

Groundwater Hydraulics

GROUNDWATER HYDRAULICS

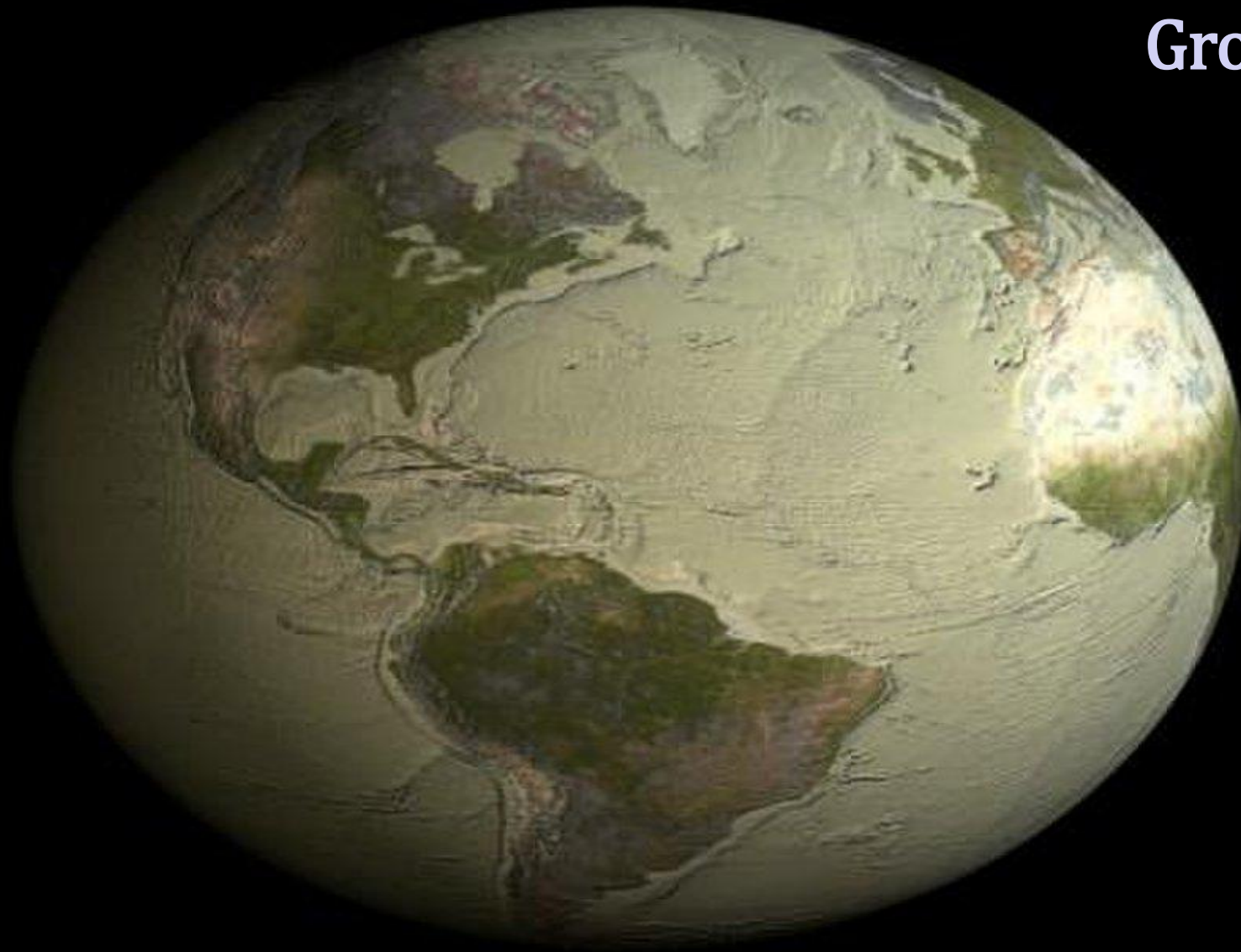
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Groundwater Hydraulics

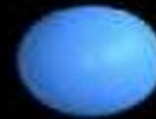
10/7/2025

Groundwater hydraulics



Fresh water

Groundwater



2022_2023



LITERATURE

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GROUNDWATER

- **ground water**: the water that **lies beneath the ground surface (groundwater level)** , **filling the pore space** between grains in bodies of sediment and clastic sedimentary rock, and **filling cracks** and **crevices** in all types of rock
- **ground water** is a **major economic resource**,
- **source of ground water** is **rain and snow** that falls to the ground a portion of which percolates down into the ground to become **ground water**
 - About half the domestic water use is from groundwater.
Varies regionally.

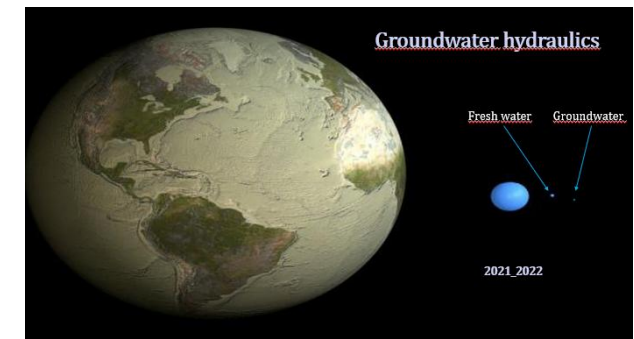


WHY GROUNDWATER IS GOOD?

- much less subject to seasonal variations in availability than surface water
- slow movement leads to high biological purity
- Temperature is remarkably constant
- Available virtually „everywhere“ if you go deep enough

