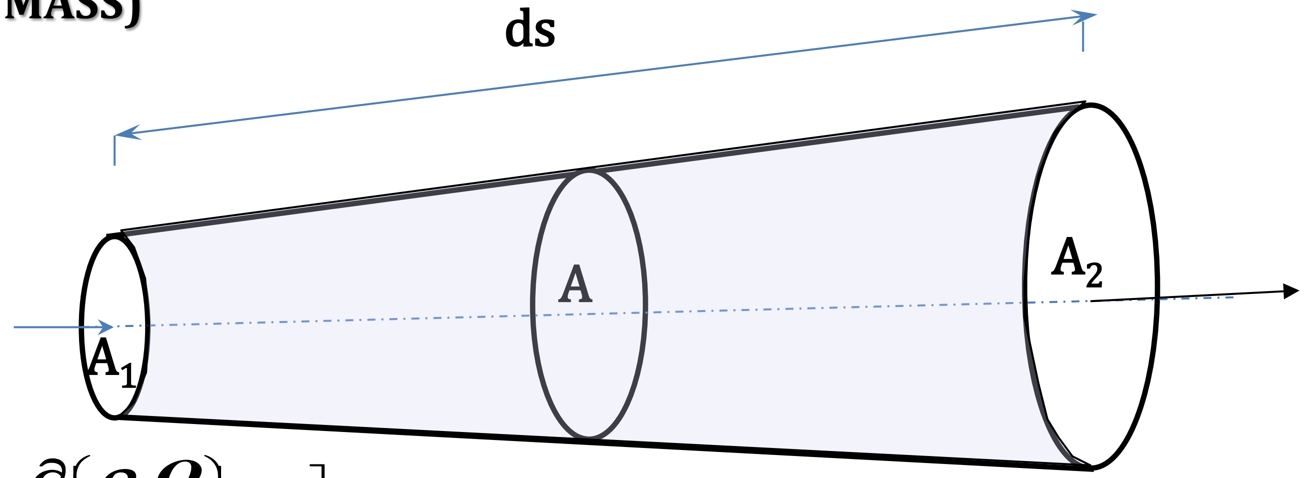
L – characteristic length:
diameter D for pipelines, hydraulic radius R
Critical Reynolds Number - for pipe $Re_{cr} = 2320$
for open channel $Re_{cr} = 580$
for groundwater flow $Re_{cr} = 1$



CONTINUITY EQUATION

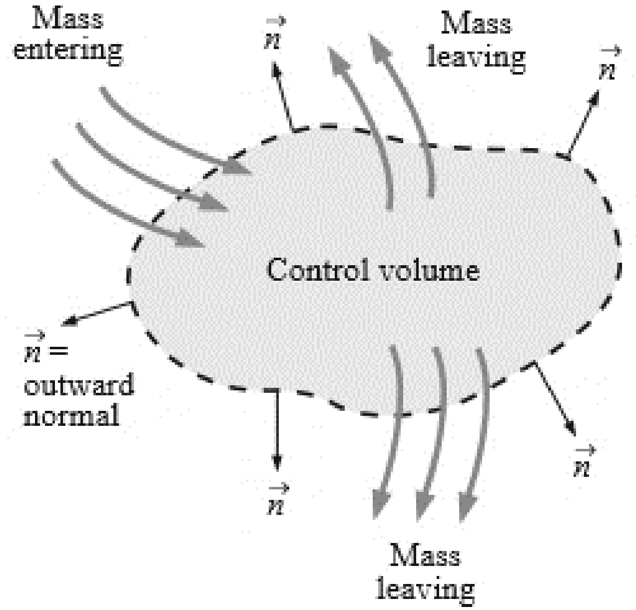
mass leaving - mass entering = - rate of increase of mass in cv

(LAW OF CONSERVATION OF MASS)



Input mass A_1 : $\rho.Q.dt$

Output mass A_2 : $\left[(\rho.Q) + \frac{\partial(\rho.Q)}{\partial s} ds \right] dt$



Change of mass inside V in time dt $\frac{\partial(\rho.A.ds)}{\partial t} dt$

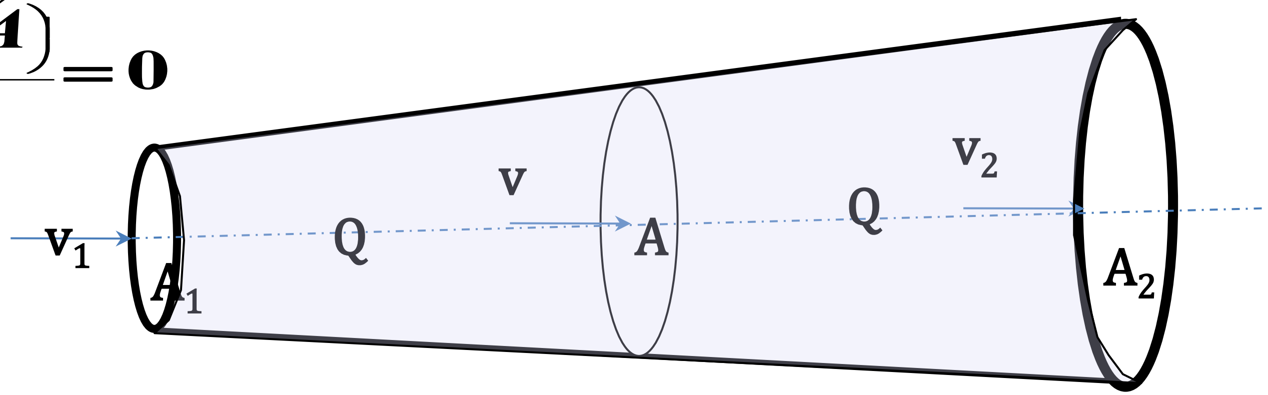
CONTINUITY EQUATION FOR UNSTEADY FLOW

$$\frac{\partial(\rho.Q)}{\partial s} + \frac{\partial(\rho.A)}{\partial t} = 0$$

CONTINUITY EQUATION - STEADY FLOW

$$\frac{\partial(\rho.Q)}{\partial s}+\cancel{\frac{\partial(\rho.A)}{\partial t}}=0$$

