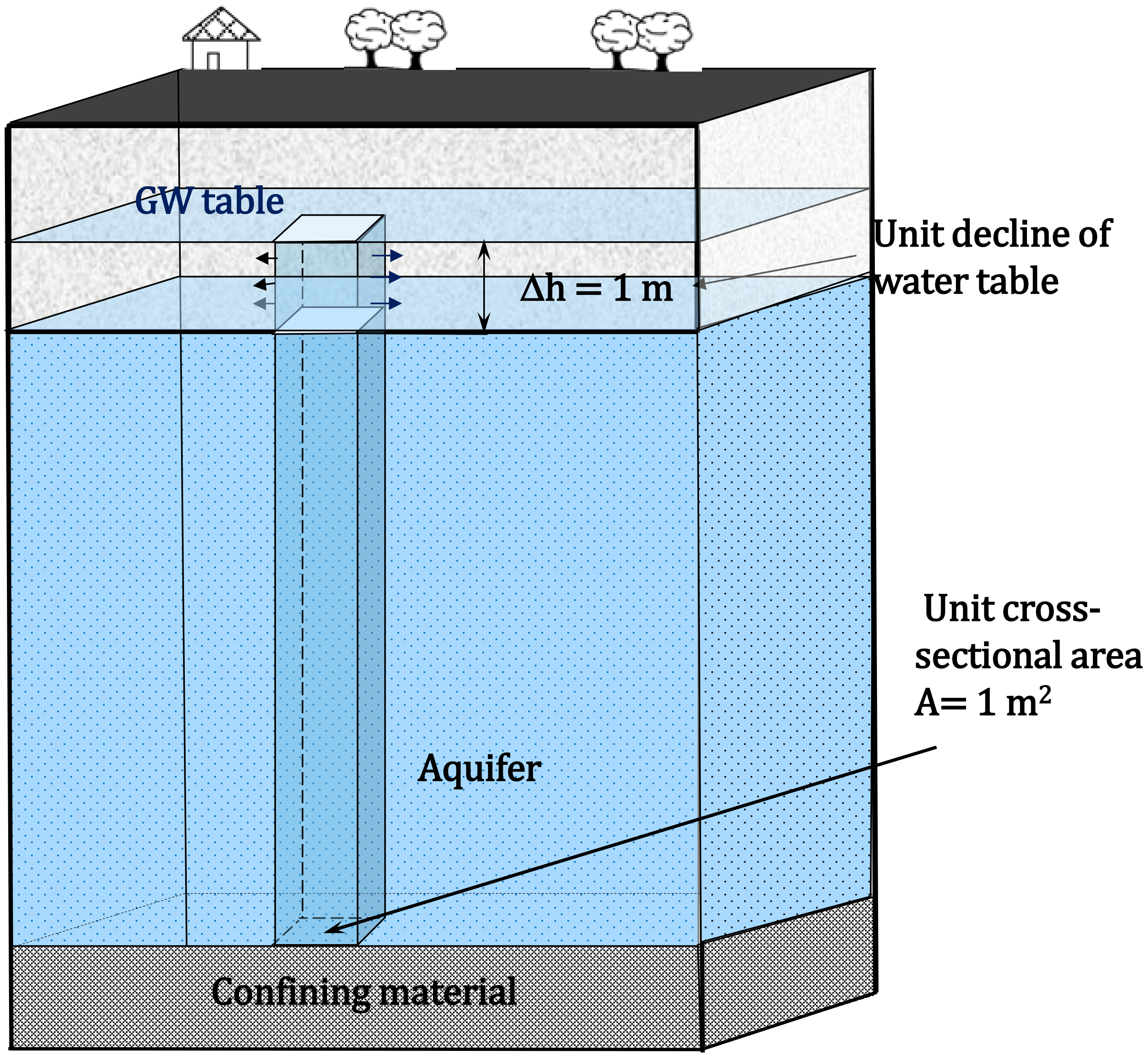

Groundwater Hydraulics
5

- **Storativity** (S_y)
- – **Specific yield** ability of an aquifer to store water
- Change in volume of stored water due to change in piezometric head.
- Volume of water released (taken up) from aquifer per unit decline (rise) in piezometric head.

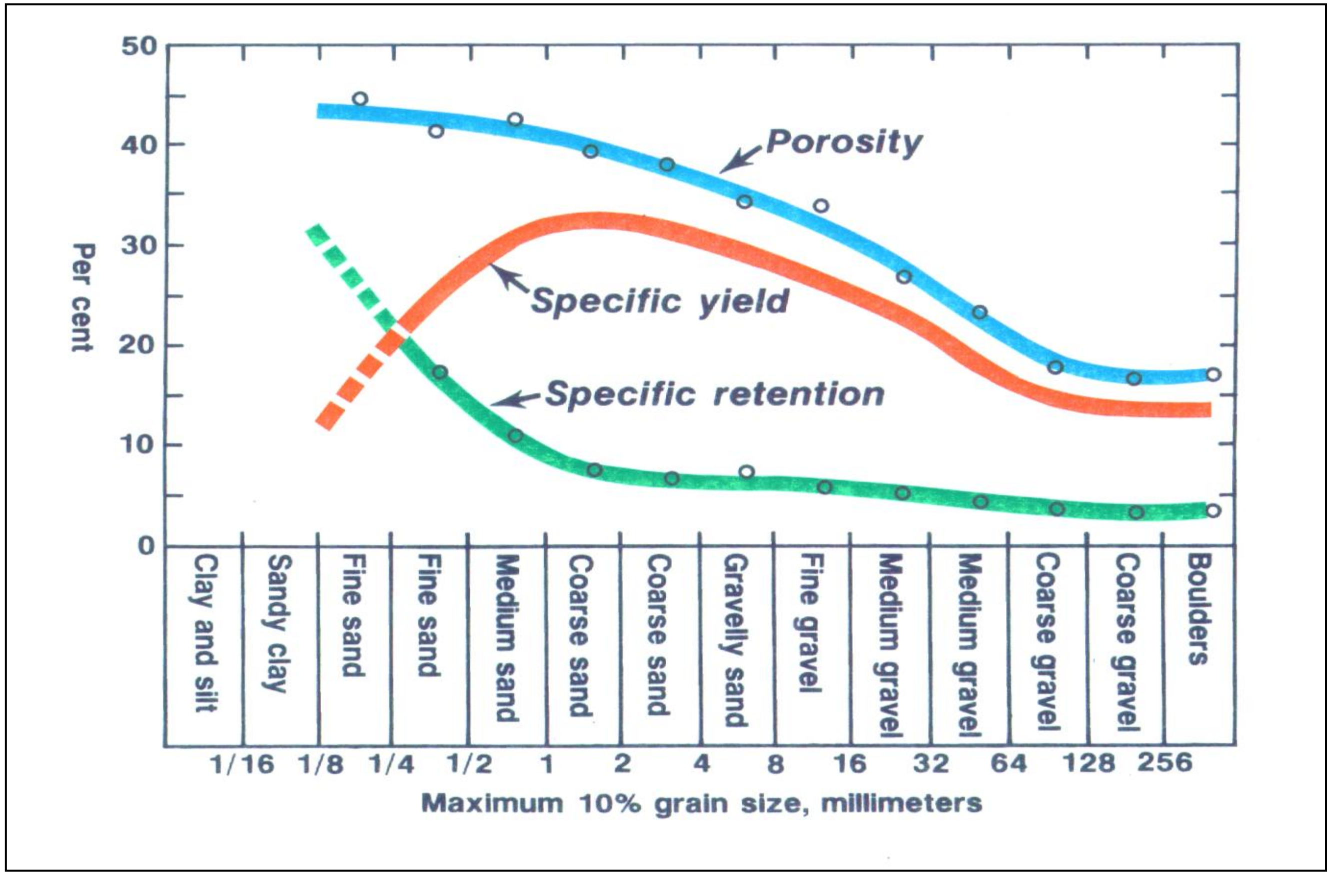
In **unconfined aquifer**, main source of water is drainage of water from pores

$$S_y = \frac{\Delta V}{A\,\Delta h}$$



POROSITY, SPECIFIC YIELD AND SPECIFIC RETENTION

- Porosity: maximum amount of water that a rock can contain when saturated.
- Portion of the GW: draining under influence of gravity: **SPECIFIC YIELD - S_y**
- Portion of the GW: retained as a film on rock surfaces and in very small openings: **SPECIFIC RETENTION - S_r**