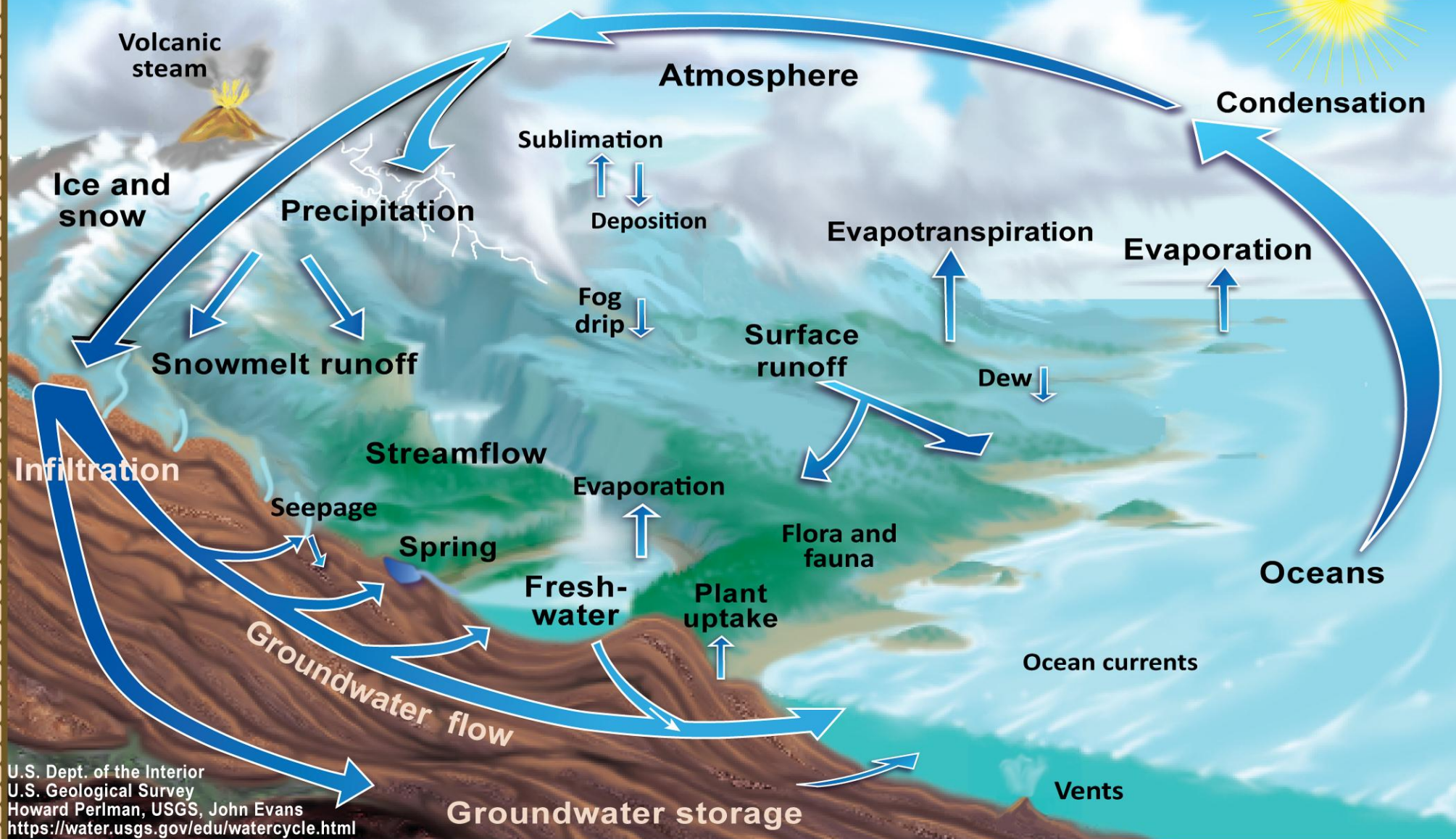
Groundwater

Estimate of the World Water Balance



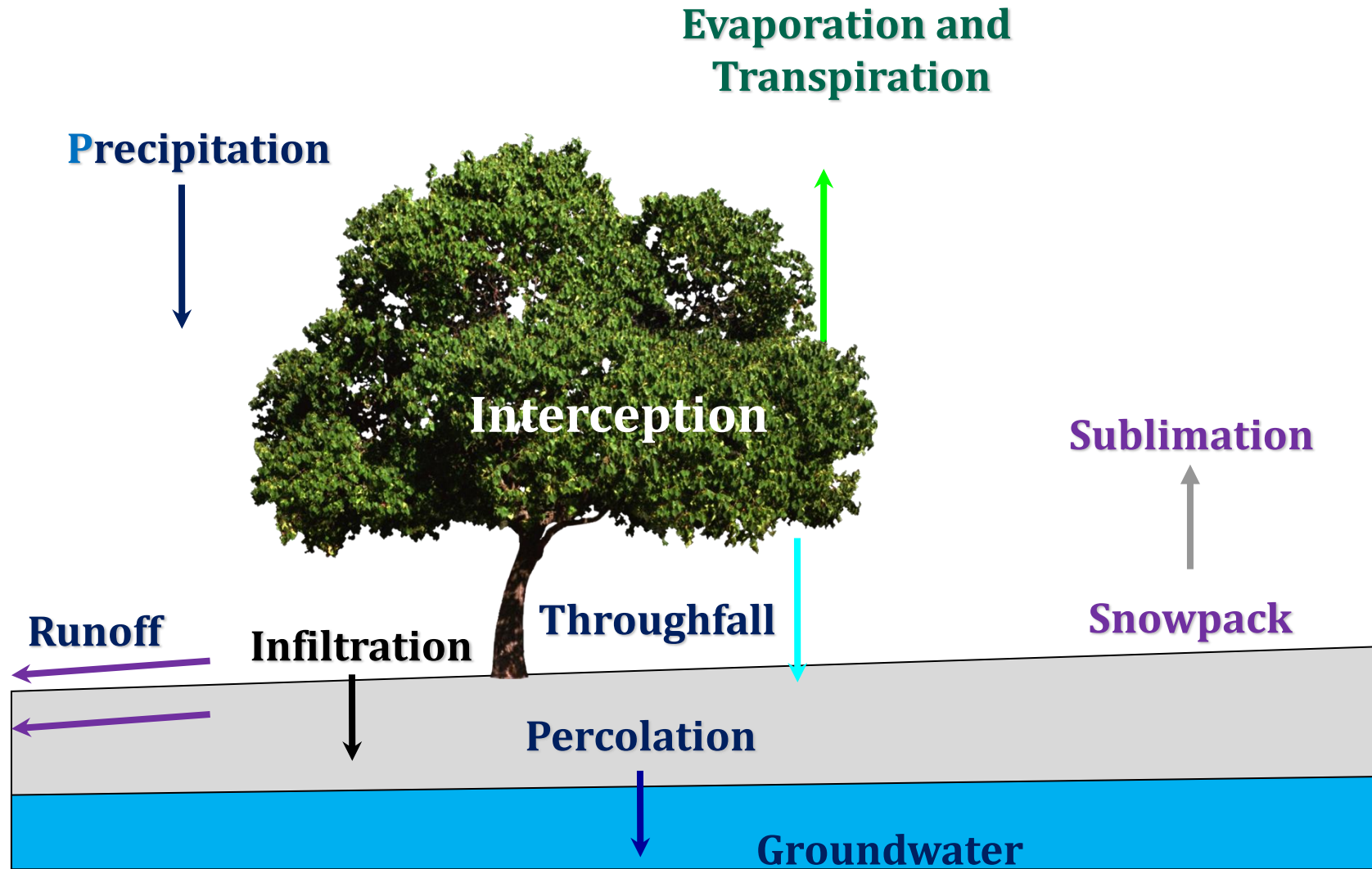
Parameter	Surface area (km ²) X 10 ⁶	Volume (km ³) X 10 ⁶	Volume %	Equivalent depth (m)	Residence Time
Oceans and seas	361	1370	94	2500	~4000 years
Lakes and reservoirs	1.55	0.13	<0.01	0.25	~10 years
Swamps	<0.1	<0.01	<0.01	0.007	1-10 years
River channels	<0.1	<0.01	<0.01	0.003	~2 weeks
Soil moisture	130	0.07	<0.01	0.13	2 weeks – 1 year
Groundwater	130	60	4	120	2 weeks – 10,000 years
Icecaps and glaciers	17.8	30	2	60	10-1000 years
Atmospheric water	504	0.01	<0.01	0.025	~10 days
Biospheric water	<0.1	<0.01	<0.01	0.001	~1 week

The Water Cycle



U.S. Dept. of the Interior
U.S. Geological Survey
Howard Perlman, USGS, John Evans
<https://water.usgs.gov/edu/watercycle.html>

HYDROLOGIC CYCLE - TERMINOLOGY



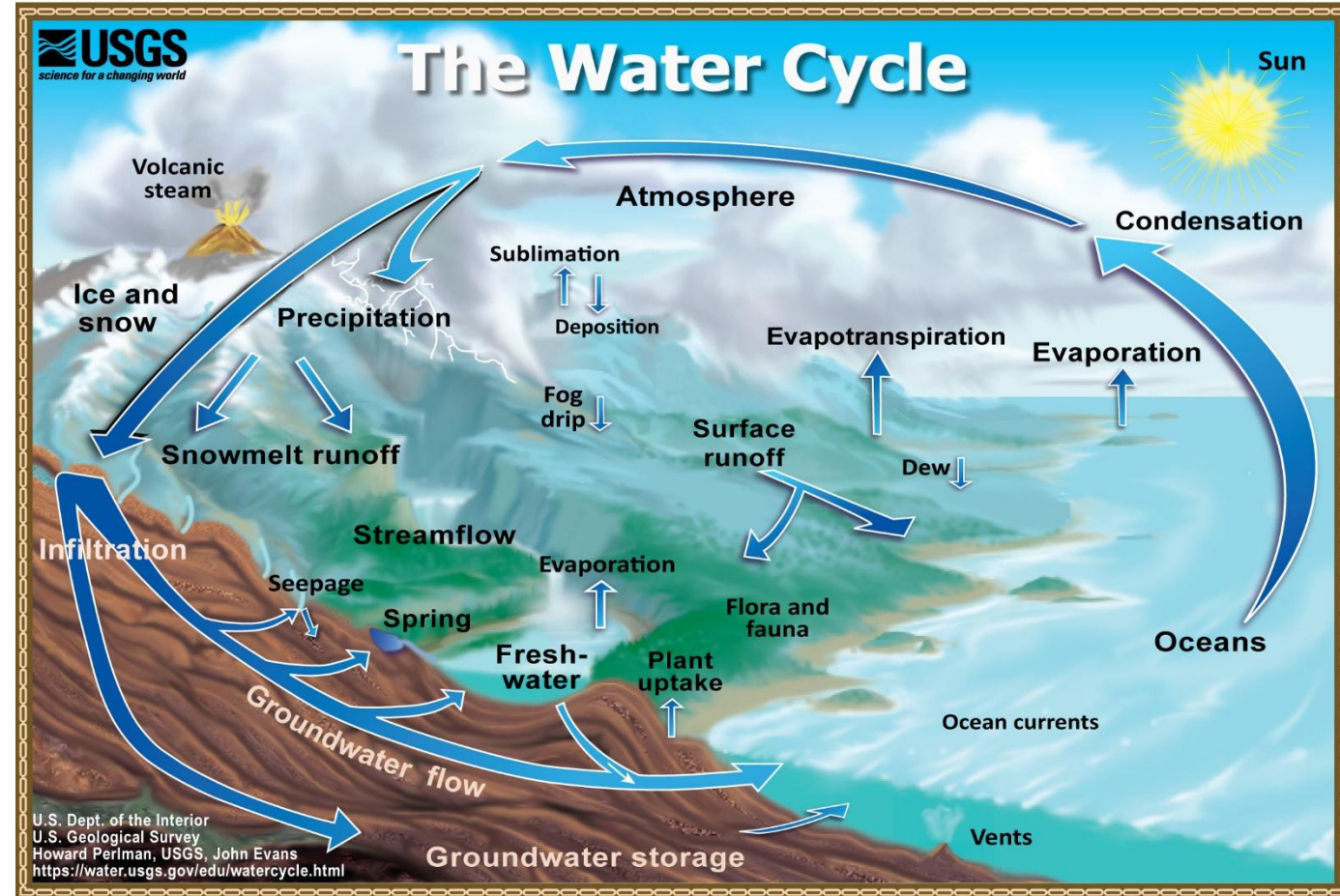
THE WATER BUDGET: **LAW OF MASS CONSERVATION**

CLOSED SYSTEM

$$P - ET - R = \Delta S$$

IMPORTANCE OF GROUNDWATER (GW)

1. It provides **water for rivers, streams** and wetlands
2. Part of the Hydrologic Cycle
3. It provides a **water resources for humans, stock and plants** (irrigation)
4. 25% of all the fresh water on Earth
5. Important Environmental Issues
6. **Groundwater** is an important source of **drinking water** for **European** countries, with even **75% of EU** inhabitants depending on **groundwater** for their **water** supply.
6. It helps **maintain lake water levels**
7. It can provide a pathway to filter, remove contaminants (but not always)

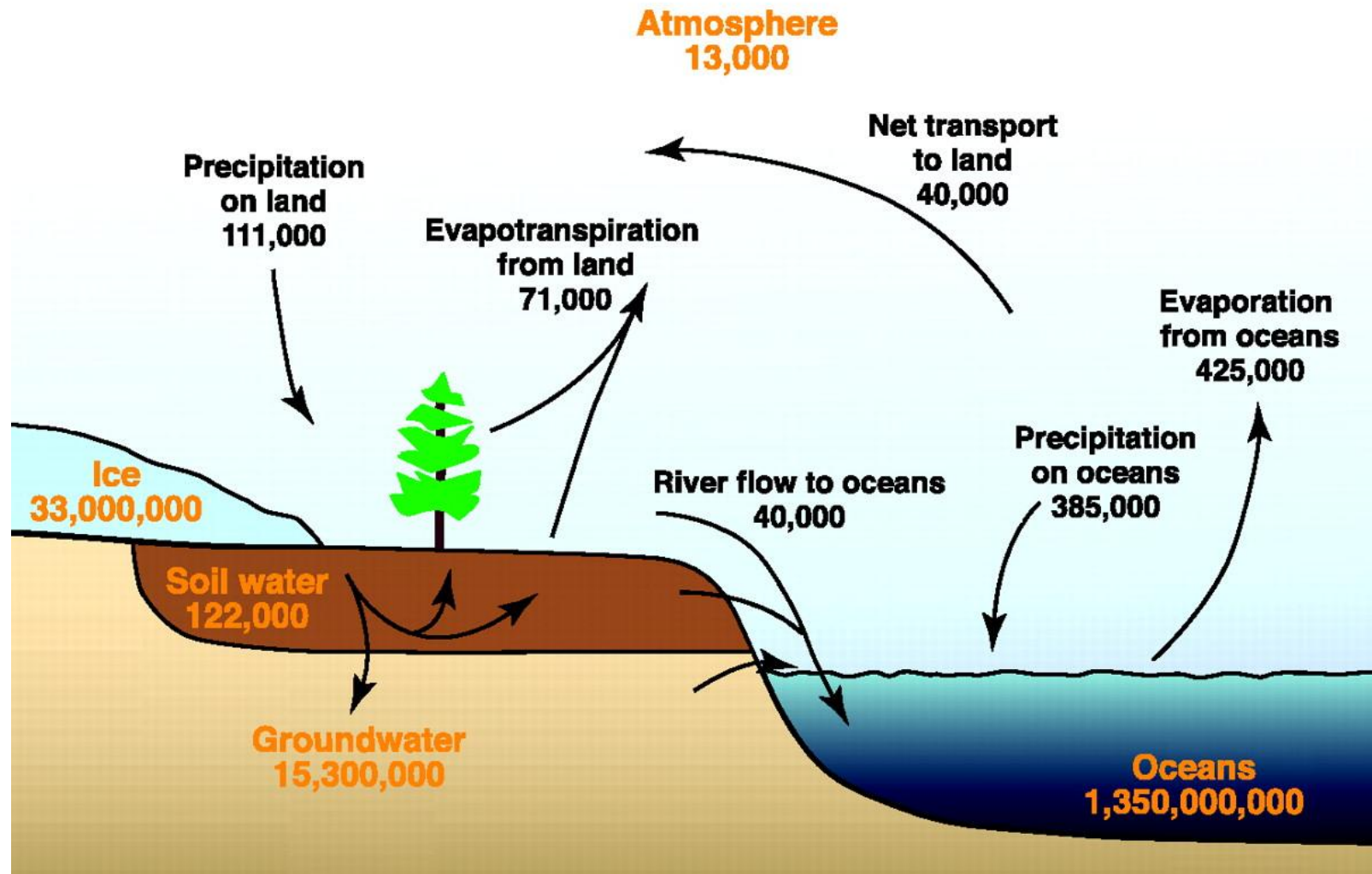


Characteristic of GW:

