



Technical note

Theoretical evaluation of high-permeability wellbore skin effect on aquifer response under pumping

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ABSTRACT

Skin zones with distinct hydraulic properties often form around wellbores during drilling operations. Conventional skin-factor models, developed for low-permeability (positive) skin, assume discontinuous hydraulic heads between the wellbore and aquifer. However, these models may be inadequate for high-permeability (negative) skin scenarios resulting from intensive well development. This study derives a new skin-factor approach for negative skin approaches based on the two-zone model approach. We derive a new skin-factor approach that maintains continuous heads between the wellbore and aquifer while allowing for discontinuous fluxes. Using Laplace and finite cos-Fourier transformations, we develop a groundwater flow model for aquifer pumping under a partially penetrating well, incorporating this new approach. An asymptotic expansion method is employed to obtain a large-time solution. The proposed model is also compared to the existing two-zone model. Sensitivity analysis reveals that high-permeability skin significantly affects drawdown, particularly when there is a sudden increase in porosity within the skin zone. This revised approach provides a more accurate framework for modeling various skin effects, offering valuable insights for groundwater management and well hydraulic modeling in both hydrogeology and petroleum engineering applications.

1. Introduction

Aquifer pumping is a valuable technique for characterizing the hydraulic properties of aquifers, such as hydraulic conductivity and specific storage. During a pumping test, water is extracted from a well while temporal drawdown data are measured at observation wells. These hydraulic parameters can be determined using either an analytical drawdown solution coupled with an optimization method or a type-curve method for analyzing the measured data. Numerous studies have developed mathematical models for drawdown induced by pumping in confined aquifers (Theis, 1935; Hantush et al., 1964; Papadopoulos and Cooper, 1967; Yeh and Chang, 2013; Lin et al., 2016).

During well development, a skin zone may form near the well due to drilling and/or development activities. A positive skin (also called a low-permeability skin) usually results from damage caused by well drilling and incomplete development, creating a zone adjacent to the wellbore with lower hydraulic conductivity than the undisturbed formation (Houben, 2015; Houben et al., 2023). Conversely, a negative skin (or high-permeability skin) is a zone around a well with higher hydraulic conductivity than the surrounding formation (Yeh et al., 2008; Valois et al., 2024). Houben et al. (2022) mentioned that this occurs when well development processes remove fine-grained

particles during lateral insertion or subsequent operations because the removal of these fines enhances hydraulic conductivity near the well. Wellbore skin is commonly introduced to account for the skin zone's influence on drawdown. The skin thickness may range from a few millimeters to several meters (Novakowski, 1989; Houben et al., 2023). The literature offers various approaches for developing solutions that consider the skin effect (Hurst, 1953; Van Everdingen, 1953; Hawkins, 1956; Agarwal et al., 1970; Sandal et al., 1978; Moench and Hsieh, 1985a; Novakowski, 1989; Moench, 1997; Cassiani et al., 1999). Two main types of approaches incorporate the skin effect on drawdown: the two-zone approach and the skin-factor approach.

The two-zone approach assumes the skin zone is independent of the aquifer formation zone, with distinct physical properties and different initial and boundary conditions. These zones are connected using continuity conditions (Barker and Herbert, 1982; Moench and Hsieh, 1985a; Butler and McElwee, 1990; Novakowski, 1989; Yang and Gates, 1997; Yang and Yeh, 2005, 2007; Barua and Bora, 2010; Tsai and Yeh, 2012; Lin et al., 2016). On the other hand, the skin-factor describes a single flow model for the aquifer with a skin-factor condition as an inner boundary condition at the well. This model introduces a dimensionless parameter called the skin factor to reflect the head loss caused by