SELECTED VALUES OF POROSITY, SPECIFIC YIELD AND SPECICIC RETENTION

(values in percent by volume)

Material	Porosity	Specific yield	Specific retention
Soil	**55**	**40**	**15**
Clay	**50**	**2**	**48**
Sand	**25**	**22**	**3**
Gravel	**20**	**19**	**1**
Limestone	**20**	**18**	**2**
Sandstone	**11**	**6**	**5**

STORATIVITY (COEFFICIENT OF STORAGE) AND SPECIFIC STORAGE

1. If water is removed from a confined aquifer:
   - **Hydraulic head decreases** - water level in wells falls
   - **Fluid pressure decreases** in the aquifer.
   - **Porosity decreases** as the granular skeleton contracts (aquifer collapses slightly)
   - The **volume of water increases**

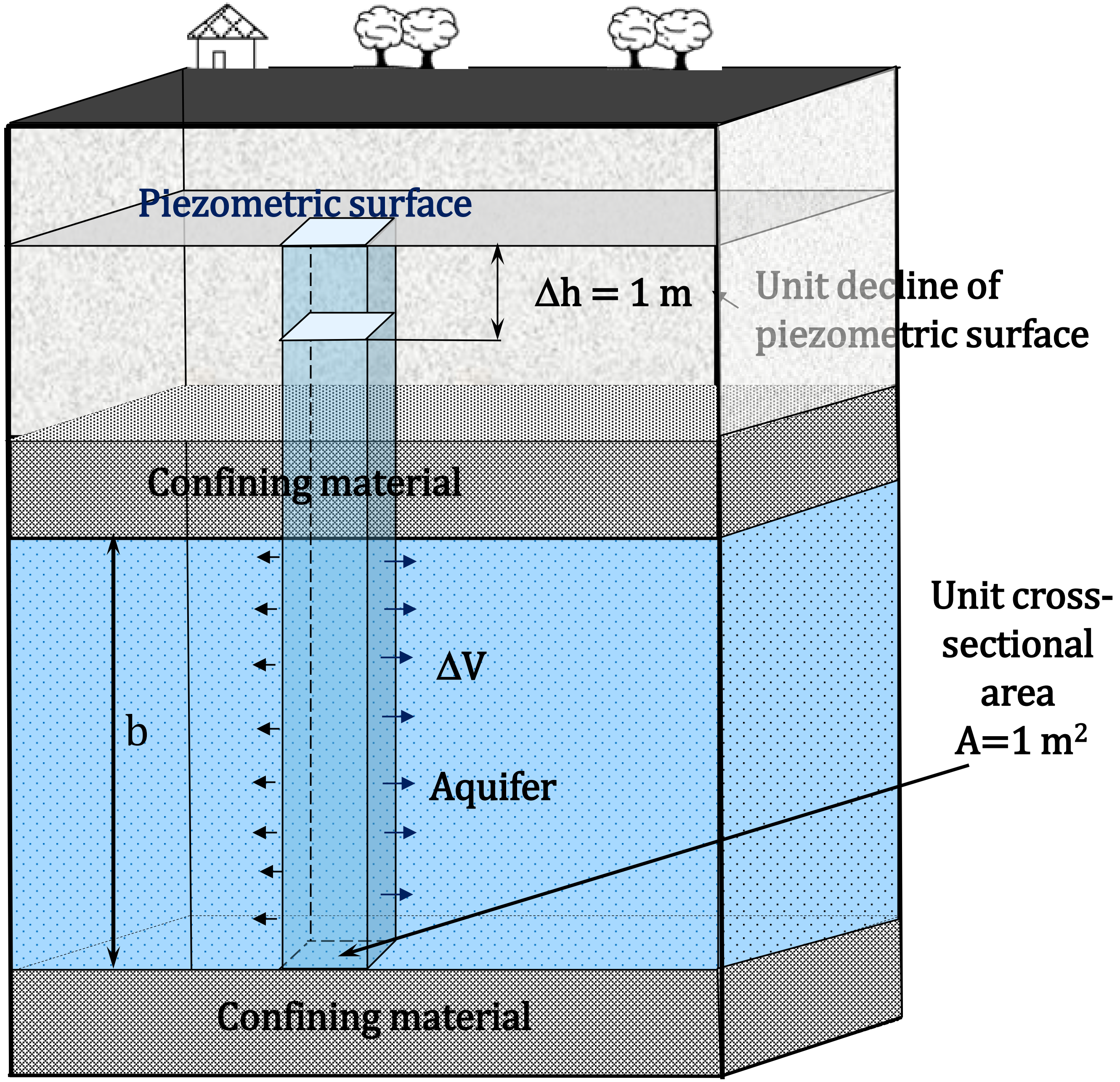
Water is released from storage via:

1. decrease in fluid pressure

2. increase in pressure from overburden

AQUIFER (CONFINED) STORAGE

- **Storativity** (S) - ability of an aquifer to store water
- Change in volume of stored water due to change in piezometric head.
- Volume of water released (taken up) from aquifer per unit decline (rise) in piezometric head.



- Storativity is a dimensionless property

S = volume of water/(unit area) (unit head change) = $L^3/(L^2 * L) = m^3/m^3$

SPECIFIC STORATIVITY, S_s

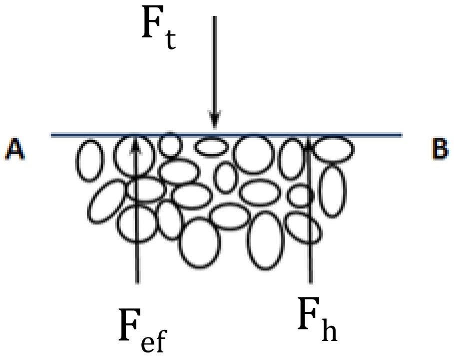
The specific storage of a saturated aquifer is defined as the volume of water released from the storage **per unit volume** of the **aquifet per unit decline in hydraulic head.**

SPECIFIC STORATIVITY, S_s

Karl Von Terzaghi

CONFINED AQUIFER

– **Terzaghi, 1925** – *effective stress* σ_{ef} – the portion of the total stress that is borne by the granular skeleton



Forces (A) – (B)

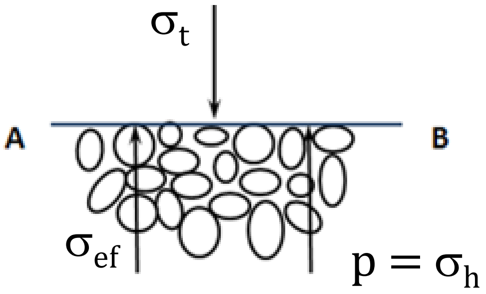
$$\mathbf{F_t = F_{ef} + F_h}$$

$\mathbf{F_t}$ – Total force above surface (A) – (B)

$\mathbf{F_{ef}}$ – effective force(A) – (B)