- stable ... reliable source of water
- slow ... once contaminated, it is very difficult to clean up

World: Groundwater (GW) represents **97 % of all unfrozen fresh water**

THE GLOBAL WATER CYCLE - GLOBAL WATER VOLUMES & FLUXES (KM³)



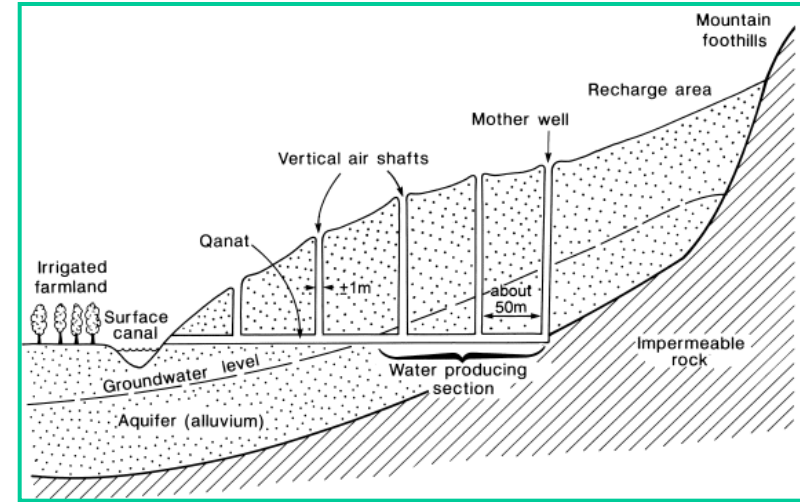
HISTORY - OLD THEORIES

- **Homer** (~1000 BC):
 - “from whom all rivers are and the entire sea and all springs and all deep wells have their waters”
- **Seneca** (3 BC -65 AD)
 - “You may be quite sure that it not mere rainwater that is carried down into our greatest rivers.”
- **Da Vinci** (1452-1519)
 - accurate representation of the **hydrologic cycle**
- **Kircher** (1615-1680):
 - Water from the ocean is vaporized by the hot earth, rises, and condenses inside mountains.

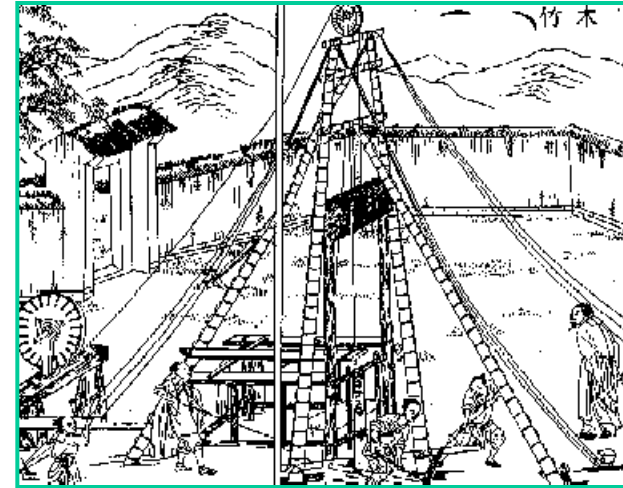


Some History

- **Qanals**
 - Subterranean tunnels used to tap and transport groundwater
 - Originally in Persia
 - Kilometers in length
 - Up to 3000 years old
 - Many still operating
- **Chinese Salt Wells**
 - 1000 years ago: Drilled wells
 - Over 300 meters deep
 - Bamboo to retrieve cuttings
 - By year 1858: 1000 meters deep
 - Called “cable tool” drilling today



Ancient Persian Qanat



Ancient Chinese Salt Well



- **PERCOLATION THEORY**

- **Vitruvius (~80-20 BC) 8th Book on Water and Aqueducts.** Rain and snow on land reappears as springs and rivers

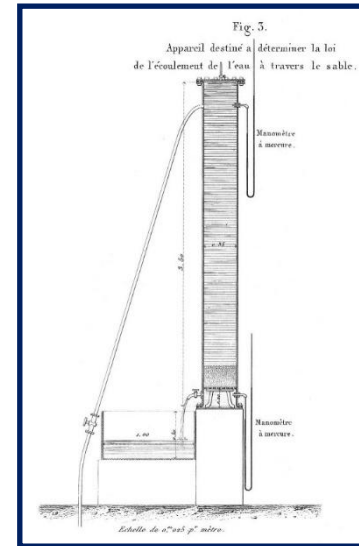


- **Perrault, Mariotte (1670):** Water balance on the Seine river. River flow explained by rainfall..

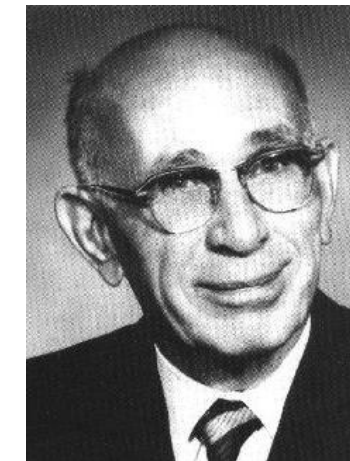
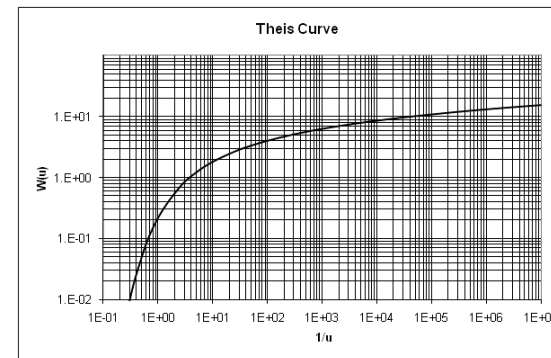


MODERN THEORY

- **Henri Darcy (1856):** Relationship for the flow through sand filters. Resistance of flow through aquifers. Solution for unsteady flow.
- **King (1899):** Water table maps, groundwater flow, cross-section
- **C.V. Theis (1930s):** Well Hydraulics
- **C. E. Jacob (1940)** Partial differential equation of transient groundwater flow



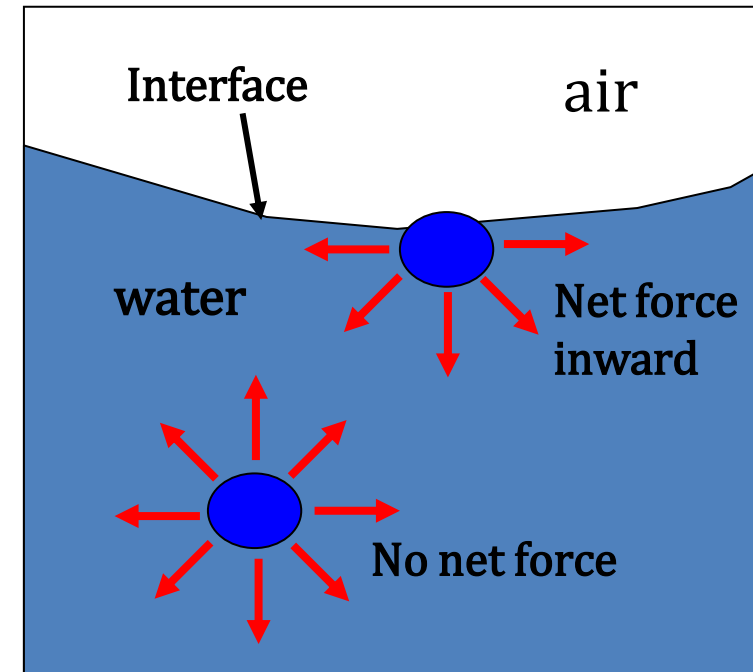
Henri Darcy



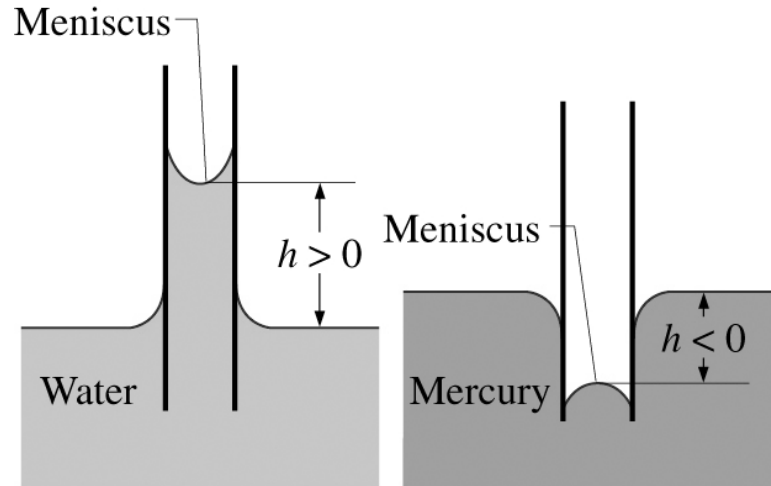
C.V. Theis

SURFACE TENSION

- **Below interface**
 - Forces act equally in all directions
- **At interface**
 - Some forces are missing
 - Pulls molecules down and together
 - Like membrane exerting *tension* on the *surface*
- **Curved interface**
 - Higher pressure on concave side
- **Pressure** increase is balanced by surface tension
 - $\sigma = 0.073 \text{ N/m}$ (@ 20°C)
- **Capillary pressure**
 - Relates pressure on both sides of interface



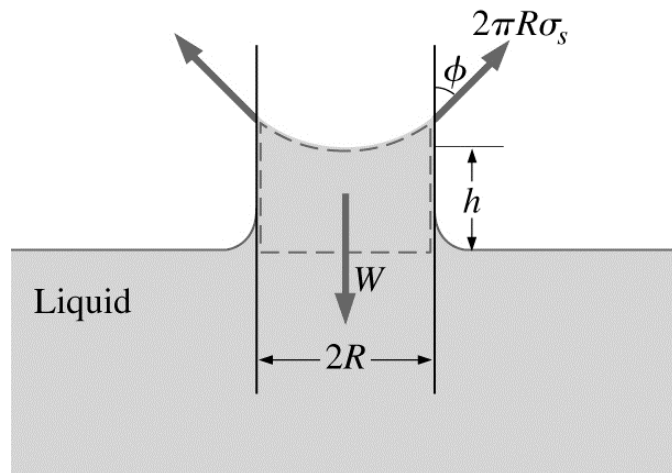
CAPILLARITY



- **Capillary effect** is the **rise** or **fall** of a liquid in a small-diameter tube.
- The curved free surface in the tube is call the **meniscus**.
- Force balance can describe magnitude of capillary rise.