$$\frac{\partial(\rho.Q)}{\partial s}=0$$



steady flow compressible liquid – no dependency on time

$$\rho.Q=const.$$

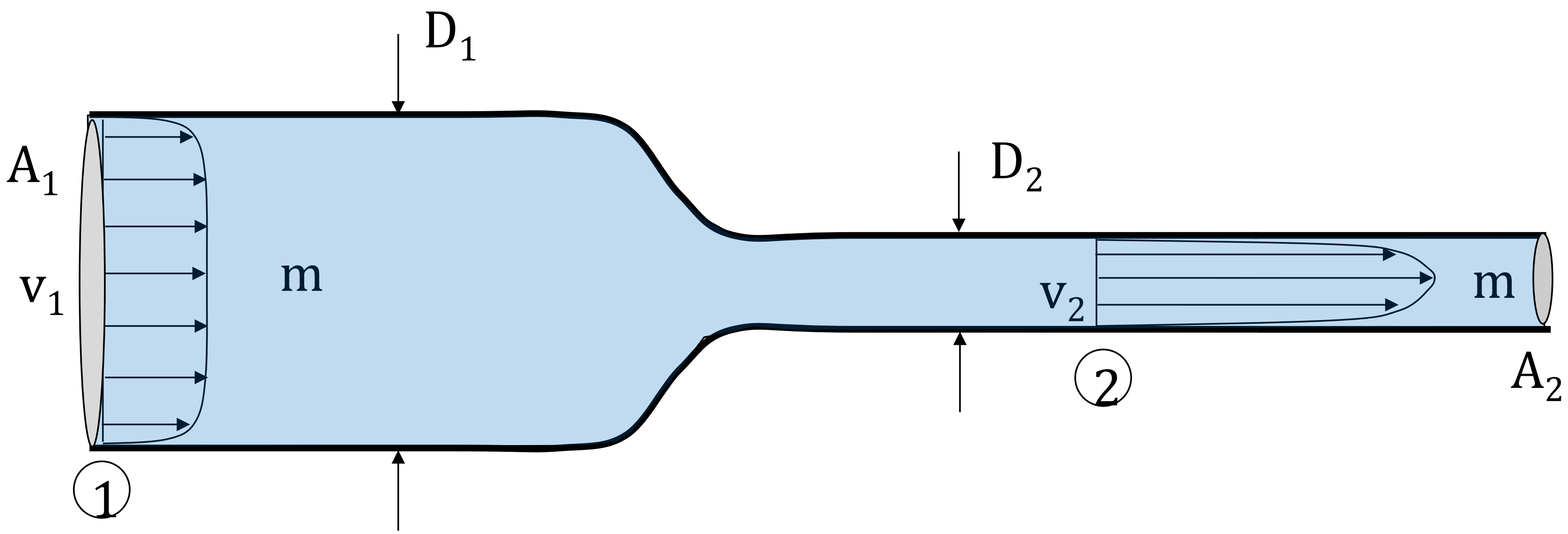
$$Q=\rho_1.A_1.v_1=\rho_2.A_2.v_2=\rho_i.A_i.v_i=\text{konst}$$

STEADY FLOW of incompressible liquid

$Q = const.$ $\rho = \text{const.}$

$$A_1.v_1=A_2.v_2=\text{konst.}=Q_v$$

- For pipes with variable diameter, $\dot{m}$ is still the same due to conservation of mass, but $v_1 \neq v_2$



$$Q = A_1.v_1 = A_2.v_2 = \text{konst.}$$

BERNOULLI EQ. FOR IDEAL FLUID

(LAW OF CONSERVATION OF ENERGY)

BERNOULLI EQUATION FOR IDEAL FLUID (ENERGY CONSERVATION)

expresses the **principle of conservation of energy**

The Bernoulli Equation is a statement of the **conservation** of **mechanical energy**

$$\underbrace{h+\frac{p}{\rho g}}_{\text{pot. e.}}+\underbrace{\frac{v^2}{2g}}_{\text{kinet.e.}}$$

$$h+\frac{p}{\rho g}+\frac{v^2}{2g}=Const.=ME$$

THE DERIVATION OF BERNOULLI EQUATION (ENERGY CONSERVATION)

$\frac{p}{\rho g}$ = **PRESSURE HEAD**

h = **ELEVATION (GEODETIC)HEAD**

$\frac{v^2}{2g}$ = **VELOCITY HEAD**

$z+\frac{p}{\rho g}=$ Piezometric head

$h+\frac{p}{\rho g}=$ **HYDRAULIC GRADE LINE** – **HGL** or **PRESSURE GRADE LINE** – **PGL**“

$h+\frac{p}{\rho g}+\frac{v^2}{2g}=$ **Total head** - **ENERGY GRADE LINE** - EGL

Each term in the BE is called **„head“**

BERNOULLI EQUATION FOR IDEAL FLUID

$$h+\frac{p}{\rho g}+\frac{v^2}{2g} = \text{const.} = E$$

PGL

EGL

PGL – pressure grade line

EGL – energy grade line

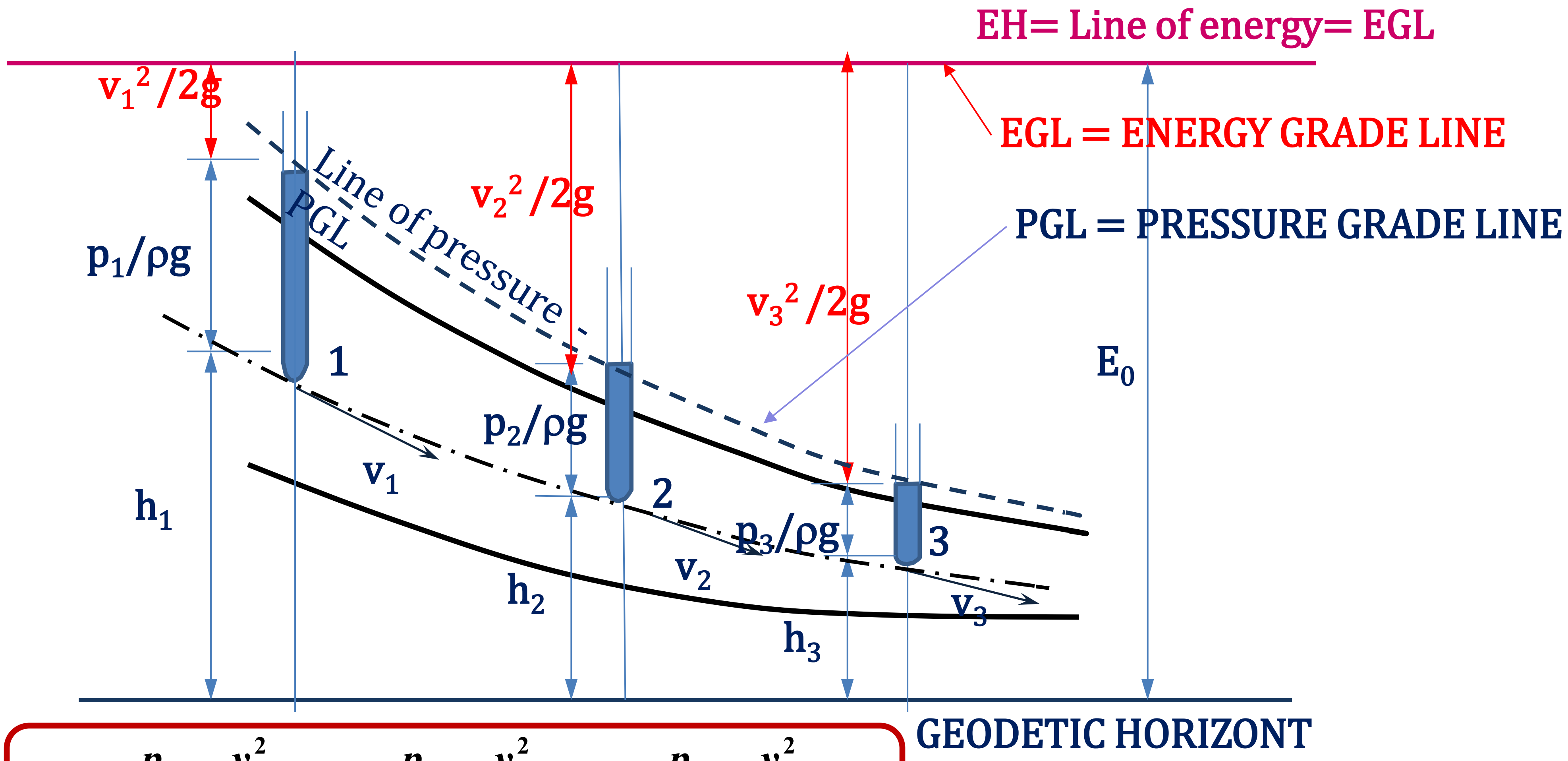
EH – energy horizont

Pressure head $(p/\rho g)$!!!! $p = p_{out} + \rho g z$!!!