$\mathbf{F_h}$ – hydrostatic force (A) – (B)

Stresses (force/area):



$$\boldsymbol{\sigma_t = \sigma_{ef} + p}$$

In terms of the changes in these parameters

$$\boldsymbol{d\sigma_t = d\sigma_{ef} + dp}$$

SPECIFIC STORATIVITY, S_s

The change in total stress …... very small ⟶ 0

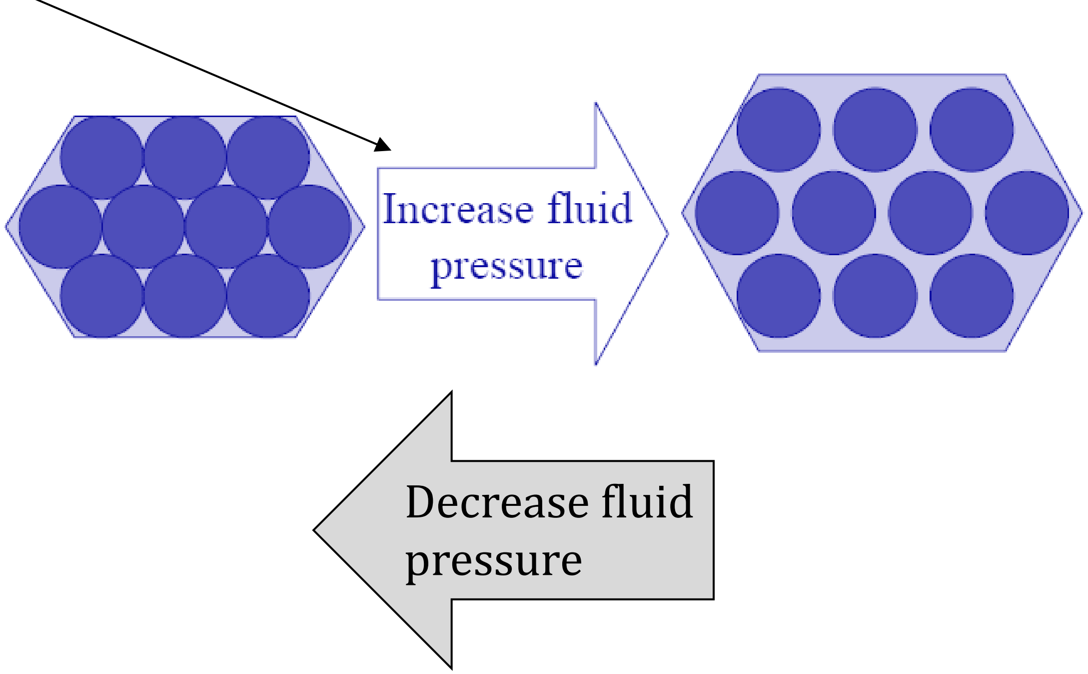
$$d\sigma_t \rightarrow 0$$

$$d\sigma_t = d\sigma_{ef} + dp \quad \rightarrow d\sigma_{ef} + dp = 0$$

Change in hydrostatic pressure ---- change in effective stress:

$$dp = -d\sigma_{ef}$$

an **increase in pressure** causes the grains to spread apart somewhat



SPECIFIC STORATIVITY, S_s

A. Compressibility of water

$$\beta_w = \frac{-1}{V_w}\frac{dV_w}{dp} \Rightarrow dV_w = -\beta_w . V_w \, dp$$

For saturated aquifer (porosity) → $n = \frac{V_w}{V_t} \Rightarrow V_w = nV_t$

$p = \rho\, g\, h \rightarrow dp = \rho\, g\, dh$

$$dV_w = -\beta_w . V_w \, dp = -\beta_w \underbrace{(n\, V_t)}_{V_w} \underbrace{(\rho\, g\, dh)}_{dp}$$

$$dV_w = \beta_w\, n\, \rho\, g$$

For specific storage and unit decline $V_T = 1$ a $dh = -1$

$$dV_w = -\beta_w\,(n\, V_t)(\rho\, g\, dh) = \beta_w\, n\, \rho\, g$$

Then

$$dV_w = \beta_w\, n\, \rho\, g$$

dV_w = volume of water produced by the expansion of water caused by decreasing hydrostatic pressure p

SPECIFIC STORATIVITY, S_s

B. Compressibility of aquifer

$$\alpha = -\frac{1}{V_t}\frac{dV_t}{d\sigma_{ef}} \Rightarrow -dV_t = \alpha V_t\, d\sigma_{ef}$$

$$\boldsymbol{V_t} = \boldsymbol{V_w} + \boldsymbol{V_s}$$

$$\boldsymbol{dV_t} = \boldsymbol{dV_w} + dV_s$$

$dV_s \rightarrow 0$

Solid part of porous media

$$\boldsymbol{dV_w} = -\boldsymbol{dV_t}$$

for confined aquifer

The negative sign is added since **the volumetric reduction dV_t** is n**egative**, but **the amount of water produced dV_w** is **possitive**

$$\boldsymbol{dV_w} = \alpha V_t\, d\sigma_{ef}$$

$$d\sigma_{ef} = -\rho\, g\, dh$$

$$\boldsymbol{dV_w} = \alpha V_t\, d\sigma_{ef} = -\alpha \boldsymbol{V_t}\, \rho g\, dh$$

Unit volume of aquifer **$V_T = 1$**

Unit decline in hydraulic head **$dh = -1$**

$$\boldsymbol{dV_w} = \alpha\, \rho\, g$$

SPECIFIC STORATIVITY, S_s

$$dV_w = \alpha \rho g \qquad\qquad dV_w = \beta_w \, n \, \rho \, g$$

The water released from the storage due to a decrease in h is produced by the two mechanism 1) expansion of the water caused by decreasing **p**
2) compaction of the aquifer caused by increasing $\boldsymbol{\sigma_{ef}}$

$$S_s = \alpha \rho g + \beta_w \, n \, \rho \, g$$

$$S_s = \rho g \, (\alpha + n\beta_w) \qquad (L^{-1})$$

where α - coef. of compressibilty of aquifer,
β_w - coef. of compressibilty of water
n - porosity
ρ – density of water
g – gravity acceleration

And storativity :

$$S = S_S \, . \, b$$

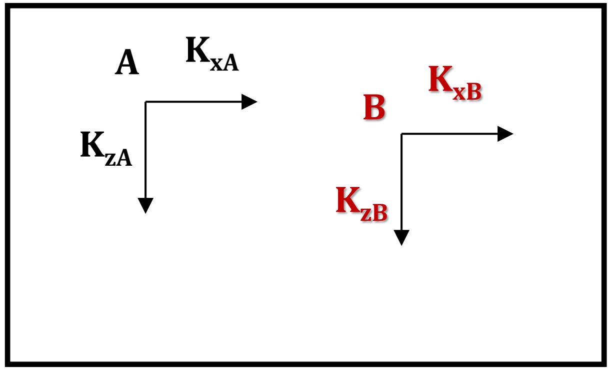
b – aquifer thickness

HOMOGENITY AND ISOTROPY
HETEROGENEITY AND ANISOTROPY

- **Homogeneous** aquifer
  - Properties are the same at every point
- **Heterogeneous** aquifer
  - Properties are different at every point
- **Isotropic** aquifer
  - Properties are same in every direction
- **Anisotropic** aquifer
  - Properties are different in different directions
- Often results from stratification during sedimentation