Equilibrium of **surface tension force** and **gravitational** pull on the water cylinder of height produces:

$$F_{\sigma,z} - W = 0$$



$$2\pi R \sigma \cos \phi = \pi R^2 h \gamma \quad \Rightarrow \quad h = \frac{2\sigma \cos \theta}{\gamma R}$$

- σ surface tension
- ϕ wetting angle (is the angle of contact)
- γ specific weight of liquid
- R radius of tube

CAPILLARY RISE

Capillary rise is a function of surface tension (σ [FL⁻¹]), fluid specific weight (γ_w [FL⁻³]) contact angle with the solid surface (θ), and pore diameter (d [L]):

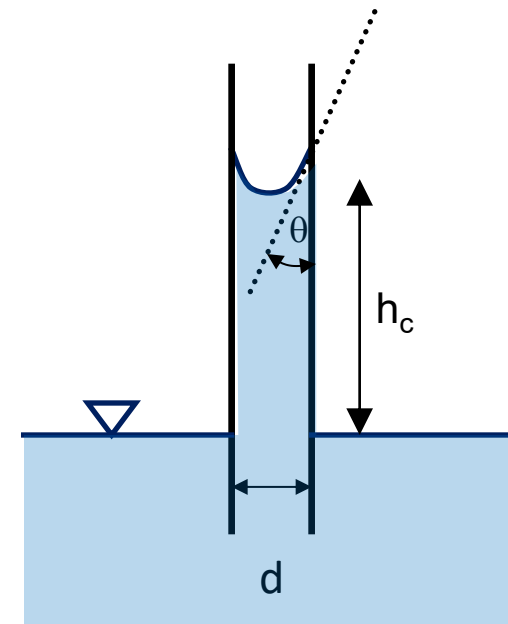
For liquid-vapour interfaces:

$$h_c = 4\sigma \cos(\theta) / \gamma_w d$$

The surface tension of water at 20°C is 7.3×10^{-2} N/m and $\gamma_w = 9.81 \times 10^3$ N/m³. For water in contact with silicates θ is close to zero so $\cos(\theta)=1$.

$$h_c \approx 3 \times 10^{-5} / d$$

where d is measured in metres.





CAPILLARY RISE OF WATER IN SOILS

<u>Soil Type</u>	<u>Capillary Rise (m)</u>
Clay	>10
Fine Silt	7.5
Coarse Silt	3.0
Very Fine Sand	1.0
Fine Sand	0.50
Medium Sand	0.25
Coarse Sand	0.15
Very Coarse Sand	0.04
Fine Gravel	0.015

$$h_c \approx 3 \times 10^{-5} / d$$

$$\text{For } d = 2 \times 10^{-3} \text{ m}$$

(coarse sand 2 mm)

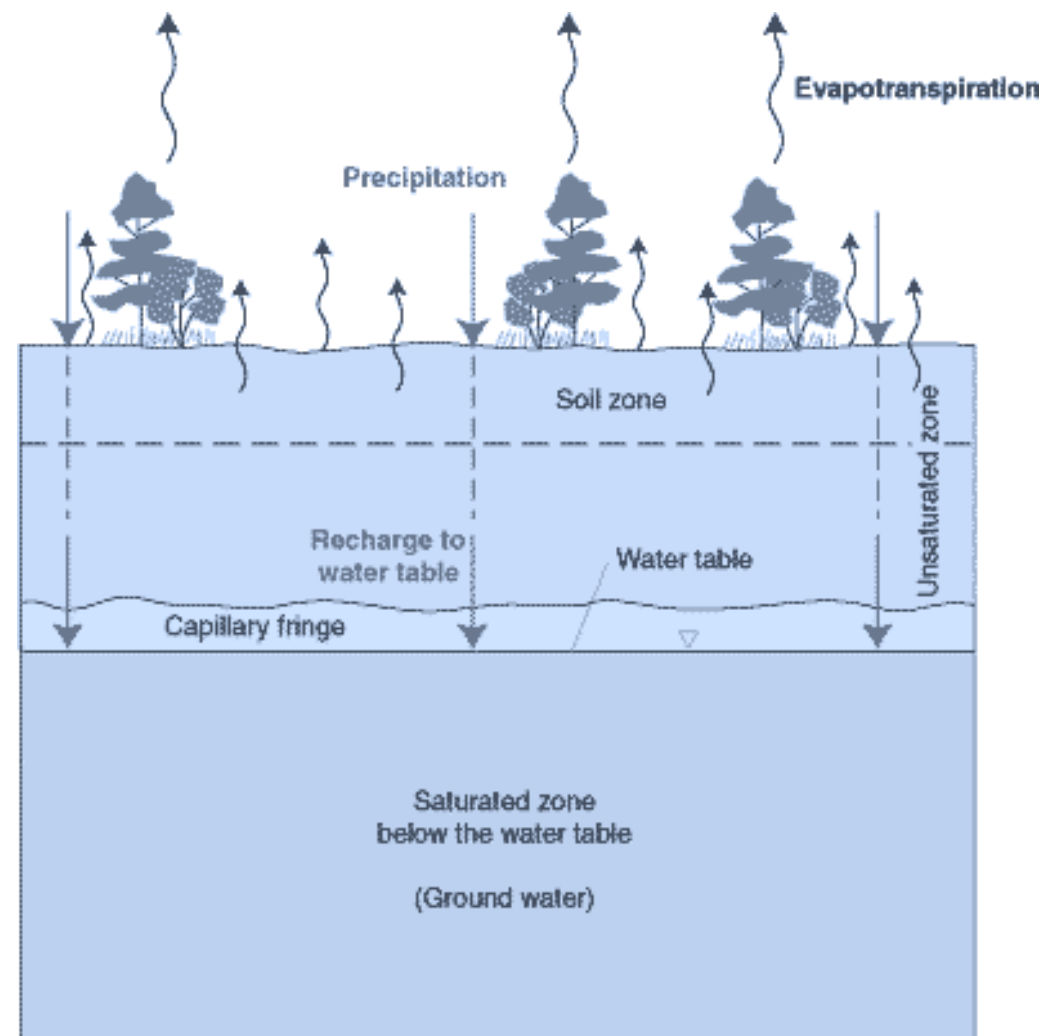
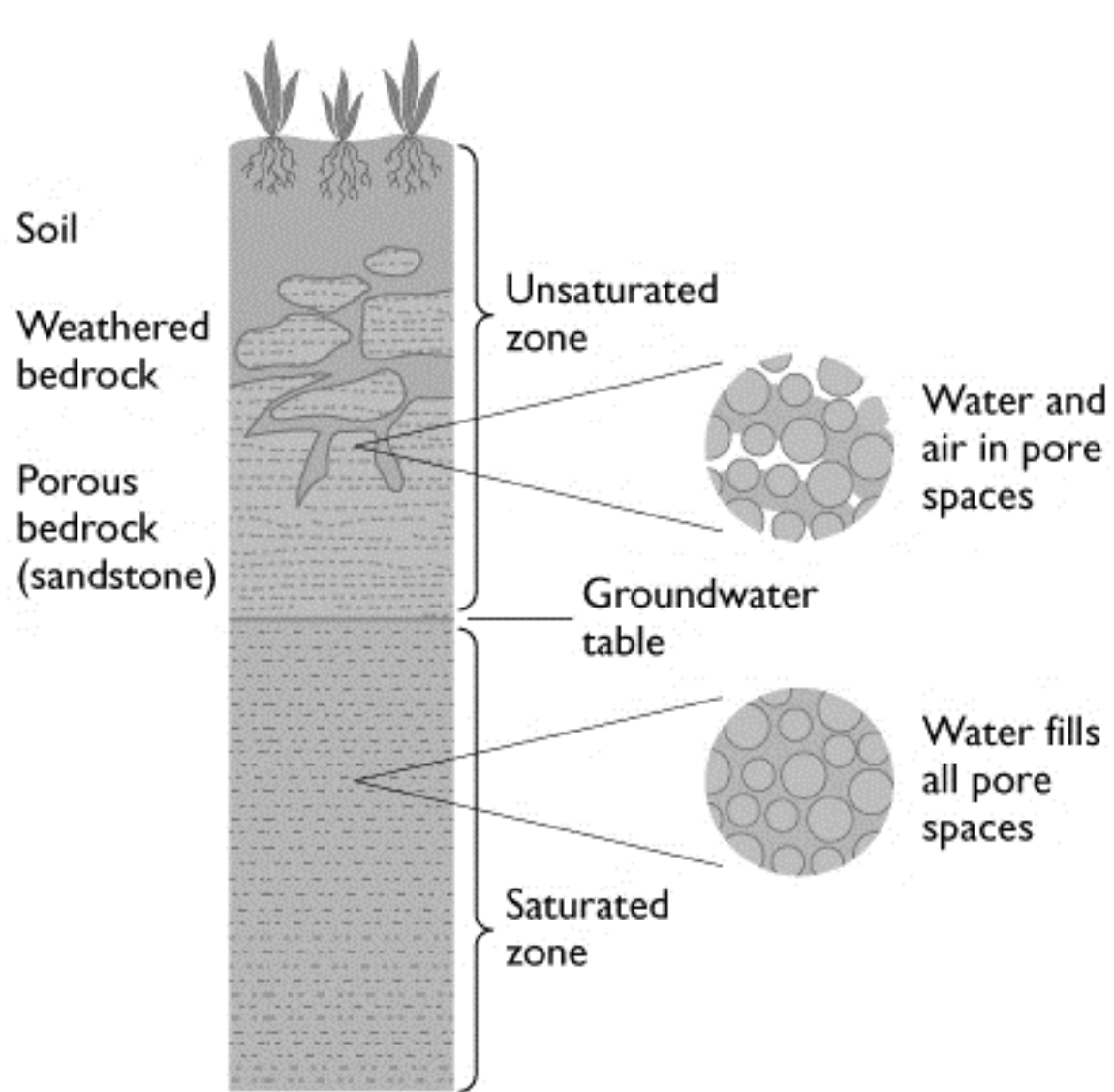
$$h_c \approx 0.015 \text{ m}$$