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Nomenclature

b	Aquifer thickness
C_D	$1/2S_s b$
C_g	Factor defined as $S_{s,skin} r_s$
c	Intercept
d	Depth to top of the well screen
d_D	d/b
K_0	Modified Bessel function of second type with orders 0
K_1	Modified Bessel function of second type with orders 1
K_r	Hydraulic conductivity of aquifer in radial direction
K_w	Hydraulic conductivity of wellbore
K_z	Hydraulic conductivity of aquifer in vertical direction
$K_{z,skin}$	Hydraulic conductivity of wellbore skin
l	Depth to bottom of the well screen
l_D	l/b
m	Slope
P	Parameter
p	Laplace parameter
Q	Pumping rate
q_r	Flux of the aquifer near the wellbore skin in radial direction
$q_{r,w}$	Flux of the wellbore
$q_{r,skin}$	Flux of the wellbore skin in radial direction
$q_{z,skin}$	Flux of the wellbore skin in vertical direction
r	Radial distance from the center of the pumping well
r_D	r/r_w
r_s	Skin zone thickness
r_w	Radius of wellbore
S_f	Skin factor defined as $K_{z,skin} r_s$
S_s	Specific storage of aquifer
$S_{s,skin}$	Specific storage of wellbore skin
s	Drawdown of aquifer
s_D	$4\pi K_r b s / Q$
s_{skin}	Drawdown of wellbore skin
s_w	Drawdown of wellbore
t	Operating time
t_D	$K_r t / r_w^2 S_s$
X	Sensitivity value
z	Depth for the ground
z_D	z/b
α	Skin factor for positive skin
β	$S_f r_w / b^2 K_r$
Δ	0.001
γ	Euler–Mascheroni constant, approximately 0.577216
κ	$r_w^2 K_z / b^2 K_z$
λ	$\sqrt{\kappa \xi_n^2 + p}$
λ_0	$\xi_n \sqrt{\kappa}$
ξ_n	Fourier parameter defined as $n\pi$

Abbreviations

DIM	Differential importance measure
PSF	Positive skin factor
NSF	Negative skin factor

the skin zone (Van Everdingen, 1953; Hawkins, 1956; Moench, 1997; Moench and Hsieh, 1985b; Cassiani et al., 1999; Chen and Chang, 2003, 2006; Lin et al., 2017; Wang et al., 2022; Lin and Yeh, 2022; Mahdavi et al., 2024). Essentially, the two-zone model and the skin-factor model can be viewed as distributed-parameter and lumped-parameter models, respectively. In practice, the skin-factor model is more commonly applied to field tests due to its relative simplicity. The fewer parameters required by the skin-factor model help reduce uncertainties associated with parameter estimation in the model.

The commonly used skin-factor approach is based on the assumption of positive skin, as evidenced by the head loss term in the head continuity equation (Moench, 1997). This positive-skin-factor (PSF) approach assumes discontinuous hydraulic heads between the wellbore and aquifer, while maintaining continuous fluxes through the skin zone. In the PSF approach, head loss is proportional to the wellbore flux and governed by a dimensionless skin factor (Van Everdingen, 1953; Hurst, 1953). However, the PSF approach is only effective when the skin zone permeability is sufficiently low. For negative skin, the assumption of head discontinuity is invalid due to the expectation of continuity. Applying the PSF approach to negative skin scenarios results in a physically impossible situation where the wellbore head exceeds the aquifer head, yet flow continues towards the pumping well. To address this issue, Brons and Miller (1961) proposed the concept of an effective radius to account for the skin effect, suggesting that the skin effect diminishes as the effective radius increases. However, this empirical approach does not fundamentally resolve the PSF approach's limitations in accurately representing flow behavior associated with negative skin.

The mathematical techniques applied in well hydraulics are closely related to those employed in heat conduction, due to the similar physical principles and mathematical frameworks governing both phenomena. For instance, the foundational (Theis, 1935) solution was developed based on the analogies between radial groundwater flow and radial heat flow (Freeze, 1990; Kurylyk and Irvine, 2019). Consequently, methodologies used in the study of heat conduction can be effectively adapted for applications in well hydraulics. In the field of thermal conduction, Lin and Chen (2023) focused on designing a thermal invisibility cloak that accounts for the effects of imperfect interfaces within a three-zone radial composite material. Their study considered the presence of low-conductivity and/or high-conductivity materials at the interface (in the middle zone). Their work demonstrates that the mathematical model of the middle zone can be simplified to interface conditions representing low- and/or high-conductivity type interfaces through an asymptotic approach using Taylor's expansion, effectively characterizing the impact of imperfections across the interface. They found that the low-conductivity type interface describes the extra temperature drop at the imperfect contact with continuous heat flux, whereas the high-conductivity type interface maintains temperature continuity while allowing for heat flux discontinuity due to the presence of the high thermal gradient. We notice that the low-conductivity interface condition is analogous to the classical positive-skin-factor (PSF) approach in well hydraulics; namely, the PSF condition describes additional head loss at the wellbore surface due to the wellbore skin resistance with continuous water flux condition. Although the high-conductivity case resembles the negative-skin-factor (NSF) approach, it lacks a comprehensive theoretical framework. Ideally, this framework would account for the continuity of the hydraulic head while addressing the potential discontinuity in water flux. Inspired by this approach, we propose that the NSF approach in well hydraulics could be derived in a similar manner.