$$K_{horizontal} > K_{vertical}$$

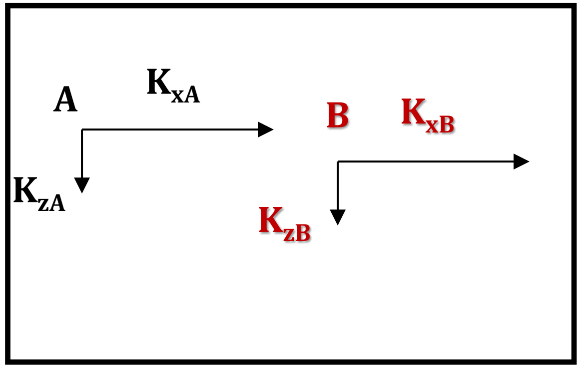
K(x,y) IN TWO DIMENSIONS



1. Homogeneous, isotropic

$K_{xA} = K_{xB}$ $K_{xA} = K_{zA}$

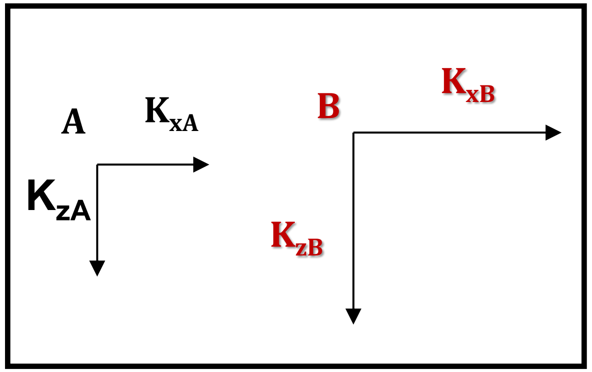
$K_{zA} = K_{zB}$ $K_{xB} = K_{zB}$



2. Homogeneous, anisotropic

$K_{xA} = K_{xB}$ $K_{xA} \neq K_{zA}$

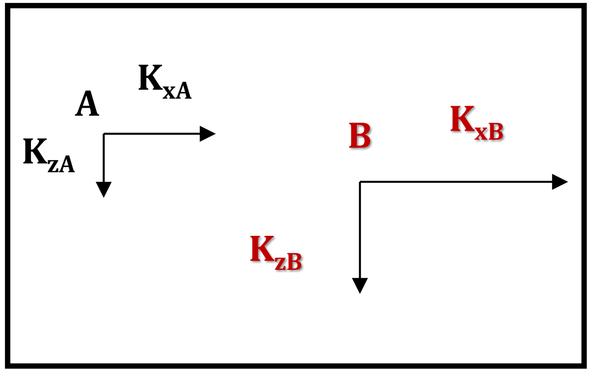
$K_{zA} = K_{zB}$ $K_{xB} \neq K_{zB}$



3. Heterogeneous, isotropic

$K_{xA} \neq K_{xB}$ $K_{xA} = K_{zA}$

$K_{zA} \neq K_{zB}$ $K_{xB} = K_{zB}$



4. Heterogeneous, anisotropic

$K_{xA} \neq K_{xB}$ $K_{xA} \neq K_{zA}$

$K_{zA} \neq K_{zB}$ $K_{xB} \neq K_{zB}$

HYDRAULIC HEAD

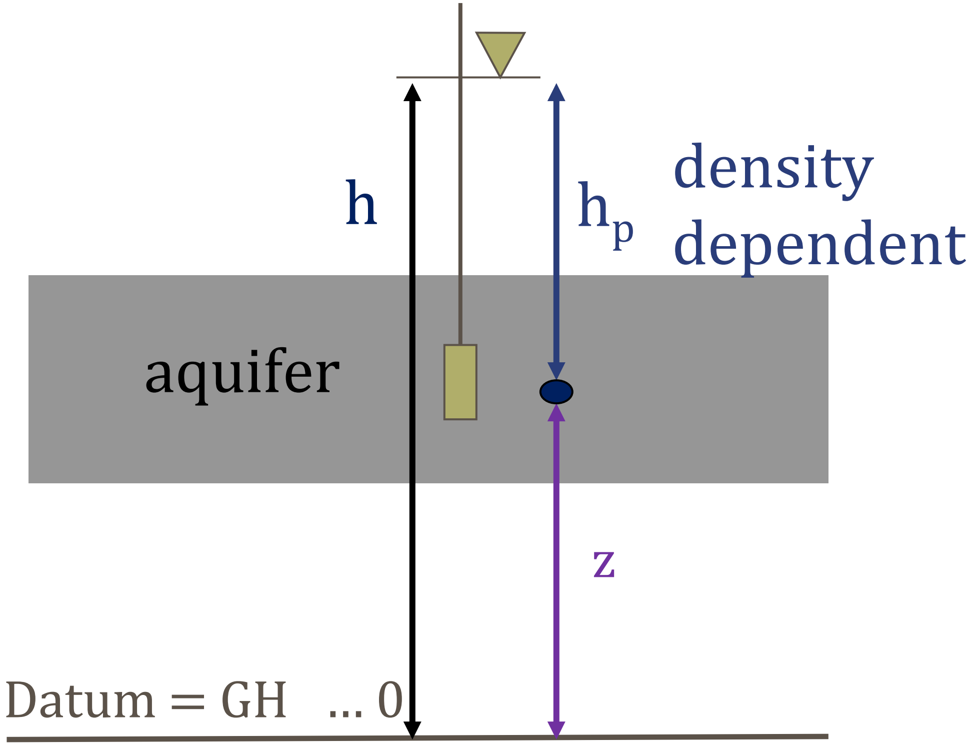
total head = elevation head + pressure head

(pressure head: varying density fluids – important in contamination or salinity)