PGL = (pressure head) + (elevation head)

EGL = (elevation head) + (pressure head)+ (velocity head)

BERNOULLI EQ. FOR IDEAL FLUID



$$h_1 + \frac{p_1}{\rho g} + \frac{v_1^2}{2g} = h_2 + \frac{p_2}{\rho g} + \frac{v_2^2}{2g} = h_3 + \frac{p_3}{\rho g} + \frac{v_3^2}{2g} = E_0$$

h – elevation (geodetic) head **p/ρg** - pressure head $v^2/2g$ - velocity head

IDEAL FLUID

HYDRAULIC CALCULATIONS OF PIPELINES

2 kinds of equations:

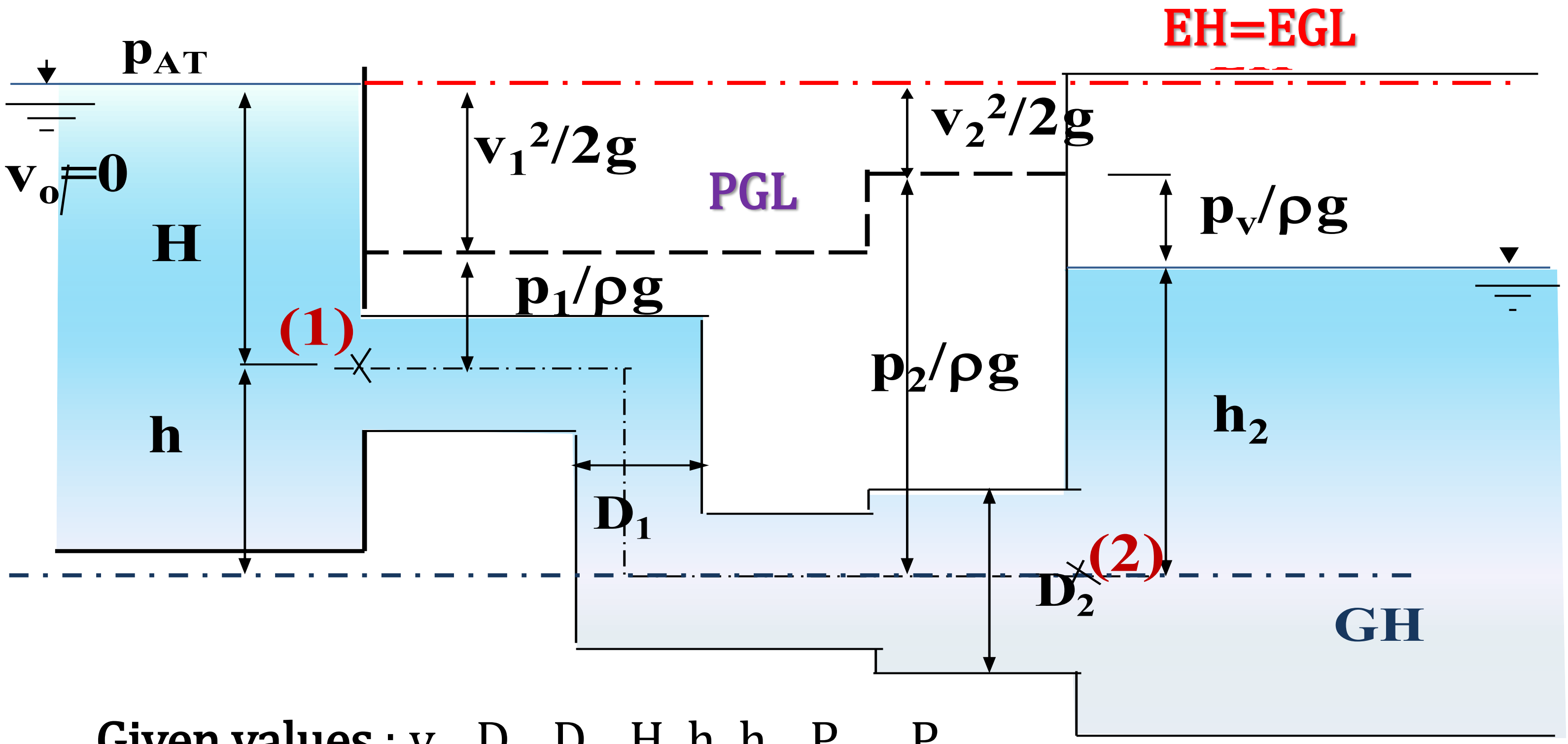
Bernoulli equation ← elevations and pressure relations,

Continuity equation - boundary conditions

calculation: **Q, v, D, L, H, p**

PGL, EGL

BERNOULLI EQUATION FOR IDEAL FLUID



Procedure:

1. Choose **GH**
2. Choose **(1)** and **(2)**
3. **BE** for **(1)** and **(2)**
4. **Continuity eq**.
5. Calculation $\mathbf{v_i}$ and **Q**
6. Graph of **EGL** and **PGL**

Given values : v_o, D_1, D_2, H, h, h_2, P_{AT} , P_V

?: **Q, v_1, v_2**

EGL, **PGL**

BE – (1) **BE – (2)**

$$h + H + \frac{p_{AT}}{\rho\ g} + \frac{v_o^2}{2\ g} = h_2 + \frac{p_V}{\rho\ g} + \frac{v_2^2}{2\ g}$$

$$v_2 = \sqrt{2g.\left[h + H + \frac{p_{AT}}{\rho\ g} + \frac{v_o^2}{2\ g} - \left(h_2 + \frac{p_V}{\rho\ g}\right)\right]}$$

Discharge: $\mathbf{Q = v_2 . A_2 = v_1 . A_1}$

$$A_2 = \frac{\pi D_2^2}{4}$$

BERNOULLI EQ. FOR REAL FLUID

REAL FLUID

HYDRAULIC CALCULATIONS OF PIPELINES

3 kinds of equations:

Bernoulli equation ← elevations and pressure relations,

Continuity equation boundary conditions

Equations of losses ← geometry and roughness of pipe, discharge

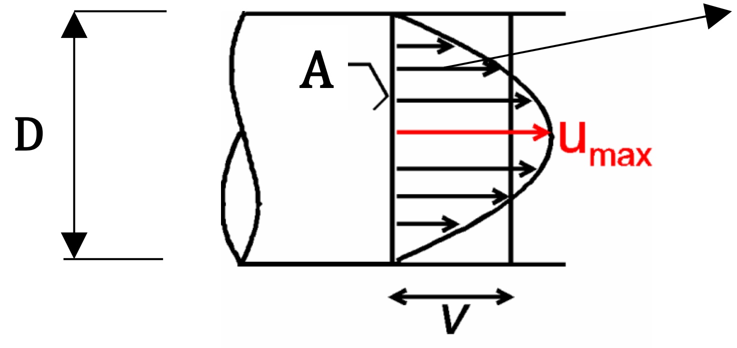
calculation: **Q, v, D, L, H, p, Z**

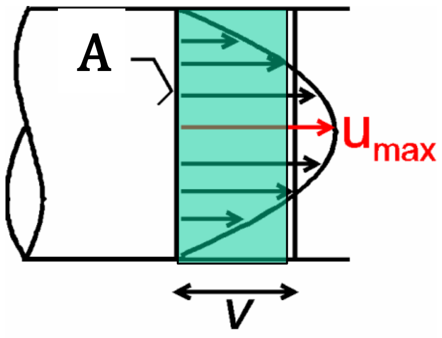
CORIOLIS NUMBER - α

point velocity u

Transfer to average (mean) velocity v

rectangle with one side D

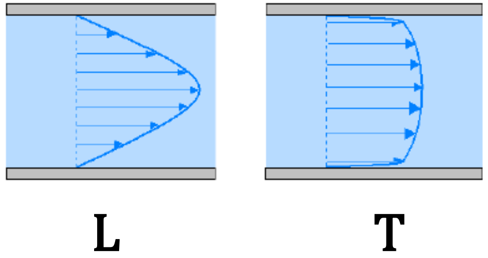




in technical calculations – kinetic energy head is expressed from **mean velocity** v

$$\frac{\alpha v^2}{2g}$$

α - coefficient of kinetic energy - **Coriolis number** depends on the shape of cross section and on form of velocity profile



circular pipelines and regular channels α = 1,05 ¸ 1,2,
LAMINAR FLOW α = 2,

current technical calculations of pipelines (**TURBULENT FLOW**) α **....1,0**

REAL FLUID

- Piping systems include fittings, valves, bends, elbows, tees, inlets, exits, enlargements, and contractions.
- These components interrupt the smooth flow of fluid and cause **additional losses** because of flow separation and mixing
- We introduce a relation for the **minor losses** associated with these components

$$h_L = K_l \frac{v^2}{2g}$$

- K_L is the loss coefficient.
- Is different for each component.
- Is assumed to be independent of Re.

CALCULATION OF HEAD (ENERGY) LOSSES:

In General:

When a fluid is flowing through a pipe, the fluid experiences some resistance due to which some of energy (head) of fluid is lost.

Energy Losses (Head losses)

- Major Losses
- Minor losses

Major Losses: loss of head **due to pipe friction** and to viscous dissipation in flowing water

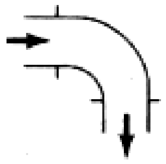
$$Darcy - Weisbach\ equation \Rightarrow \quad h_F = f\frac{L}{D}\frac{v^2}{2g}$$

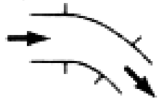
Minor losses: Loss due to the **change of the velocity** of the flowing fluid in the **magnitude** or in **direction** as it moves through fitting like Valves, Tees, Bends and Reducers.

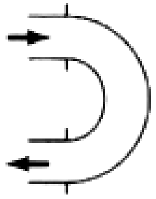
$$h_L = K_l\frac{v^2}{2g}$$

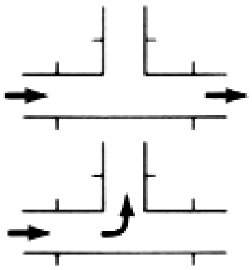
MINOR LOSSES

Component	K_L
Elbows	
Regular 90°, flanged	0.3
Regular 90°, threaded	1.5
Long radius 90°, flanged	0.2
Long radius 90°, threaded	0.7
Long radius 45°, flanged	0.2
Regular 45°, threaded	0.4
180° return bends	
180° return bend, threaded	0.2
180° return bend, flanged	1.5
Tees	
Line flow, flanged	0.2
Line flow, threaded	0.9
Branch flow, flanged	1.0
Branch flow, threaded	2.0

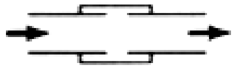








Component	K_L
Union, threaded	0.8
Valves	
Globe, fully open	10
Angle, fully open	2
Gate, fully open	0.15
Gate, ¼ closed	0.26
Gate, ½ closed	2.1
Gate, ¾ closed	17
Ball valve, fully open	0.05
Ball valve, 1/3 closed	5.5
Ball valve, 2/3 closed	210



Source: Munson et al. (1998)

BERNOULLI EQ. FOR REAL FLUID

$$h + \frac{p_1}{\rho\ g} + \frac{\alpha v_1^2}{2\ g} = h + \frac{p_2}{\rho\ g} + \frac{\alpha v_2^2}{2\ g} + \sum_{i=1}^{2}(h_{zmi} + h_{zti})$$

HEAD LOSS

$$h_L = h_{LF,major} + h_{LM,minor}$$

If the piping system has constant diameter

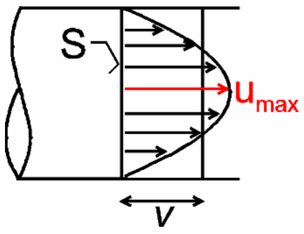
FRICTION FACTOR f

$$h_L = \left(f\ \frac{L}{D} + \sum K_L\right)\frac{v^2}{2g}$$

CORIOLIS NUMBER - α

point velocity u

average velocity v



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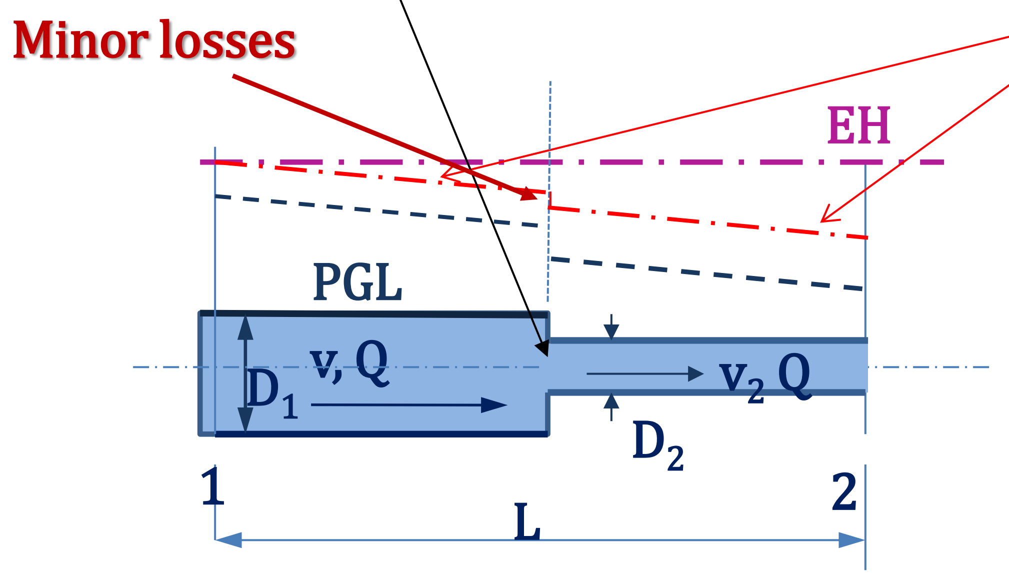
circular pipelines and regular channels α = 1,05 ¸ 1,2,
LAMINAR FLOW α = 2,