In this study, we revisit the skin factor theory and derive a new skin-factor approach based on modifying two-zone model, building upon the approach presented by Lin and Chen (2023). This two-zone model provides a general framework describing the flow interaction among the wellbore skin, aquifer formation, and wellbore. We derive the new NSF approach using a Taylor expansion of the two-zone model. We

then incorporate the NSF approach into a groundwater flow model to describe drawdown distribution induced by constant-rate pumping in a partially penetrating well within a confined aquifer. The model solution is obtained using Laplace and finite cos-Fourier transform techniques, with the large-time solution derived through asymptotic expansion in the Laplace domain. We compare our solution to the classical PSF solution and large-time solution. We explore the effects of PSF and NSF approaches on pump-induced drawdown to understand their influence on the aquifer system's hydraulic behavior under varying skin factor values. Moreover, we compare the proposed NSF model to the existing two-zone model. A sensitivity analysis is conducted to investigate the impact of various aquifer and wellbore parameters. Finally, we discuss the implications of this newly developed skin-factor model for groundwater management and well hydraulic modeling.

2. Methods

2.1. Groundwater flow model

Fig. 1 presents a schematic representation of the conceptual model illustrating the aquifer drawdown distribution $s(r, z, t)$ [L] induced by a partially penetrating well in confined aquifers with a uniform thickness of b [L]. The variable r [L] represents the radial distance from the center of the pumping well, z [L] denotes the depth, and t [T] represents the operating time. The pumping well has a radius of r_w [L] and operates at a constant pumping rate Q [L³T⁻¹]. The screen of the well is installed from d to l [L]. The aquifer exhibits anisotropic hydraulic conductivities, denoted as K_r and K_z [LT⁻¹] in the radial (r) and vertical (z) directions, respectively. The water extracted by the well is assumed to be instantaneously sourced from the specific storage S_s [L⁻¹] of the aquifer.

By integrating Darcy's Law with the continuity equation, the governing equation for drawdown response can be expressed in cylindrical coordinates as (Bear, 2012)

$$S_s \frac{\partial s}{\partial t} = K_r \frac{1}{r} \frac{\partial}{\partial r} \left(r \frac{\partial s}{\partial r} \right) + K_z \frac{\partial^2 s}{\partial z^2}, (r, z) \in [r_w, \infty) \times [0, b] \quad (1)$$

The associated initial condition and infinite-distance boundary condition can be respectively expressed as

$$s = 0, t = 0 \quad (2)$$

and

$$\lim_{r \rightarrow \infty} s = 0 \quad (3)$$

The aquifer is bounded by two impermeable layers, so no-flux conditions are applied to the top and bottom of the aquifer:

$$\frac{\partial s}{\partial z} = 0, z = 0, b \quad (4)$$

2.2. Development of skin-factor approach for negative skin

Consider a skin zone surrounding the wellbore with a uniform thickness of r_s (Fig. 2a). The drawdowns in the wellbore and skin zone are denoted as s_w and s_{skin} , respectively. Two interfaces are present: one between s_w and s_{skin} , and another between s_{skin} and the drawdown in the aquifer (s). These interfaces are subject to continuity in both drawdown and flux.

The continuity equation, expressed in terms of Darcy fluxes within the skin zone, is given as follows (Chiu et al., 2007):

$$S_{s,skin} \frac{\partial s_{skin}}{\partial t} = -\frac{1}{r} \frac{\partial}{\partial r} (r q_{r,skin}) - \frac{\partial}{\partial z} (q_{z,skin}), (r, z) \in [r_w, r_s] \times [0, b], \quad (5)$$

where $S_{s,skin}$ is the specific storage of the skin zone [L⁻¹], and $q_{r,skin}$ and $q_{z,skin}$ are the water fluxes in the skin zone [LT⁻¹] in the r and z directions, respectively.