HYDRAULIC HEAD

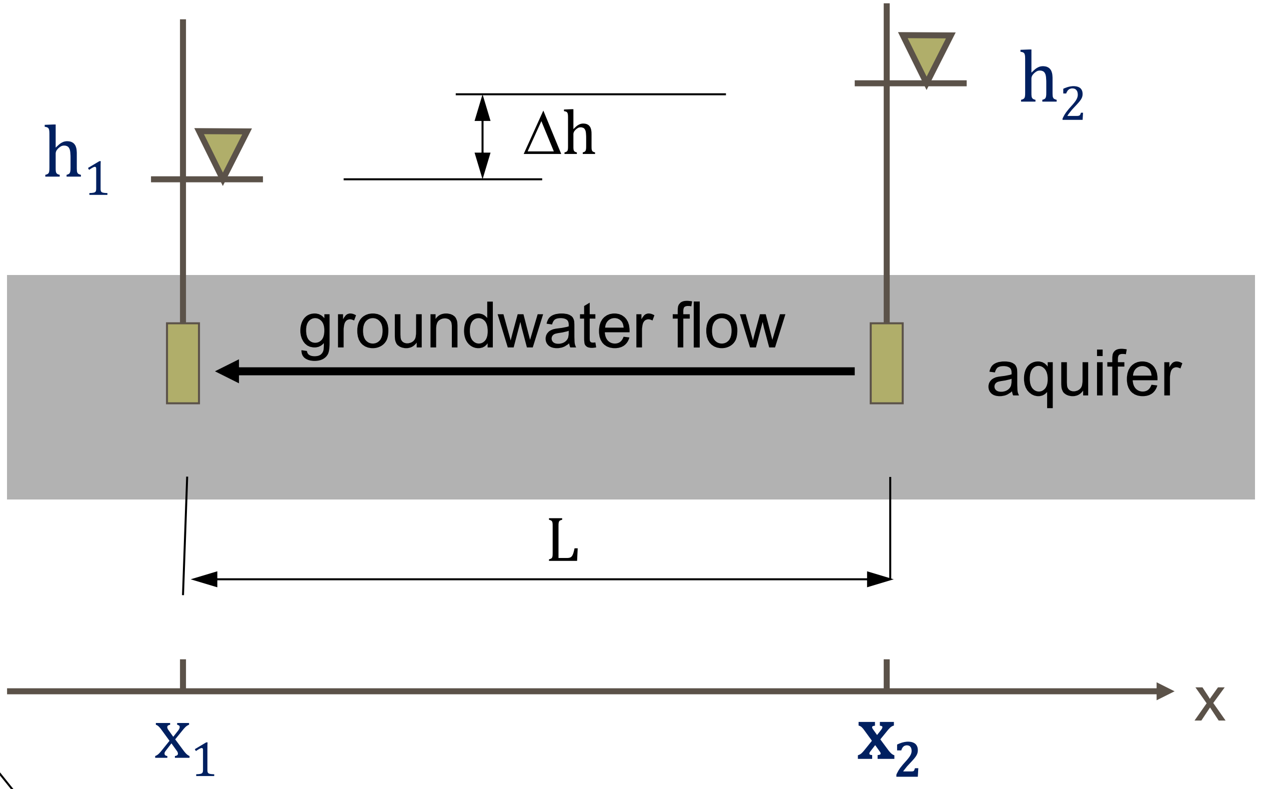
$$\boldsymbol{h} = z + h_p$$

$$h = z + \frac{p}{\gamma} = z + \frac{p}{\rho g}$$



HYDRAULIC GRADIENT

$$I = \frac{h_2 - h_1}{L}$$



$$\nabla h = (h_2 - h_1)/(x_2 - x_1)$$

KINDS AND FORMS OF FLOW (GROUNDWATER)

Unsteady flow ….. $Q = Q(t)$, $v = v(t)$

Steady flow ….. $Q\ (x, y, z) = const.$

uniform flow … $A = const. \quad v = const.$

non – uniform flow $A \neq const. \quad v \neq const.$

•with **free level** – flow limited by solid walls, free level on surface, motion caused by own weight of liquid

• **pressure** – flow limited by solid walls from all sides, motion caused by difference of pressures (**hydraulic heads**)

Solution of flow:

• space flow (3D numeric models)

• planar flow (2D – simplified solution)

• one dimensional (1D)

Flow regime :

- laminar
- turbulent

HEAD LOSS IN POROUS MEDIA

- Piezometric head $$h_1 = \frac{p_1}{\gamma} + z_1$$

- Energy is lost in the flow through the porous medium due to friction

- Energy equation $$\frac{v_1^2}{2g} + \frac{p_1}{\gamma} + z_1 = \frac{v_2^2}{2g} + \frac{p_2}{\gamma} + z_2 + h_L$$

- Neglect velocity terms $$h_L = \left(\frac{p_1}{\gamma} + z_1\right) - \left(\frac{p_2}{\gamma} + z_2\right) = h_1 - h_2 = \Delta h$$

- Flow is always from higher head to lower head

HEAD LOSS IN POROUS MEDIA

- Piezometric head
- Energy is lost in the flow through the porous medium
  due to friction
- Energy equation
- Neglect velocity terms
- Flow is always from higher head to lower head

TERMS TO REMEMBER

Pressure head: water pressure at a given point, which can be measured by a piezometer
Elevation head: height above GH (z=0)
Total head: the sum of pressure and elevation head
Potential energy: product of the total head and the gravitational constant
Hydraulic gradient: change in the total head per unit distance
Hydraulic conductivity: water flux density per unit volume of water and per unit hydraulic gradient
Macroscopic velocity: the speed of water flow through the cross-sectional area of solid matrix and interstices

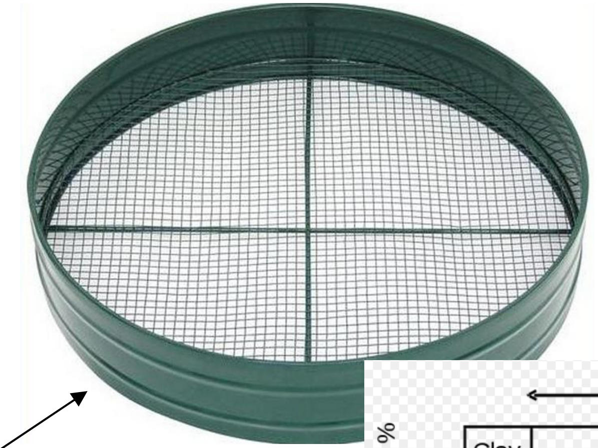
LAMINAR AND TURBULENT FLOW

- **laminar – particles of liquid move at parallel paths**
- **turbulent – motion of particles of liquid: irregular and** inordinate, fluctuations of velocity vector in time and space, mixing inside flow
- Criterion – **Reynolds number**