current technical calculations of pipelines (TURBULENT FLOW) α 1,0

LOCAL (MINOR) LOSSES IN PIPELINES



$$i_E = \frac{h_{LF}}{L} \Rightarrow h_{LF} = i_E . L$$

MINOR LOSSES

$$h_{lM} = \boxed{K_{lM}} \frac{v^2}{2\,g}$$

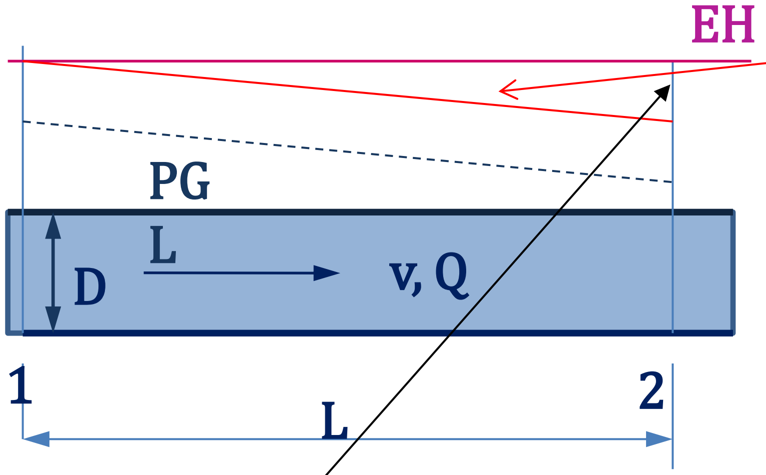
Coef. for minor loss

Reynolds number

$$Re = \frac{v . D}{\upsilon}$$

FRICTION (MAJOR) LOSSES IN PIPELINES

MAJOR LOSSES



i_E Slope of EGL

$$i_E = \frac{h_{lt}}{L} \Rightarrow h_{lt} = i_E . L$$

f – friction coefficient

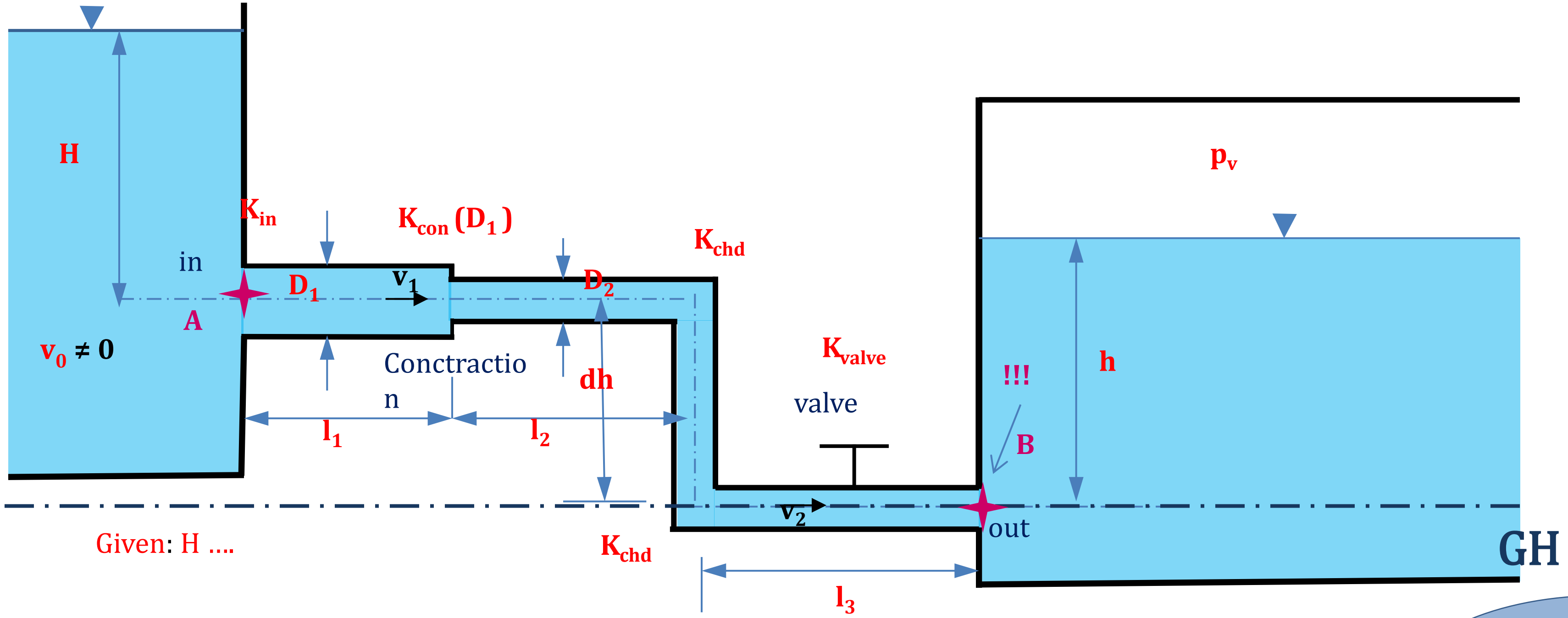
Darcy – Weisbach equation

$$h_{lf} = f \frac{L}{D} \frac{v^2}{2g} \quad (m)$$

Reynolds number $\mathbf{Re} = \frac{v.D}{\upsilon}$

TWO RESERVOIRS ARE CONNECTED BY A PIPE

??? -Q, v_1, v_2, TČ.ČE



Procedure:

1. Choose GH.
2. Choose (A) and (B).
3. BE for (A) and (B).
4. Divide into sections.
5. Express losses
6. Calculation v_i and Q
7. EGL and PGL

BERNOULLI EQ. for (A) a (B)

$$dh + H + \frac{p_{at}}{\rho g} + \frac{v_0^2}{2g} = h + \frac{p_v}{\rho g} + \frac{v_2^2}{2g} + \sum_{i=1}^{k} h_{LMi} + \sum_{i=1}^{l} h_{LFi}$$

Sections of pipe

1. sec $$\left(\boldsymbol{K_{inlet}} + \boldsymbol{K_{change}} + \boldsymbol{f_1}\frac{\boldsymbol{l_1}}{\boldsymbol{D_1}}\right)\frac{v_1^2}{2g} = \boldsymbol{n1}\frac{v_1^2}{2g}$$

2. sec $$\left(\boldsymbol{2.K_{ch.\_of\_dir}} + \boldsymbol{K_{ch\_of\_D}} + \boldsymbol{f_2}\frac{\boldsymbol{(l_2+dh+l_3)}}{\boldsymbol{D_2}}\right)\frac{v_2^2}{2g} = \boldsymbol{n2}\frac{v_2^2}{2g}$$

Continuity eq.