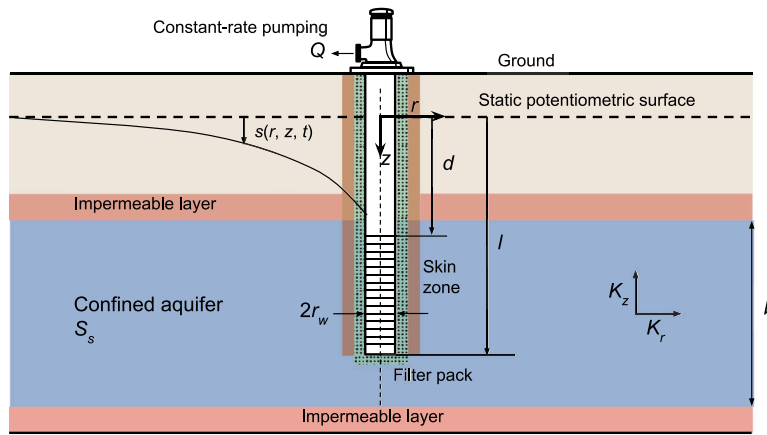


Fig. 1. Schematic representation of drawdown distribution induced by a constant-rate pumping at partially penetrating well.

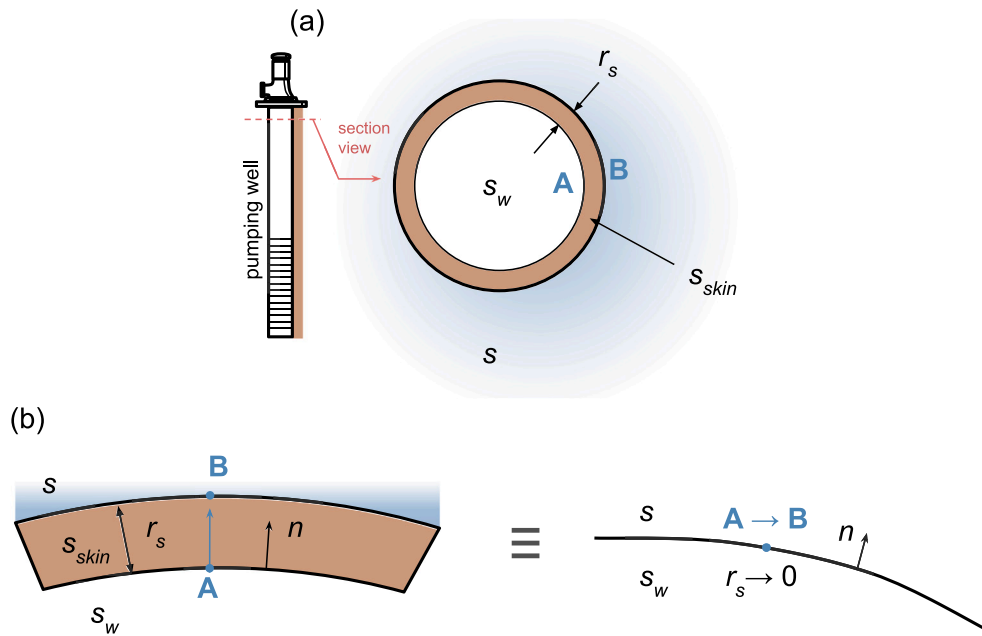


Fig. 2. (a) The cross-section of a composite aquifer with aquifer formation and skin zone. (b) The geometry of the skin zone at the interface between the wellbore and aquifer. Here, n represents the normal vector of the interface.

Since r_s is relatively small compared to the overall aquifer system (Fig. 2b), the flux in the aquifer near the skin zone surface (q_r) [LT^{-1}] can be approximated using a first-order Taylor series expansion:

$$q_r(r_w + r_s^+) = q_{r,w}(r_w + r_s^-) \approx q_{r,w}(r_w) + r_s \frac{\partial q_{r,w}(r_w)}{\partial r} + O(r_s^2), \quad (6)$$

yielding:

$$\frac{q_r - q_{r,w}}{r_s} = \frac{\partial q_{r,w}}{\partial r}. \quad (7)$$

where $q_{r,w}$ is the flux at the wellbore surface [LT^{-1}].