$$\boldsymbol{Re_f = \frac{vd_{10}}{\nu}}$$

v - velocity

d_{10} = effective grain size diameter

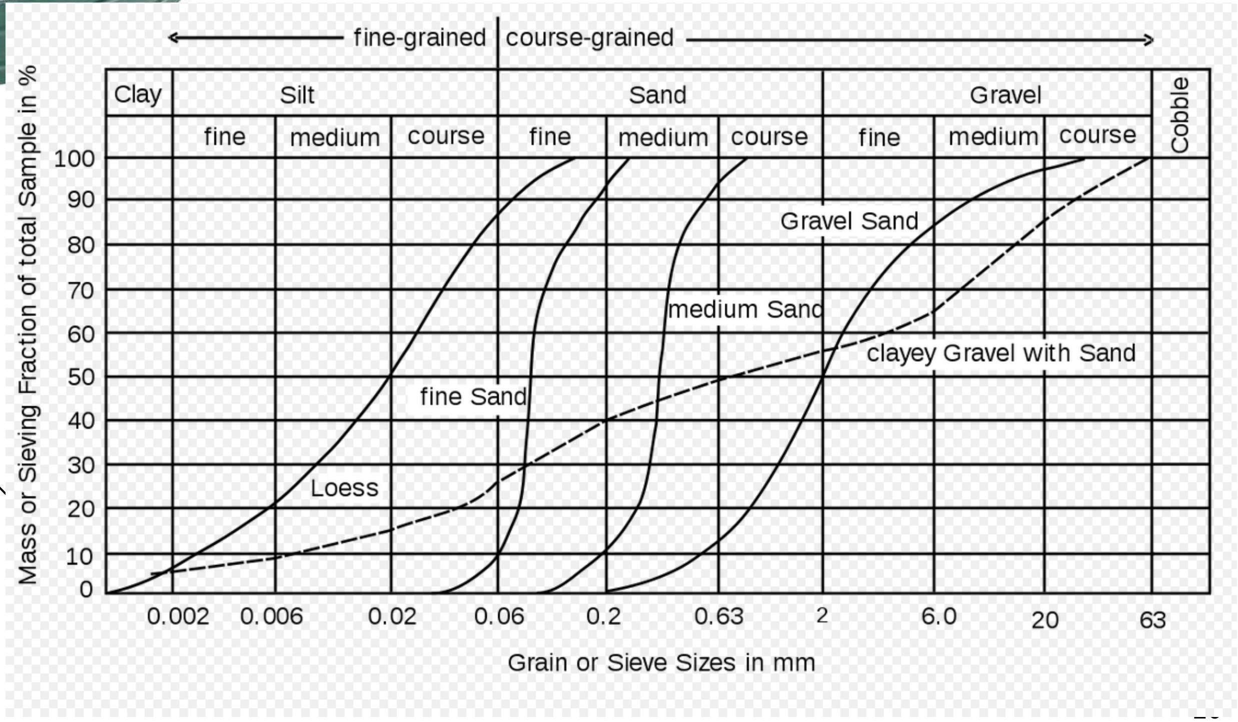


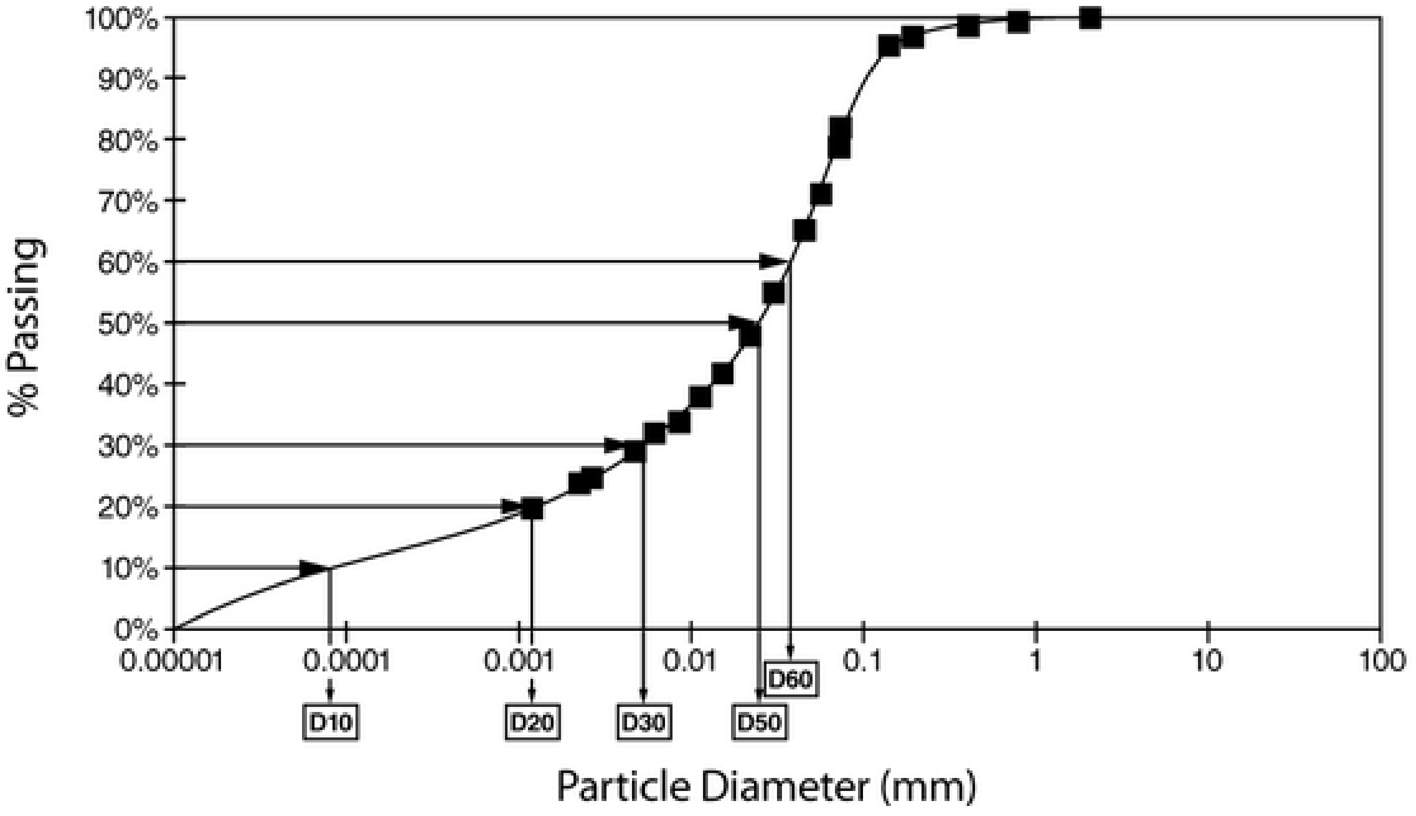
CRITICAL REYNOLDS NUMBER

for groundwater flow $Re_{fcr} = 1$

The *Reynolds number* can be used as a criterion to distinguish between laminar and turbulent flow:

A sieve analysis (or gradation test) is a practice or **procedure used** (commonly used in civil engineering) **to assess the particle size distribution** (also called gradation) of a granular material **by allowing the material to pass through a series of sieves of progressively smaller mesh size** and weighing the amount of material that is stopped by each sieve as a fraction of the whole mass...




100%
90%
80%
70%
60%
50%
40%
30%
20%
10%
0%
% Passing
0.00001
0.0001
0.001
0.01
0.1
1
10
100
D10
D20
D30
D50
D60
Particle Diameter (mm)

DARCY’S LAW

- Water flow through an aquifer.
- Darcy's law (conservation of momentum) was determined experimentally by Darcy, it can be derived from the Navier-Stokes equations
- Analogous to Fourier's law, Ohm's law, or Fick's law
- Darcy's law (conservation of momentum) and the continuity equation (conservation of mass) are used to derive the groundwater flow equation

LINEAR TRANSPORT LAWS

- Fourier’s Law – **Heat** is transferred from a region of higher temperature to a region of lower temperature

Jean B. J. Fourier
1768-1830

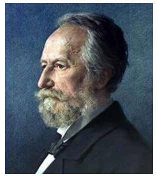
$$Q = -kA\frac{dT}{dx}$$

- Ohm's law – **Electricity** is transferred from a region of higher voltage to a region of lower voltage

Georg Simon Ohm
1789-1854

$$I = -\frac{1}{\rho}A\frac{dV}{dx}$$

- Fick's law – **Mass** is transferred from a region of higher concentration to a region of lower concentration



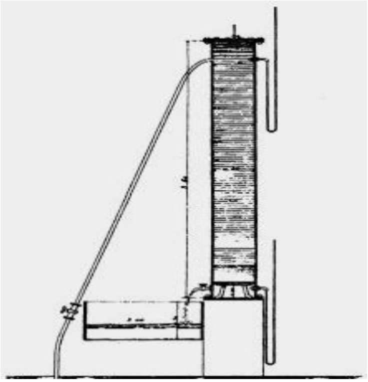
Adolf Eugen Fick
1829-1901

$$J = -DA\frac{dC}{dx}$$

- **Darcy´s law** - ???