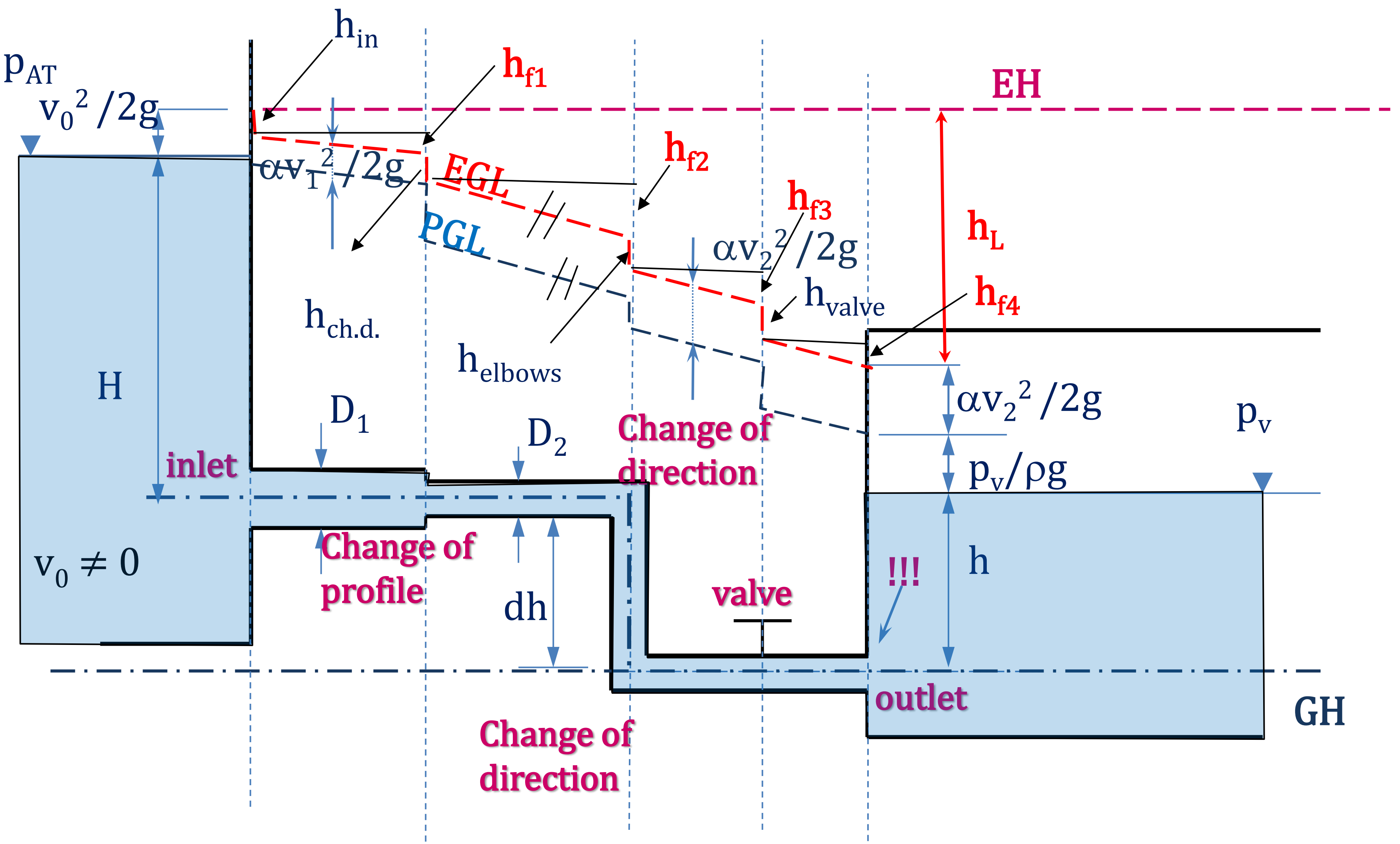
$$Q = v_1.S_1 = v_2.S_2 \Rightarrow v_2 = v_1\frac{S_1}{S_2}$$

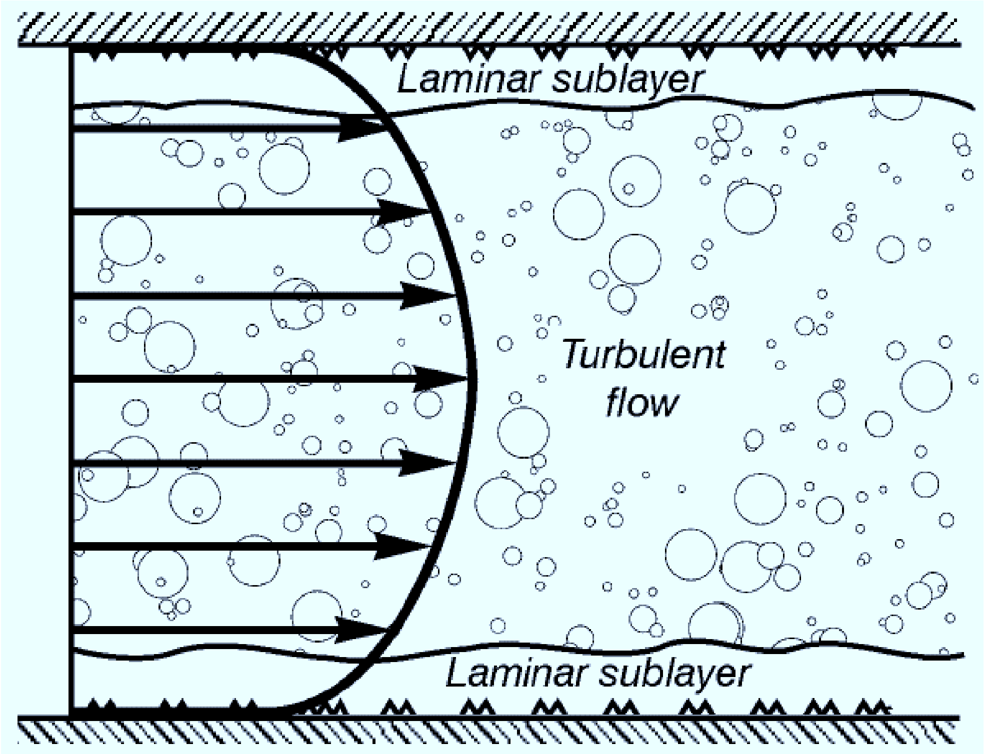
BERNOULLI EQ.

$$dh + H + \frac{p_{at}}{\rho g} + \frac{v_0^2}{2g} = h + \frac{p_v}{\rho g} + \frac{v_2^2}{2g} + \boldsymbol{n1}\frac{\boldsymbol{v_1^2}}{2g} + \boldsymbol{n2}\frac{\boldsymbol{v_2^2}}{2g}$$