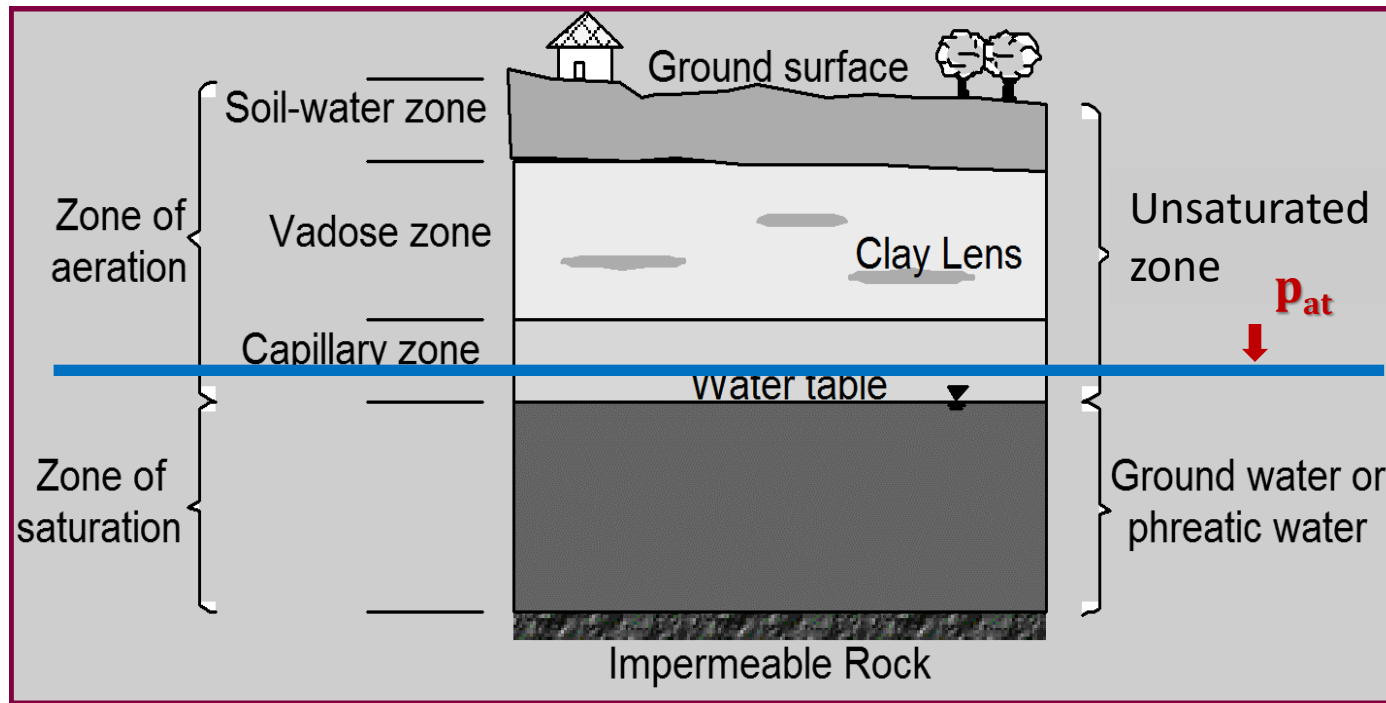
$$\text{For } d = 2 \times 10^{-6} \text{ m}$$

(clay 2 mm)

$$h_c \approx 15 \text{ m}$$

VERTICAL DISTRIBUTION OF „WATER“





VERTICAL DISTRIBUTION OF GROUND WATER

- **Soil water zone**: extends from the ground surface down through the major root zone, varies with soil type and vegetation but is usually a m thickness
- **Vadose zone (unsaturated zone)**: extends from the surface to the water table through the root zone, intermediate zone, and the capillary zone
- **Capillary zone**: extends from the water table up to the limit of capillary rise, which varies inversely with the pore size of the soil and directly with the surface tension
- * **Water table**: the level to which water will rise in a well drilled into the saturated zone
- * **Saturated zone**: occurs beneath the water table where porosity is a direct measure of the water contained per unit volume

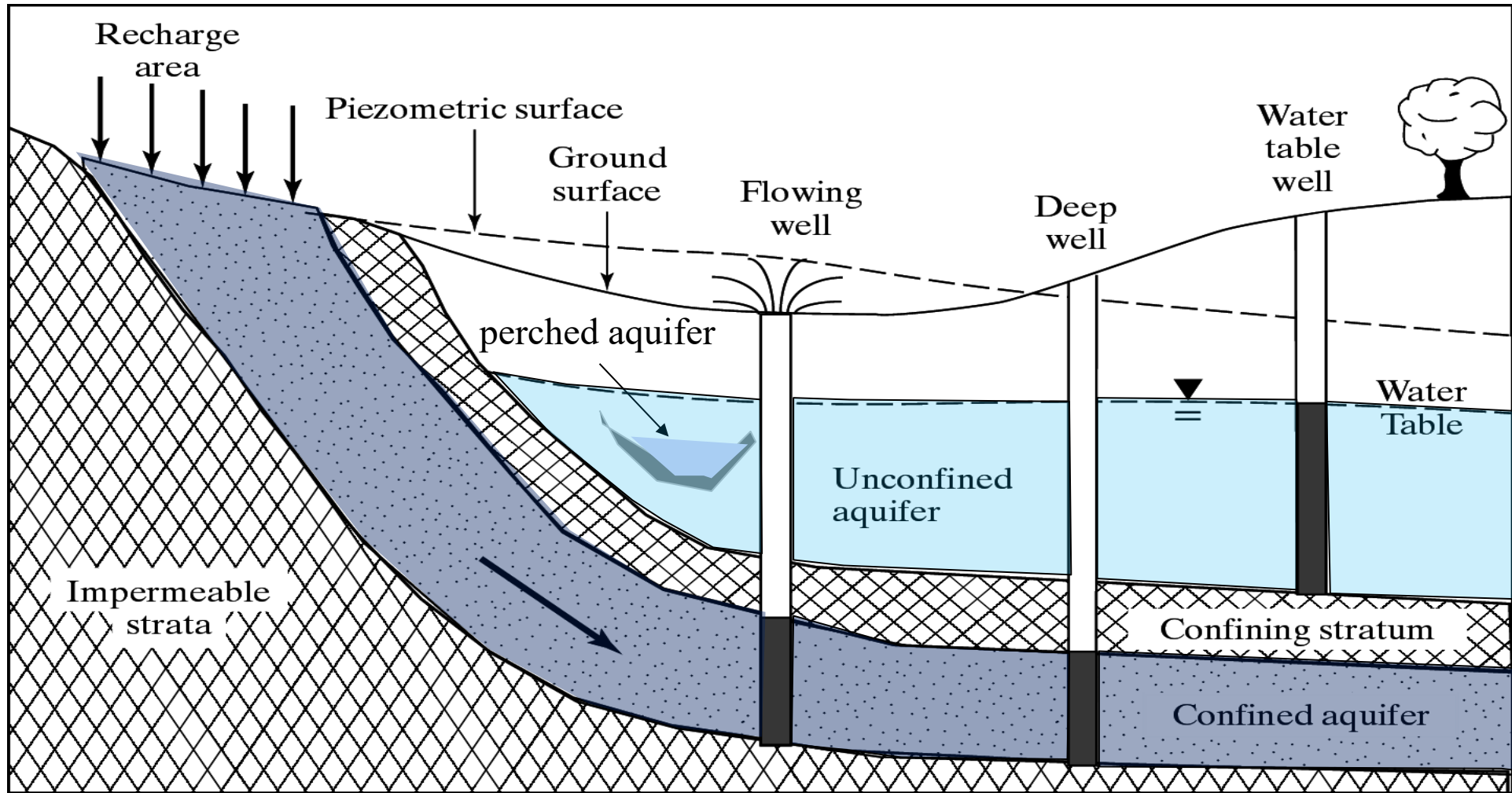
TYPICAL AQUIFER SYSTEMS

- **Aquifer** – store and transmit water , unconsolidated deposits – sand, gravel, sandstone etc.
 - **Unconfined aquifer**: an aquifer where the water table exists under atmospheric pressure as defined by levels in shallow wells
 - **Confined aquifer**: an aquifer that is overlain by a relatively impermeable unit such that the aquifer is under pressure and the pressure level rises above the confined unit
 - **Artesian aquifer**: are confined under hydraulic pressure, resulting in free-flowing water, either from a spring or from a well.
 - **Aquiclude**: store , don't transmit water; clays and less shale, impervious boundaries of aquifer
 - **Aquitard**: transmit don't store water; shale and less clay; leaky confining layers of aquifers
 - **Piezometric surface**: in a confined aquifer, the hydrostatic pressure level of water in the aquifer, defined by the water level that occurs in a lined penetrating well
- Water table**: the level to which water will rise in a well drilled into the saturated zone

SPECIAL AQUIFER SYSTEMS

- **Leaky confined aquifer**: represents a stratum that allows water to flow from above through a leaky confining zone into the underlying aquifer
- **Perched aquifer**: occurs when an unconfined water zone sits on top of a clay lens, separated from the main aquifer below

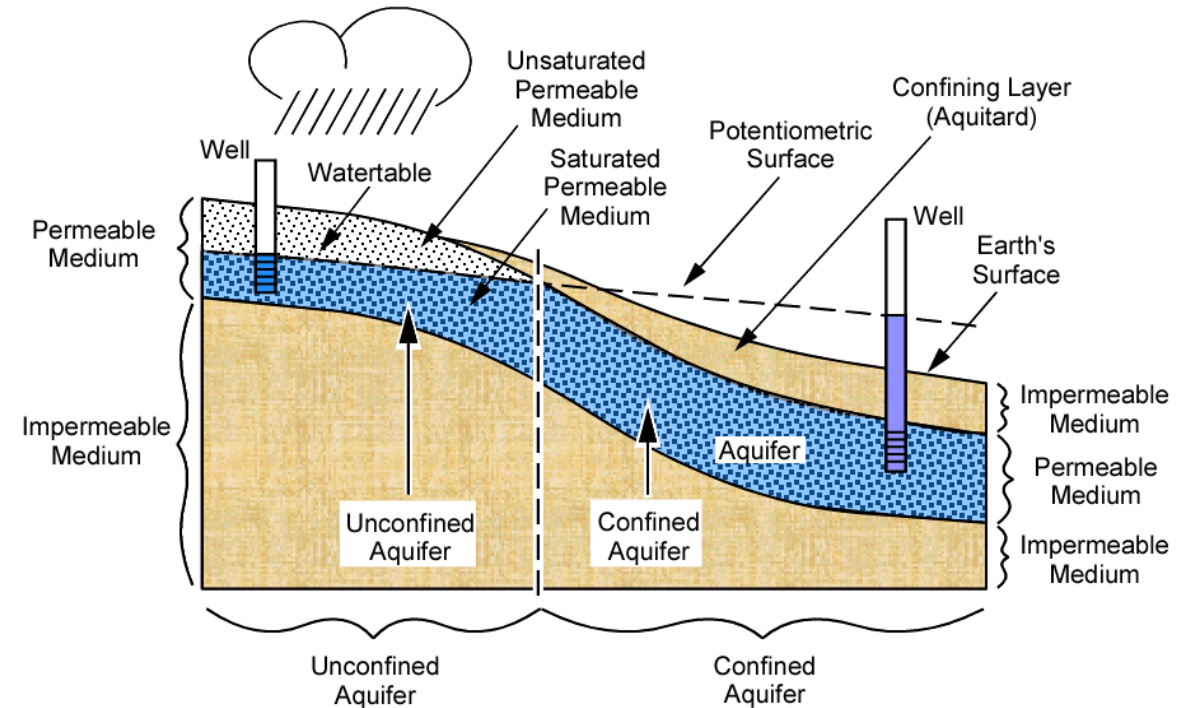
CROSS SECTION OF UNCONFINED AND CONFINED AQUIFERS



Schematic cross section illustrating unconfined and confined aquifers.