Henry Darcy
1803 - 1858

Henry Darcy 1856

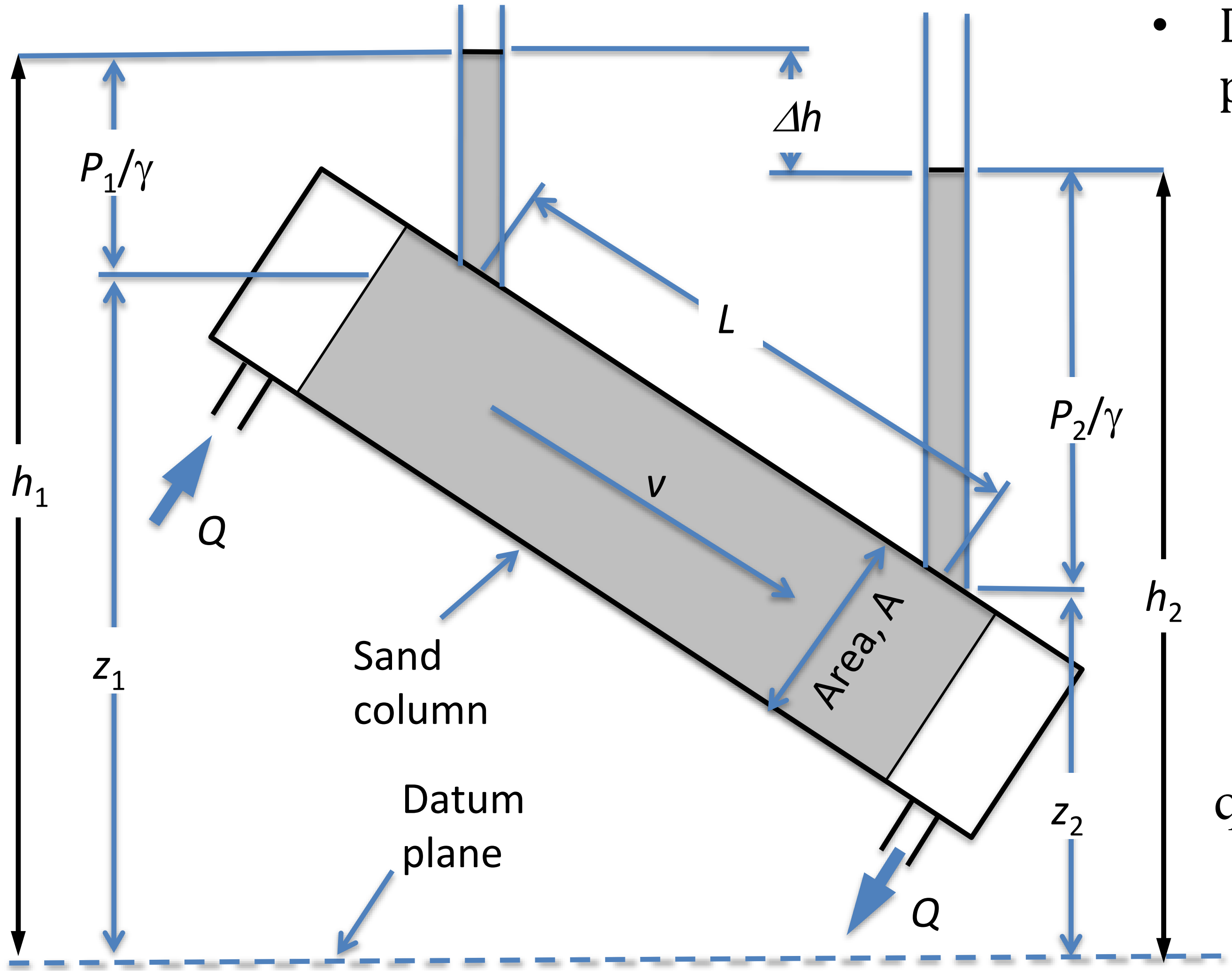


Experimental equipment

Darcy's Experimental Data

NUMÉRO de L'EXPÉRIENCE	DURÉE.	DÉBIT MOYEN par minute.	PRESSION MOYENNE SUR LE FILTRE	PRESSION MOYENNE SOUS LE FILTRE	DIFFÉRENCE des PRESSIONS.	RAPPORT des VOLUMES aux pressions.	OBSERVATIONS.
1	2	3	4	5	6	7	8
		l.	m.	m.	m.		
1	15'	18,8	P + 9,48	P − 3,60	13,08	1,44	Fortes oscillations dans le manomètre supérieur.
2	15'	18,3	P + 12,88	P 0	12,88	1,42	Id.
3	10'	18,0	P + 9,80	P − 2,78	12,58	1,43	Id.
4	10'	17,4	P + 12,87	P + 0,46	12,41	1,40	Faibles.
5	20'	18,1	P + 12,80	P + 0,49	12,33	1,47	Assez faibles.
6	16'	14,9	P + 8,86	P − 0,83	9,69	1,54	Presque nulles.
7	15'	12,1	P + 12,84	P + 4,40	8,44	1,43	Très-fortes.
8	15'	9,8	P + 6,71	P 0	6,71	1,46	Très-faibles.
9	20'	7,9	P + 12,81	P + 7,03	5,78	1.37	Très-fortes.
10	20'	8,63	P + 5,58	P 0	5,58	1,55	Presque nulles.
11	20'	4,5	P + 2,98	P 0	2,98	1,51	Id.
12	20'	4,15	P + 12,86	P + 9,88	2,98	1,39	Assez fortes. On a déjà expliqué la cause de ces oscillations.

DARCY'S EXPERIMENT



- Flow through sand filters
- Discharge (Q) proportional to
  - Area, A
  - Head drop, h_1-h_2
  - Inverse of length, L

$$\mathbf{Q \propto K.A \frac{h_1 - h_2}{L}}$$

$$\Delta h = h_2 - h_1$$

$$q = v = \frac{Q}{A} = -K\frac{\Delta h}{L}$$

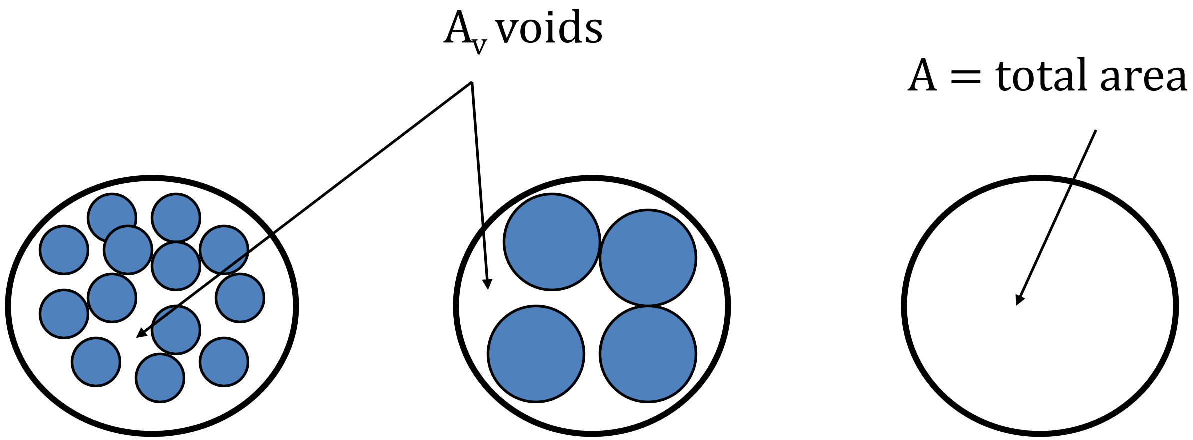
- **GROUNDWATER FLOW**

  - Direction controlled by hydraulic gradient
  - Rate controlled by gradient and hydraulic conductivity

- **Hydraulic gradient** (change in head)
  - flow occurs from high to low head
  - flow is down the hydraulic gradient
  - dh/dz, $\partial h/\partial x$, ∇h etc.