$Unkonown: \boldsymbol{Q}; \boldsymbol{v_1}; \boldsymbol{v_2};$

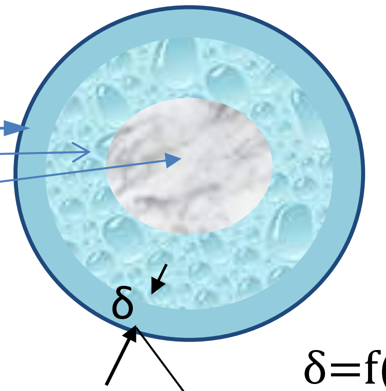
Draw EGL and PGL




Laminar sublayer
Turbulent
flow
Laminar sublayer

TURBULENT FLOW

a) **Viscous sublayer** -laminar flow($\tau = \tau_L ; \tau_T = 0$)
b) **Overlap layer**
c) **Turbulent layer** -turbulent flow ($\tau = \tau_T ; \tau_L = 0$)

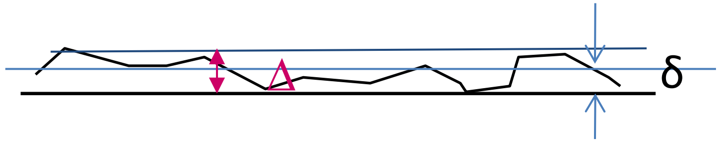


δ=f(D,Re, f)

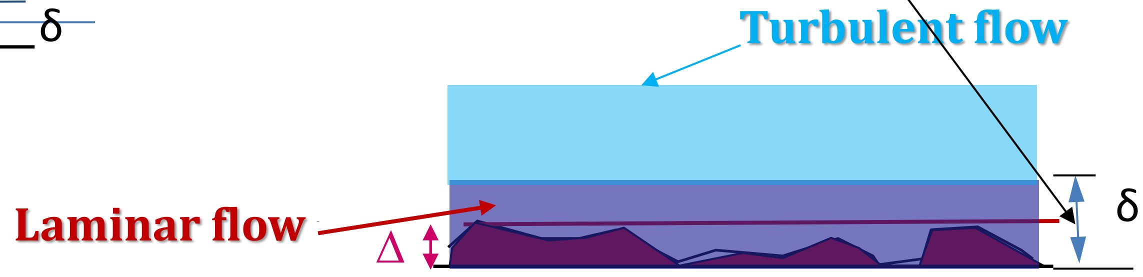
Thickness of the **viscous sublayer** $$\delta = 33{,}4 \frac{D}{\mathrm{Re}\, f^{1/2}}$$

Thickness of **the viscous sublayer** depends on **D**, **Re** and **f**:

Roughness of pipe wall
1) **Absolute roughness (Δ)**

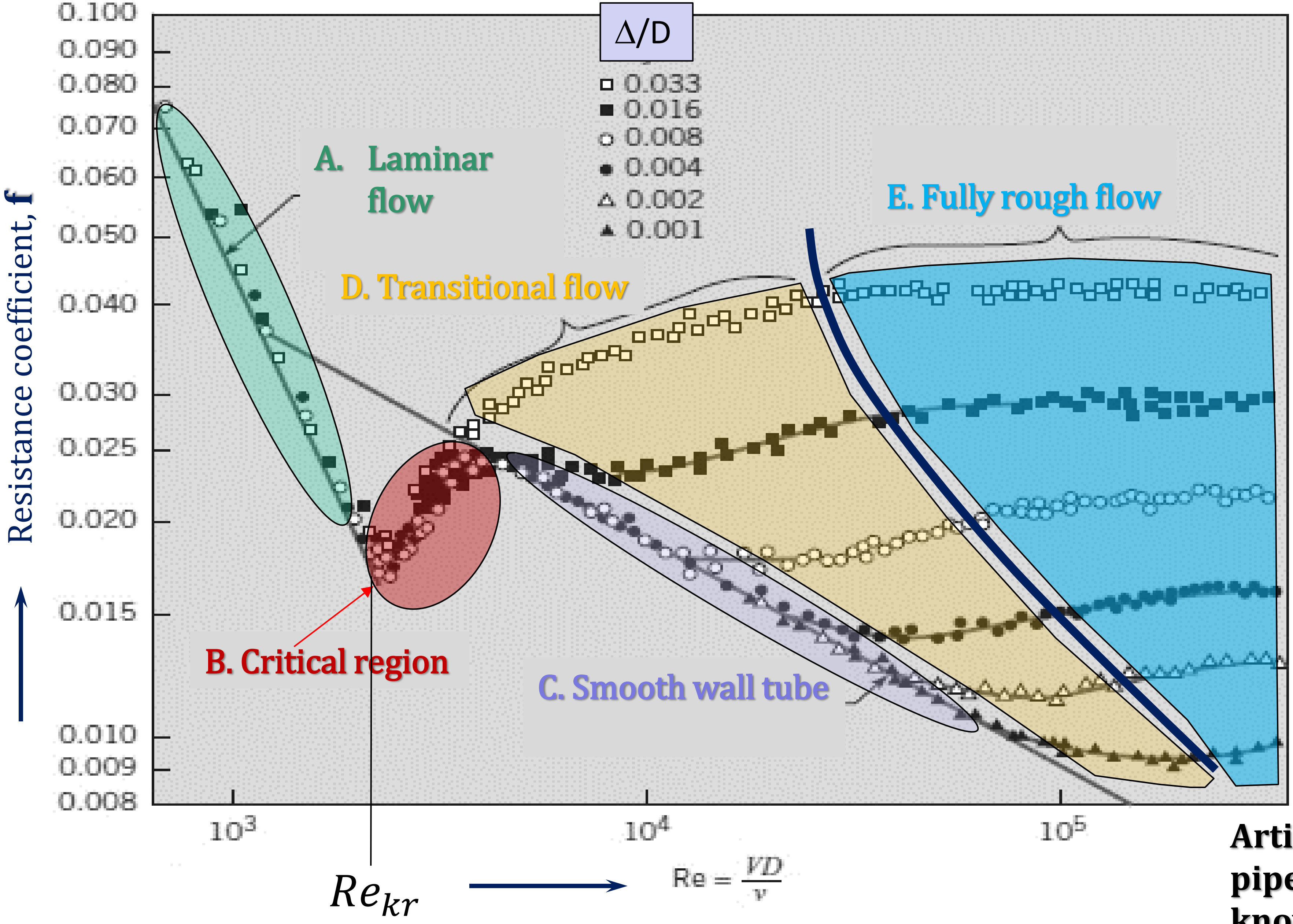


2) Hydraulics roughness
3) **Relative roughness**
Δ/D , Δ/r, Δ/R, D/Δ …….