AQUIFER CHARACTERISTICS

1. Matrix type
2. Soil classification
3. Porosity (n)
4. Confined or unconfined
5. Vertical distribution
(stratigraphy or layering)
 1. Hydraulic conductivity (K)
 2. Permeability (k)
 3. Transmissivity (T)
 4. Storage coefficient or Storativity (S)

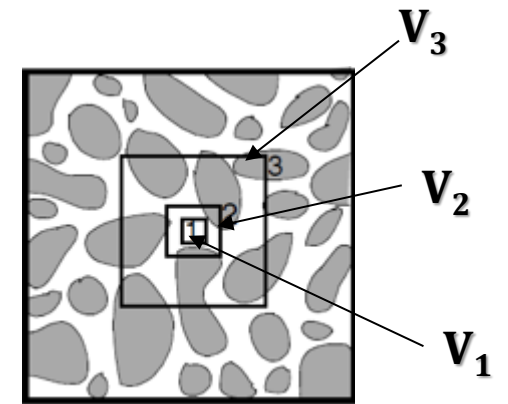
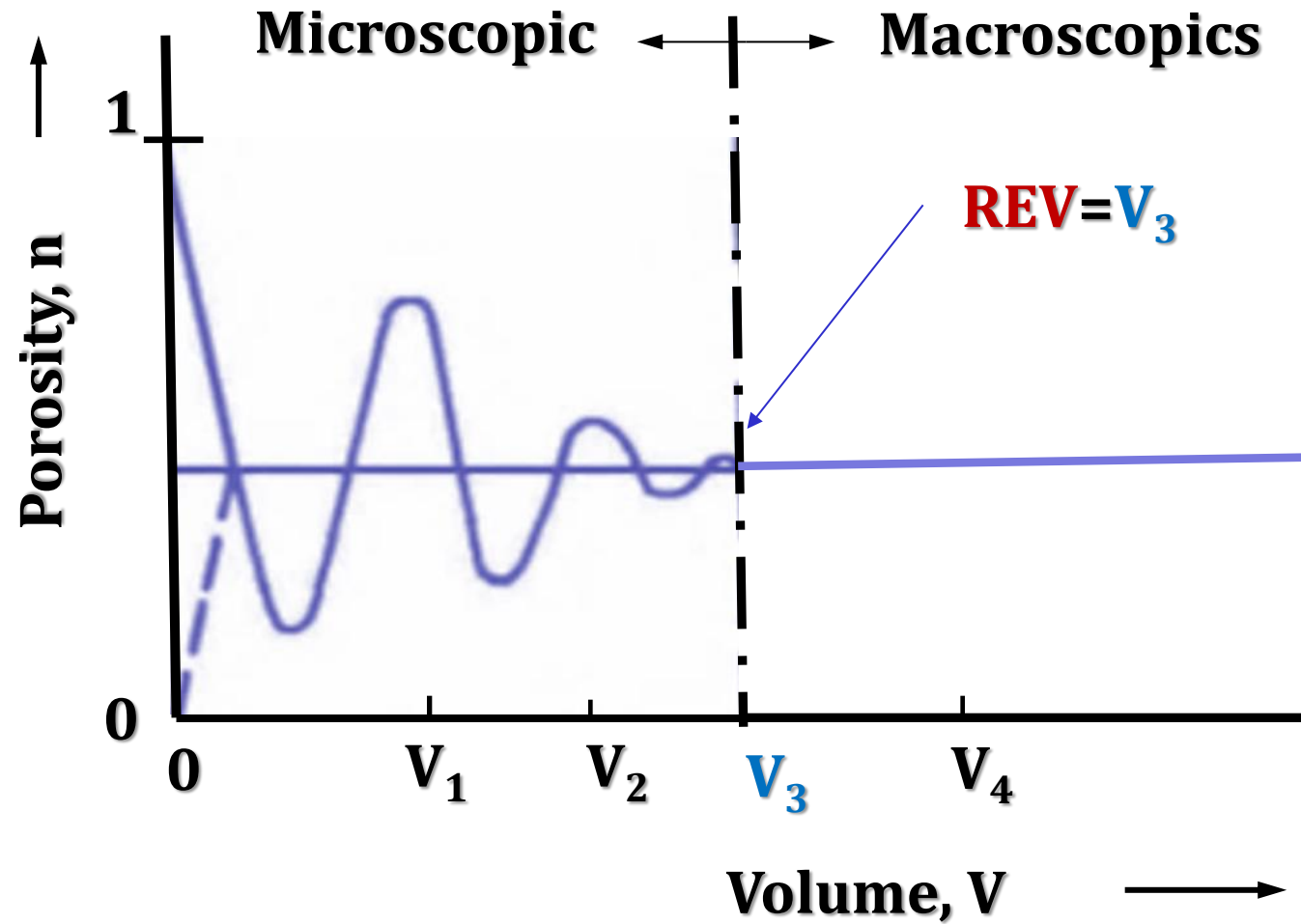




CONTINUUM APPROACH TO POROUS MEDIA - REV

- Pressure, density etc. apply to fluid elements that are large relative to molecular dimensions, but small relative to the size of the flow problem
- We adopt a **Representative Elementary Volume** (REV) approach
- **REV must be large enough to contain enough pores to define the average value of the variable** in the fluid phase and to ensure that the pore-to-pore fluctuations are smoothed out
- **REV** must be small enough that larger scale heterogeneities do not get averaged out (layering, etc.)

REV - Representative Elementary Volume

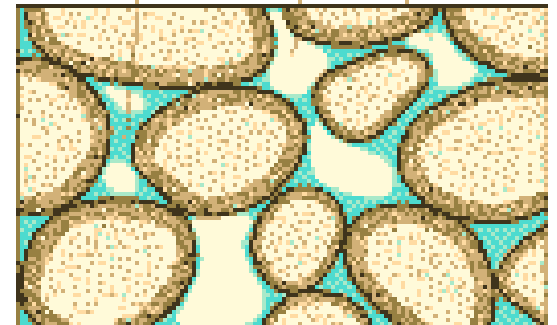
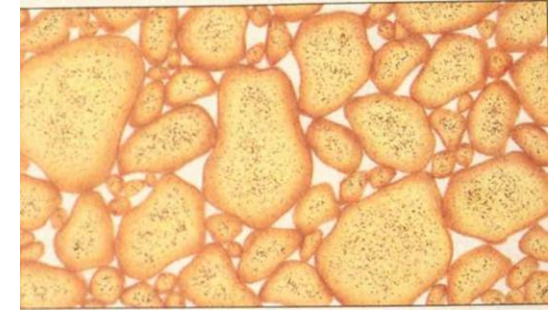


Representative
Elementary
Volume

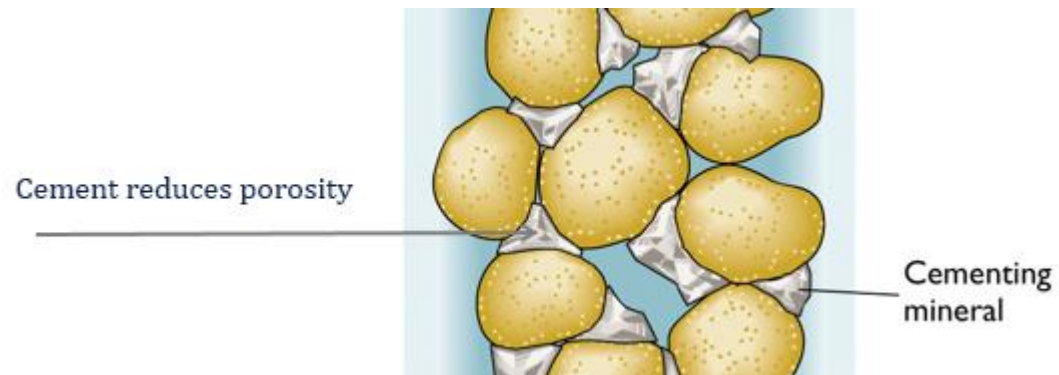
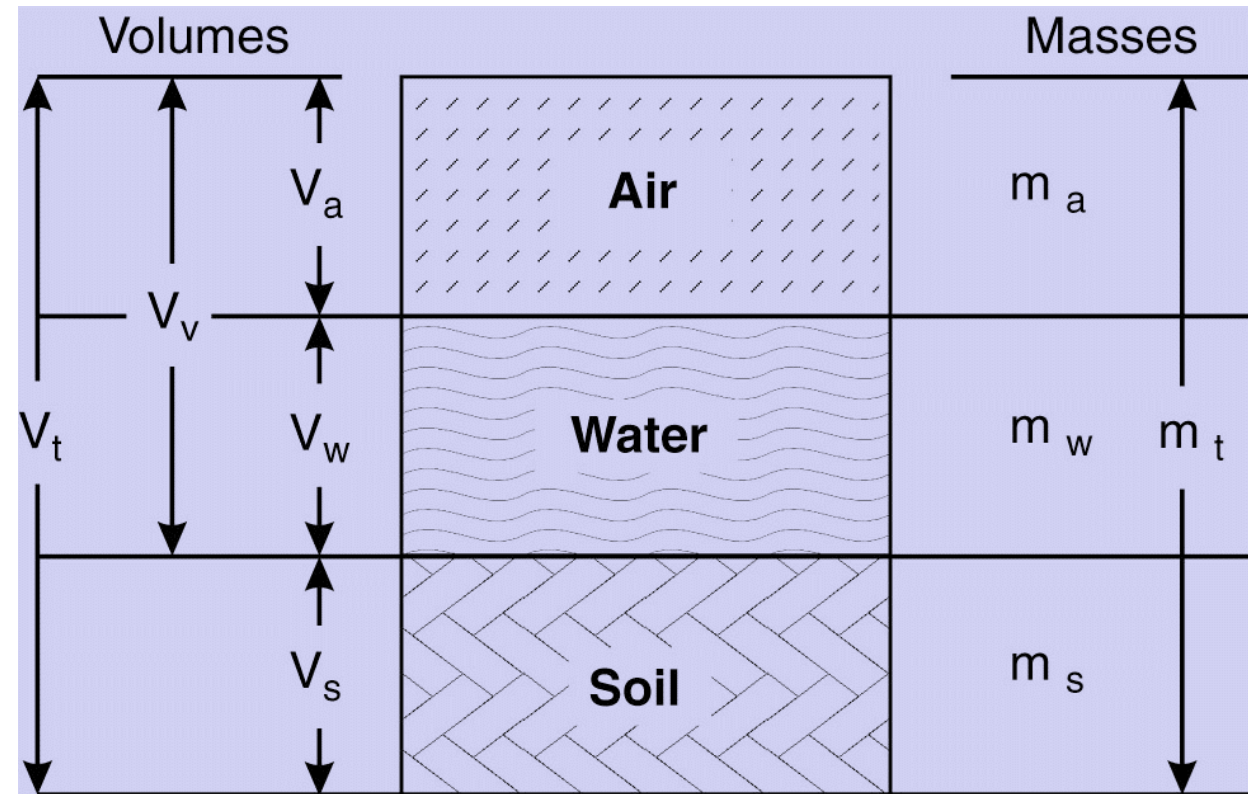
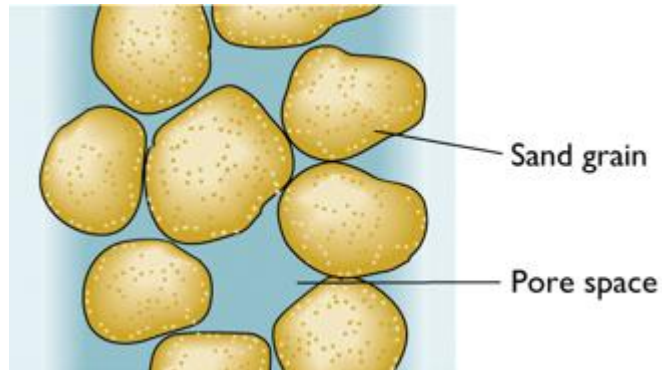
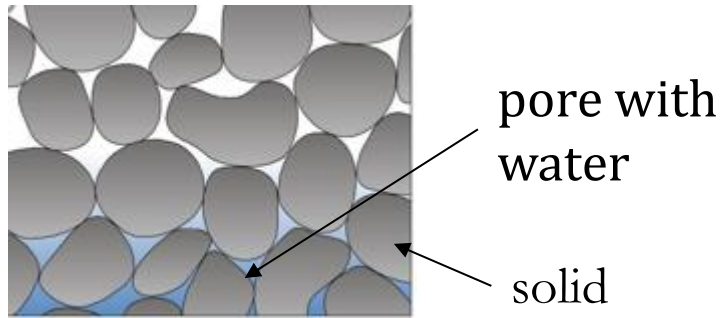
AQUIFER CHARACTERISTICS