- **DARCY VELOCITY v_D** is a fictitious velocity since it assumes that flow occurs across the entire cross-section of the sediment sample. Flow actually takes place only through interconnected pore channels (voids), at the seepage velocity v_s
- Effective porosity, n_{ef} for

ACTUAL GROUNDWATER VELOCITY (seepage velocity) - v_s



DARCY'S LAW

- v = K * i
  - V : Flow velocity

  - K : Hydraulic conductivity

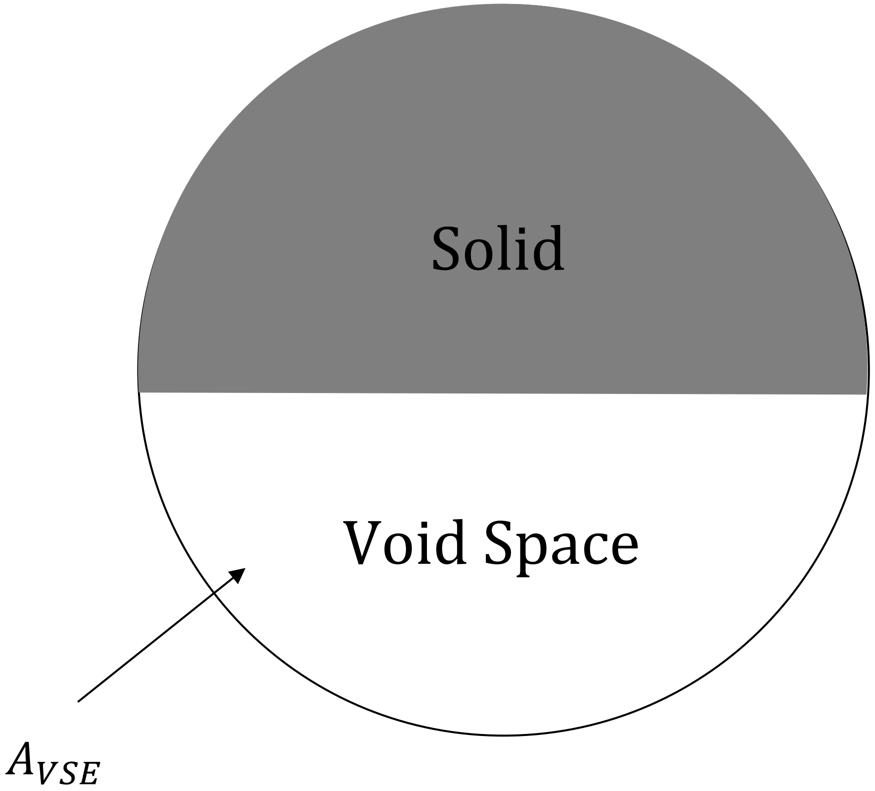
  (The rate at which a soil allows water to move through it)

  - i : Hydraulic gradient; i = Δh / L

  (Change in hydraulic head per unit of horizontal distance)

VELOCITY THROUGH POROUS MEDIUM

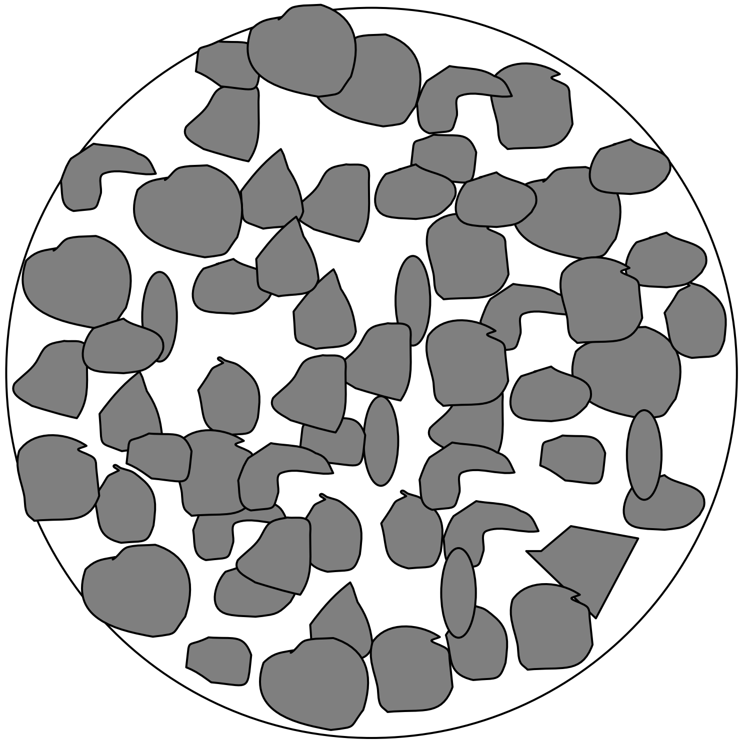
Pipe



“Porosity” = 0.5

$$v_R = \frac{Q}{A_{VSE}} \text{ (real velocity)}$$

Porous Medium



Porosity = 0.5

DARCY & SEEPAGE VELOCITY

- From the Continuity eq.:

$$Q = A\, v_D = A_{VS}\, v_s = A_{VSE}\, v_R$$

– where:

$\boldsymbol{Q}$ = flow rate

$\boldsymbol{A}$ = total cross-sectional area of material

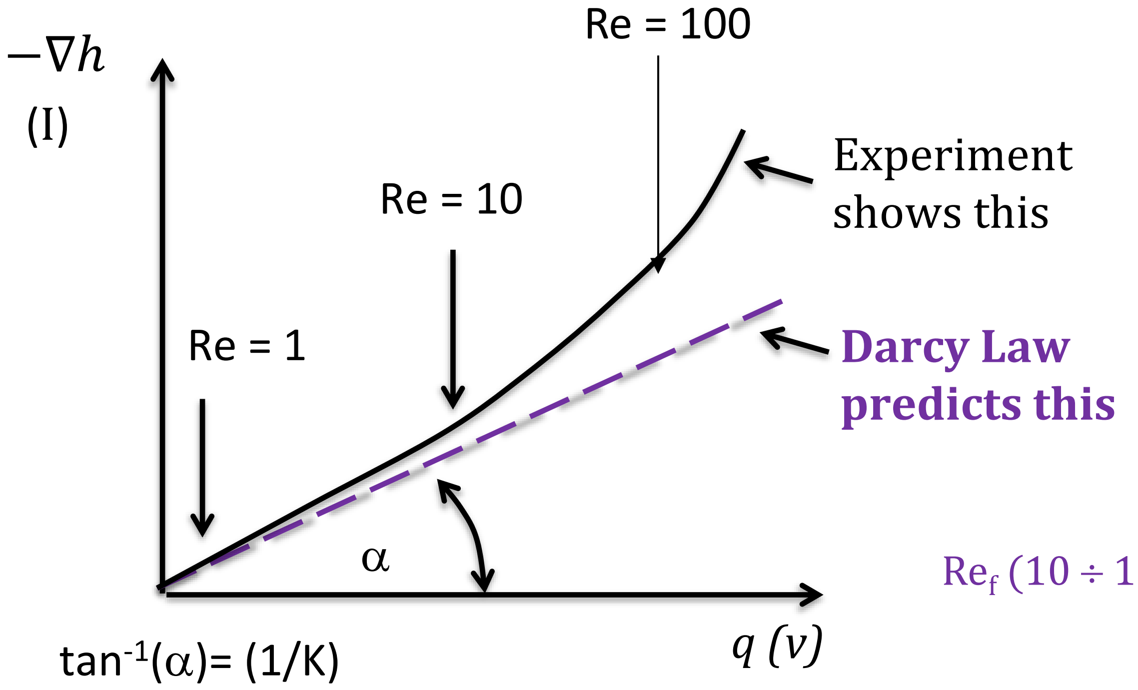
$\boldsymbol{A_{VS}}$ = area of voids

$\boldsymbol{v_s}$ = seepage velocity

$\boldsymbol{v_D}$ = Darcy velocity

$$v_R = v_D \frac{A}{A_{VSE}} \quad \dots\dots\dots. \quad \rightarrow \quad v_D = n_{ef}\, v_R$$

VALIDITY OF DARCY'S LAW



Re_f (0 ÷ 1) – Darcy eq. is valid

$$v = -K\,I \quad \Rightarrow \quad I = av$$

kde $a = 1/K$

Re_f (1 ÷ 10) Darcy eq. is also valid

Re_f (10 ÷ 100) -Nondarcian flow (Darcy eq. is not valid)

$$I = av + b.\,v^m$$

where **m** = 1,6 ÷2,0

Re_f >100 turbulent flow (Darcy eq. is not valid)

$$I = b\,v^2$$