δ - **laminar sublayer**

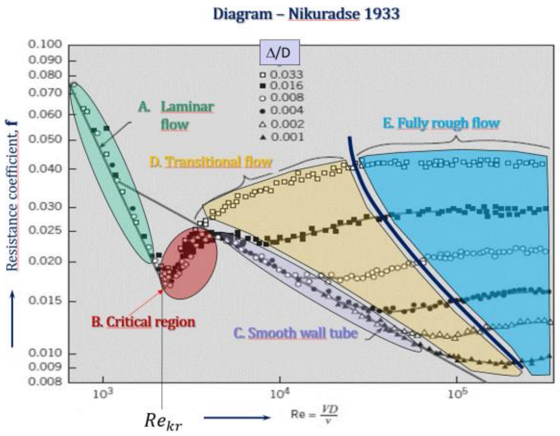
Diagram – Nikuradse 1933



Johann Nikuradse (1894-1979)

1930´s Nikuradse made great progress

Artificially roughened pipes with sand of know size, D



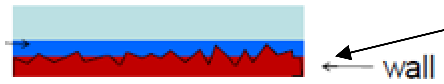
A. (1) LINEAR ZONE – Hagen-Poiseuille´s law

$f = 64/Re$ - line 1

f = f(Re)

B. (2) CRITICAL ZONE (Re =2320 - 4000) **f = f (Re)**

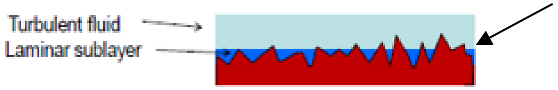
instability zone - lamin. ???? turb. Flow .. jump - Frenkel $f = 2{,}7/\mathrm{Re}^{0{,}53}$



Laminar sublayer is greater than roughness

C. (3) SMOOTH PIPES ZONE – - **f = f (Re)** δ > 5.Δ

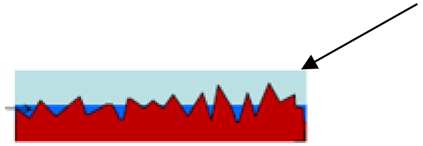
Blasius $f = 0{,}3164/\mathrm{Re}^{0{,}25}$ $\mathrm{Re} \ldots 4000 \ldots\ldots (10^5)$



Laminar sublayer nearly covers roughness

D. (4) TRANSITIONAL ZONE from Blasius - up to δ= Δ/5 **f = f (Re, r/Δ)**

Frenkel $$\frac{1}{\sqrt{f}} = -2\log\left[\frac{\Delta}{3{,}71.D} + \left(\frac{6{,}81}{\mathrm{Re}}\right)^{0{,}9}\right]$$

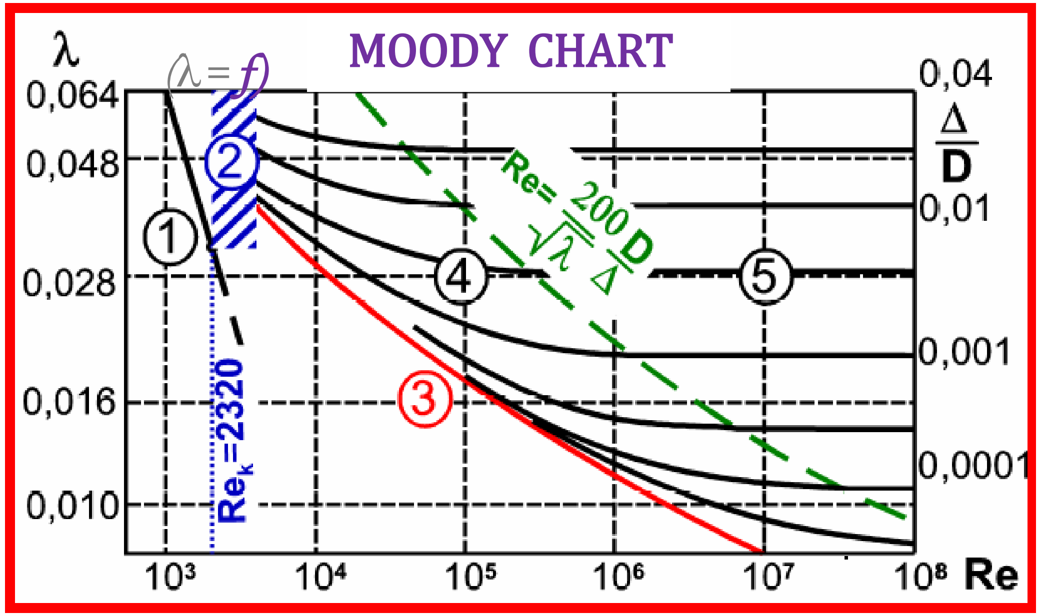


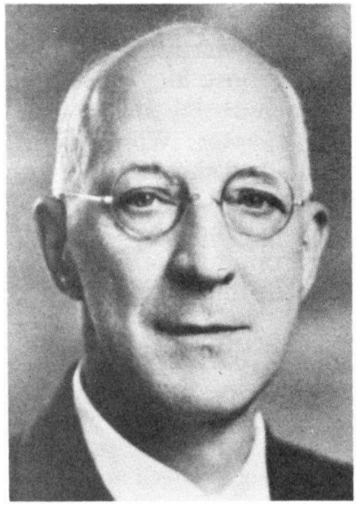
Laminar sublayer is less than roughness

E. (5) FULLY ROUGH TURBULENT ZONE – .... δ < Δ/5 **f = f(r/Δ)**

Nikuradse $$\frac{1}{\sqrt{f}} = 2\log\left[\frac{3{,}71D}{\Delta}\right]$$

COMMERCIALLY AVAILABLE PIPES





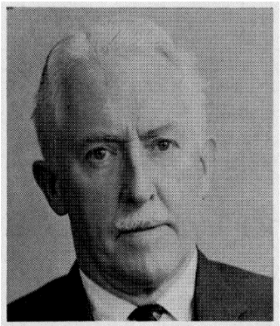
Lewis Moody, 1944

Moody chart presents the friction factor **f** for pipe flow as a function of the Re and relative roughness (Δ/D)

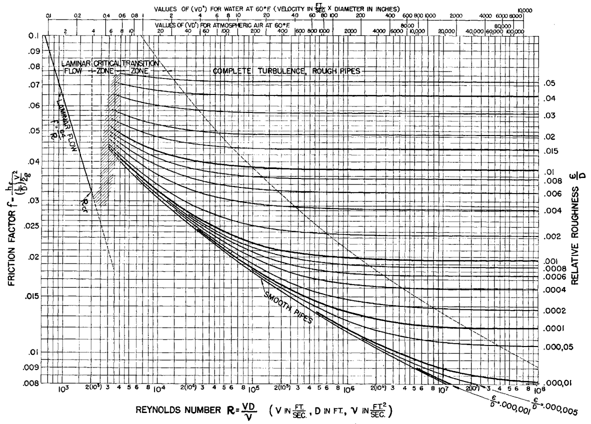
for commercial pipe in transition zone:

COLEBROOK-WHITE EQUATION (region 3,4,5)

$$\frac{1}{\sqrt{f}} = -2\log\left[\frac{2,51}{\mathrm{Re}\sqrt{f}} + \frac{\Delta}{3,7\,D}\right]$$



Cyril F. Colebrook, 1939



END