At the surface of the skin zone, the first term on the right side of Eq. (5) can be rewritten as:

$$\frac{1}{r_w + r_s} \frac{\partial}{\partial r} ((r_w + r_s) q_{r,skin}) = \frac{\partial q_{r,skin}}{\partial r}. \quad (8)$$

By letting $q_r \approx q_{r,skin}$ and applying Eq. (7), Eq. (5) can be rewritten as:

$$S_{s,skin} \frac{\partial s_{skin}}{\partial t} = -\frac{q_r - q_{r,w}}{r_s} - \frac{\partial q_{z,skin}}{\partial z}, \quad (9a)$$

or:

$$r_s S_{s,skin} \frac{\partial s_{skin}}{\partial t} = q_{r,w} - q_r - r_s \frac{\partial q_{z,skin}}{\partial z}. \quad (9b)$$

Note that the final term on the right side of Eq. (9b) results from the surface Laplacian operator.

Based on Darcy's Law, the fluxes $q_{r,w}$, q_r , and $q_{z,skin}$ can be expressed as:

$$q_{r,w} = -K_w \frac{\partial s_w}{\partial r}, \quad (10a)$$

$$q_r = -K_r \frac{\partial s}{\partial r}, \quad (10b)$$

and

$$q_{z,skin} = -K_{z,skin} \frac{\partial s_{skin}}{\partial z}, \quad (10c)$$

where K_w is the hydraulic conductivity of the wellbore [LT^{-1}] and $K_{z,skin}$ is the hydraulic conductivity of the skin zone in z direction

[LT⁻¹]. Substituting Eqs. (10a) and (10b) into Eq. (9b) yields:

$$r_s S_{s,skin} \frac{\partial s_{skin}}{\partial t} = -K_w \frac{\partial s_w}{\partial r} + K_r \frac{\partial s}{\partial r} + r_s K_{z,skin} \frac{\partial^2 s_{skin}}{\partial z^2}. \quad (11)$$

Finally, we obtain the following expression for the wellbore flux:

$$K_w \frac{\partial s_w}{\partial r} = K_r \frac{\partial s}{\partial r} + r_s K_{z,skin} \frac{\partial^2 s_{skin}}{\partial z^2} - r_s S_{s,skin} \frac{\partial s_{skin}}{\partial t}, \quad (12)$$

or

$$K_w \frac{\partial s_w}{\partial r} = K_r \frac{\partial s}{\partial r} + S_f \frac{\partial^2 s_{skin}}{\partial z^2} - C_g \frac{\partial s_{skin}}{\partial t}, \quad (13)$$

where S_f [L²T⁻¹] is the skin factor for negative skin, defined as $K_{r,skin} r_s$, and C_g [-] is a factor defined as $S_{s,skin} r_s$. This result corresponds with studies on high-conductivity interface conditions (Lipton and Vernescu, 1996; Lipton, 1997; Miloh and Benveniste, 1999; Lin and Chen, 2023), with the parameters adapted for common groundwater literature. Previous research has shown that this condition produces results closely aligned with the two-zone model when $r_s/r_w < 0.01$ (Chen and Lin, 2022b,a).

Eq. (13) implies that the water flux in the wellbore depends on the water flux through the aquifer, diffusion within the skin zone along the wellbore surface, and water storage in the skin zone. Given that the wellbore drawdown is spatially averaged and independent of distance, the left-hand term of Eq. (13) can be disregarded (i.e., $\partial s_w/\partial r = 0$ at $r = r_w$). In typical aquifer setups, the factor C_g can also be neglected (i.e., storage in the skin layer is very small), as both $S_{s,skin}$ and r_s are small, rendering its effects negligible (i.e., $C_g = 0$). This leads to:

$$K_r \frac{\partial s}{\partial r} = -S_f \frac{\partial^2 s_{skin}}{\partial z^2}, \quad r = r_w. \quad (14)$$

Assuming continuity of drawdown between the skin zone and aquifer formation (i.e., $s_{skin} = s$ at $r = r_w$), the equation can be reformulated as:

$$K_r \frac{\partial s}{\partial r} = -S_f \frac{\partial^2 s}{\partial z^2}, \quad r = r_w. \quad (15)$$

This equation establishes a new skin-factor approach for the negative skin, i.e., NSF approach. To incorporate the flux due to pumping and the effects of wellbore storage (Papadopoulos and Cooper, 1967), Eq. (15) is modified to:

$$2\pi r_w(l-d)K_r \frac{\partial s}{\partial r} = -Q + \pi r_w^2 \frac{\partial s_w}{\partial t} - 2\pi r_w(l-d)S_f \frac{\partial^2 s}{\partial z^2}, \quad r = r_w. \quad (16)$$