SOIL CLASSIFICATION BASED ON PARTICLE SIZE

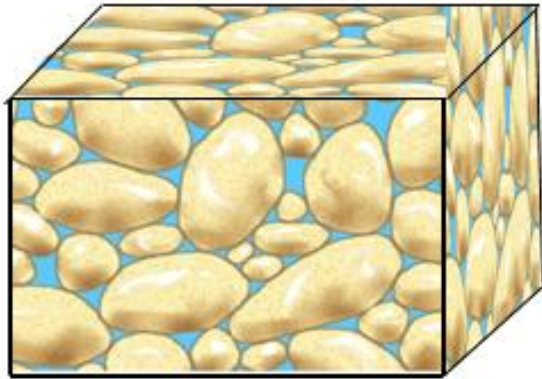
Material	Particle Size, mm
Clay	<0.004
Silt	0.004 - 0.062
Very fine sand	0.062 - 0.125
Fine sand	0.125 - 0.25
Medium sand	0.25 - 0.5
Coarse sand	0.5 - 1.0



AQUIFER CHARACTERISTICS



POROSITY

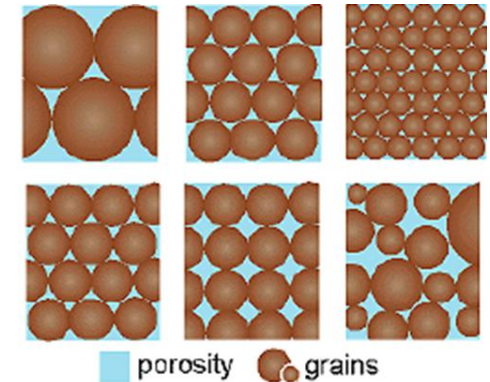


- **Volume of pores** is also the **total volume – the solids volume**
- **porosity** is a measure of the capacity of the medium to hold water
- a volume V_T of soil or rock is divided up into the volume of voids V_v and volume of solids V_s
- n_{ef} – effective pores
- V_{ef} – volume of effective pores

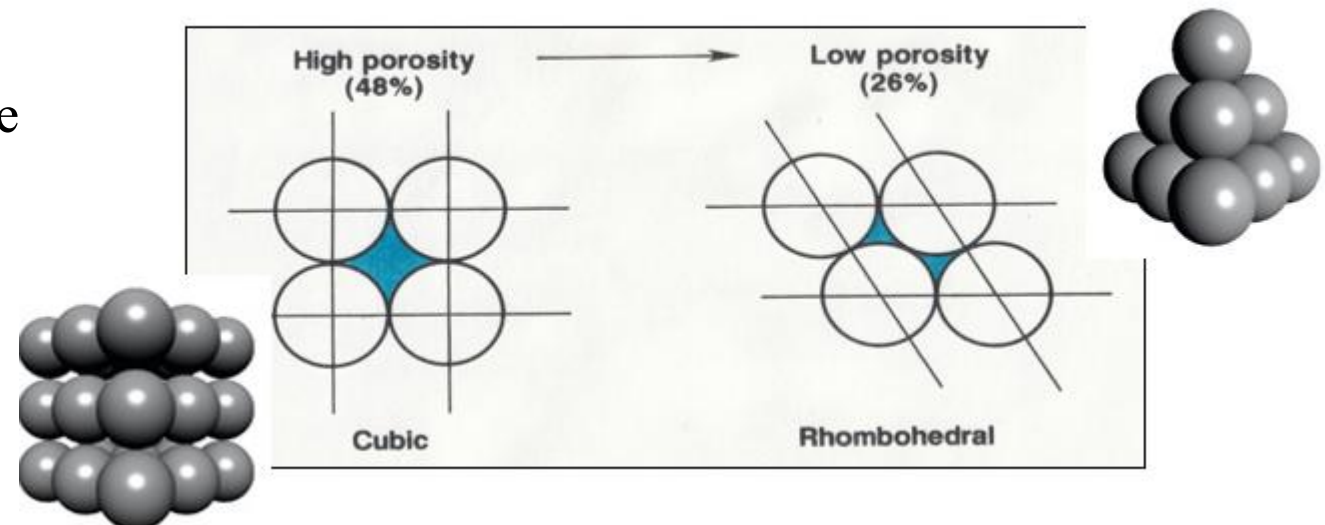
- Porosity - n

$$n = \frac{V_{pores}}{V_{total}}$$

$$n = \frac{V_{total} - V_{solids}}{V_{total}}$$



$$n_{ef} = \frac{V_{ef.pores}}{V_{total}}$$



RANGE OF POROSITY VALUES FOR MATERIALS

Material	Porosity (%)
Clay	40 – 70
Silt	35 – 50
Fine Sand	40 – 50
Medium Sand	35 – 40
Coarse Sand	25 – 40
Gravel	20 – 40
Sand and Gravel mix	10 – 30
Limestone	0 – 50
Sandstone	5 – 30
Shale	0 – 10
Crystalline Rock	0 – 10

Porosity & Effective Porosity Ranges

Material	Porosity (%)	Eff. Porosity (%)
Silt	34 - 61	0.1 – 10
Clay	34 - 60	0.1 – 10
Sand/Gravel	24 – 55	10 - 55
Limestone/dolomite	5 - 15	0.1 – 5
Shale	1 - 10	0.5 – 5
Sandstone	5 - 15	0.5 – 10



PERMEABILITY, k_p

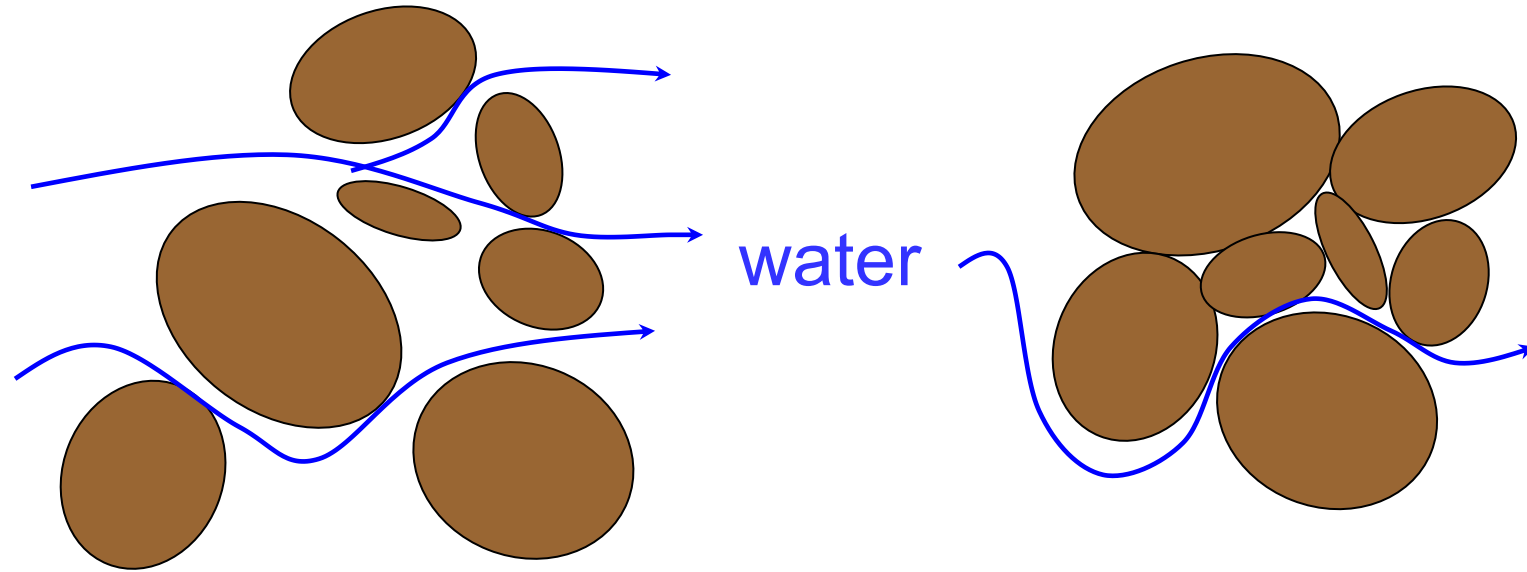
- **Permeability** - the capacity of a rock to transmit fluid through pores and fractures
 - Interconnectedness of pore spaces
 - Most sandstones and conglomerates are porous and permeable
- with dimensions of area [L^2], depends only on the properties of the porous medium: (How well the pores are connected, and how straight a path a fluid follows - is a **property of the rock**)

$$k_p = C d^2$$

$$k_p = \frac{n_{ef} \cdot D^2}{32}$$

where **C** is a dimensionless constant (sometimes called **tortuosity**) and **d** is a characteristic **pore diameter** with dimensions of length, n_{ef} – effective porosity

PERMEABILITY, k_p



Loose soil

- easy to flow
- **high** permeability

Dense soil

- difficult to flow
- **low** permeability



HYDRAULIC CONDUCTIVITY, **K**

- The **hydraulic conductivity K** is a measure of how easy the water can flow through the soil.
- The hydraulic conductivity is expressed in the units of velocity (such as cm/sec and m/sec).
- **Hydraulic conductivity** of soils **depends on** several factors:
 - Fluid viscosity (μ): **as the viscosity increases, the hydraulic conductivity decreases**
 - Pore size distribution
 - Temperature
 - Grain size distribution
 - Degree of soil saturation

HYDRAULIC CONDUCTIVITY, K

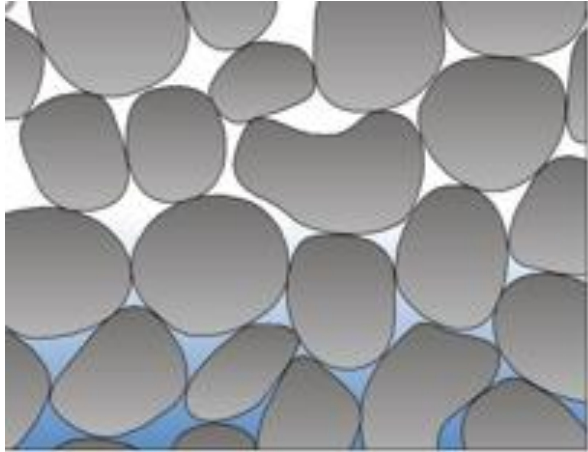
- Specific discharge (q) per unit hydraulic gradient
- Ease with which fluid is transported through porous medium
- ***Depends on both matrix and fluid properties***
 - Fluid properties:
 - Density ρ , and
 - Dynamic viscosity μ
 - Gravitational constant - g
 - Matrix properties
 - Pore size distribution
 - Pore shape
 - Tortuosity
 - Specific surface area
 - Porosity

$$K = k_p \frac{\rho g}{\mu}$$

Range of values of K

Medium	K in m/s
Gravel	10^{-3} to 1
Sand	3×10^{-6} to 10^{-2}
Typical BC Forest soil	10^{-7} to 10^{-5}
Bog soils	10^{-9} to 10^{-7}
Marine clay	10^{-12} to 10^{-9}
Basal till	10^{-12} to 10^{-10}
Igneous rock, shale	10^{-13} to 10^{-10}
Sandstone	10^{-10} to 10^{-6}

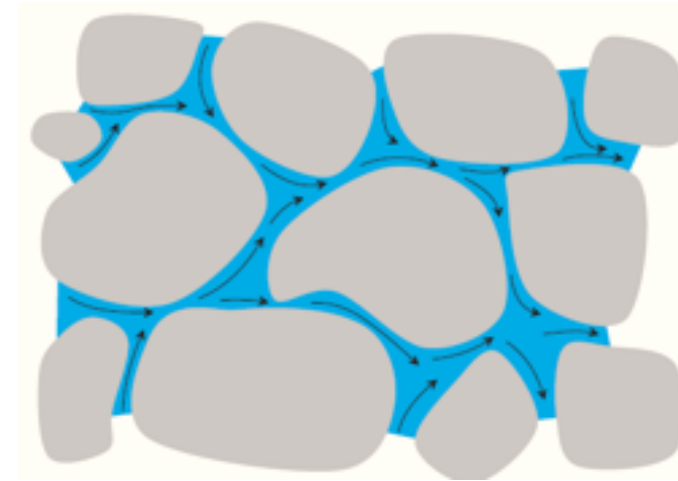
POROSITY VS. PERMEABILITY



Porosity

Ability to hold water

VS



Permeability

Ability to transmit water

Size, shape, interconnectedness

Porosity \neq Permeability

Some rocks have high porosity, but low permeability!!



TRANSMISSIVITY

Ease with which water moves through an aquifer
(rate at which water is transmitted through a unit width of aquifer
under a unit hydraulic gradient)

The product of **K** and the saturated
thickness of the aquifer, **b**

$$T = K.b$$

T: **Transmissivity**, $[L^2/T]$ e.g., m^2/d

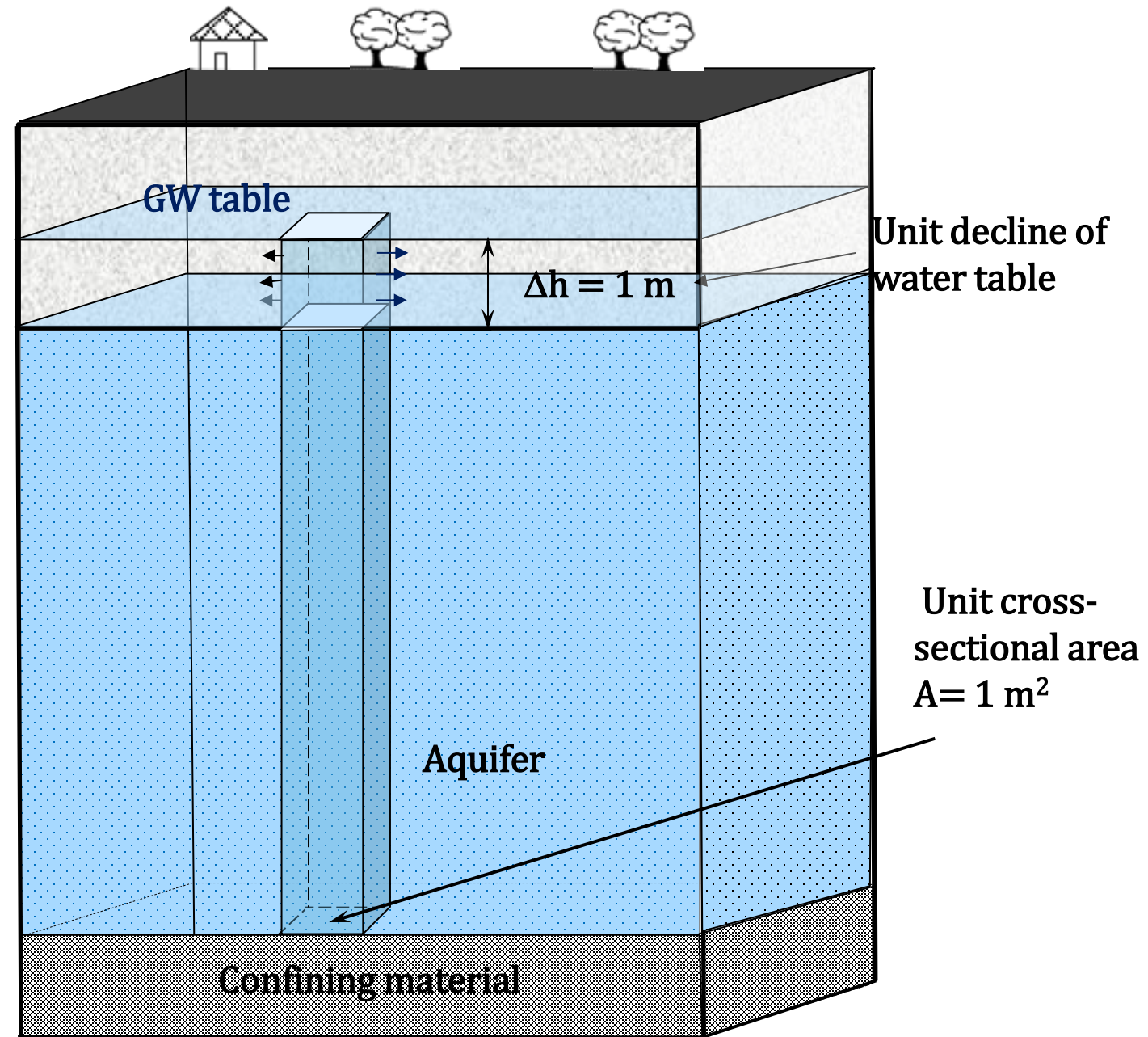
K: Hydraulic conductivity, $[L.T^{-1}]$

b: aquifer thickness, $[L]$

- **Storativity** (S_y)
- – **Specific yield** ability of an aquifer to store water
- Change in volume of stored water due to change in piezometric head.
- Volume of water released (taken up) from aquifer per unit decline (rise) in piezometric head.

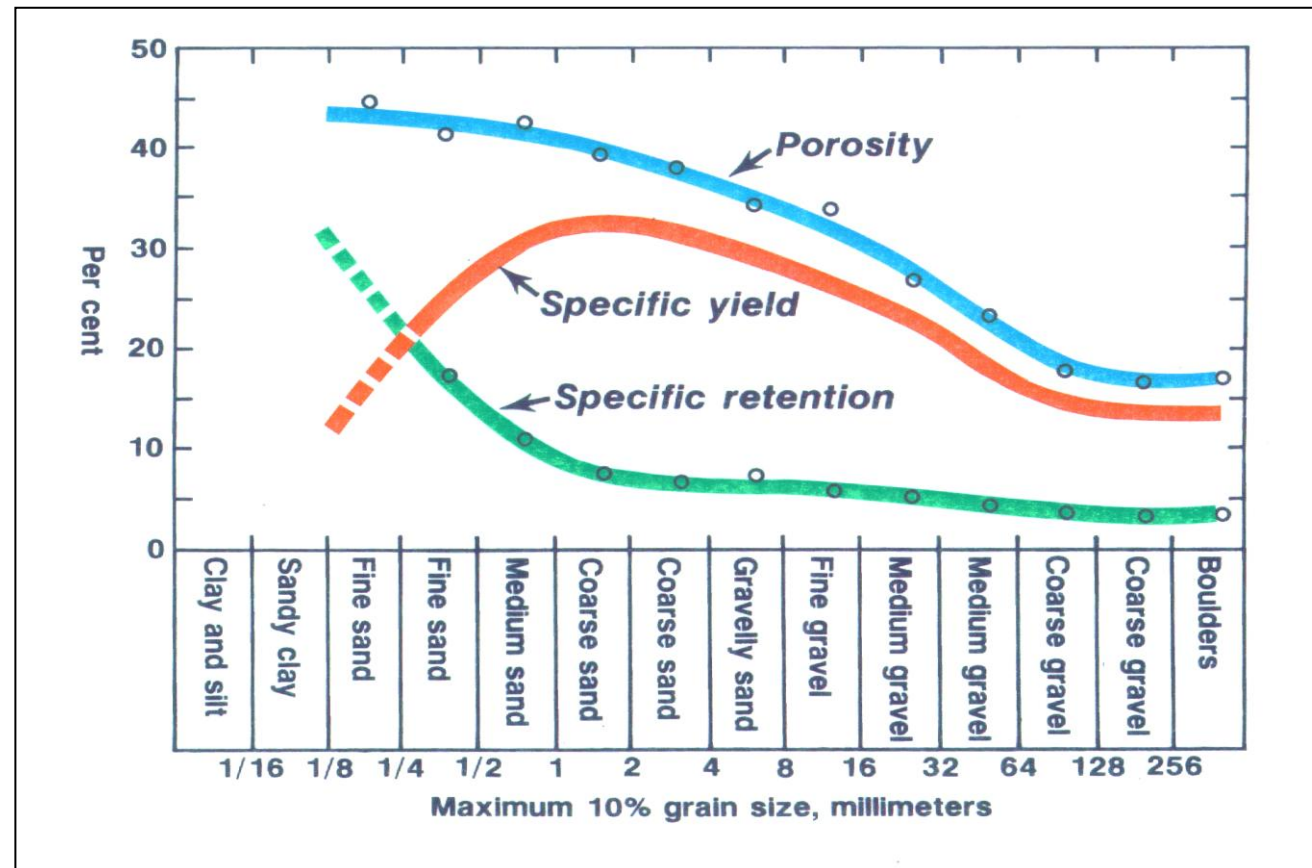
In **unconfined aquifer**, main source of water is drainage of water from pores


$$S_y = \frac{\Delta V}{A \Delta h}$$



POROSITY, SPECIFIC YIELD AND SPECIFIC RETENTION

- Porosity: maximum amount of water that a rock can contain when saturated.
- Portion of the GW: draining under influence of gravity: **SPECIFIC YIELD - S_y**
- Portion of the GW: retained as a film on rock surfaces and in very small openings: **SPECIFIC RETENTION - S_r**





SELECTED VALUES OF POROSITY, SPECIFIC YIELD AND SPECIFIC RETENTION

(values in percent by volume)

Material	Porosity	Specific yield	Specific retention
Soil	55	40	15
Clay	50	2	48
Sand	25	22	3
Gravel	20	19	1
Limestone	20	18	2
Sandstone	11	6	5