The continuity condition for drawdown between the wellbore and aquifer is required, i.e., $s = s_w$ at $r = r_w$. The inner boundary condition incorporating the new skin factor, expressed in terms of s , can be written as:

$$2\pi r_w(l-d)K_r \frac{\partial s}{\partial r} = -Q + \pi r_w^2 \frac{\partial s}{\partial t} - 2\pi r_w(l-d)S_f \frac{\partial^2 s}{\partial z^2}, \quad r = r_w. \quad (17)$$

Another condition is required to ensure drawdown continuity:

$$\frac{1}{l-d} \int_{z=d}^{z=l} s dz = s_w, \quad r = r_w. \quad (18)$$

Eqs. (17) and (18) ensure continuity of drawdown at the wellbore surface while maintaining a discontinuity in water flux. In highly permeable media, the permeability is extremely high, leading to very high flow velocities. However, when fluid flows from the aquifer medium into the highly permeable medium, the sudden increase in permeability causes a sharp rise in flow velocity. If the skin layer is very thin, this can result in an apparent discontinuity in flow behavior at the macroscopic level.

2.3. Comparing to skin-factor approach for positive skin

In the existing PSF approaches, the effect of the skin zone is incorporated into a dimensionless factor α [-], which is derived based on a

very thin skin zone around the wellbore. The skin factor condition at the well surface ($r = r_w$) can be expressed as (Moench, 1997)

$$\frac{1}{l-d} \int_{z=d}^{z=l} s dz - r\alpha \frac{\partial s}{\partial r} = s_w, \quad r = r_w \quad (19)$$

The skin factor α is proportional to the ratio of skin permeability and skin thickness, i.e., $\alpha \propto r_s/K_{r,skin}$. This approach implies that the well drawdown is the sum of the aquifer drawdown and a head loss (proportional to the wellbore flux) due to the well skin. Therefore, it will result in discontinuous drawdown drop at the wellbore surface. In addition, this boundary condition is limited to the positive skin case.

2.4. Dimensionless equations and solutions

Dimensionless parameters and variables are defined as follows: $s_D = 4\pi K_r b s / Q$ (dimensionless drawdown); $t_D = K_r t / r_w^2 S_s$ (dimensionless time); $r_D = r / r_w$ (dimensionless radial distance); $z_D = z / b$ (dimensionless vertical distance); $d_D = d / b$ (dimensionless depth to top of well screen); $l_D = l / b$ (dimensionless depth to bottom of well screen); $C_D = 1/2 S_s b$ (dimensionless wellbore storage coefficient); $\kappa = r_w^2 K_z / b^2 K_r$ (anisotropy ratio); and $\beta = S_f r_w / b^2 K_r$ (dimensionless skin factor for negative skin). Accordingly, the governing equations of Eq. (1) becomes

$$\frac{\partial s_D}{\partial t_D} = \frac{1}{r_D} \frac{\partial}{\partial r_D} \left(r_D \frac{\partial s_D}{\partial r_D} \right) + \kappa \frac{\partial^2 s_D}{\partial z_D^2}, \quad (r_D, z_D) \in [1, \infty) \times [0, 1] \quad (20)$$

The dimensionless expressions of the initial and boundary conditions are

$$s_D = 0, t_D = 0 \quad (21a)$$

$$\frac{\partial s_D}{\partial r_D} = \frac{-2}{l_D - d_D} + \frac{C_D}{l_D - d_D} \frac{\partial s_D}{\partial t_D} - \beta \frac{\partial^2 s_D}{\partial z_D^2}, r_D = 1 \quad (21b)$$

$$\lim_{r_D \rightarrow \infty} s_D = 0 \quad (21c)$$

and

$$\frac{\partial s_D}{\partial z_D} = 0, z_D = 0, 1 \quad (21d)$$

To obtain the solution to the proposed model, the Laplace transform technique can be applied to Eqs. (20)–(21d) and the Laplace domain expressions of the model are then obtained as

$$p \bar{s}_D = \frac{1}{r_D} \frac{\partial}{\partial r_D} \left(r_D \frac{\partial \bar{s}_D}{\partial r_D} \right) + \frac{\partial^2 \bar{s}_D}{\partial z_D^2} \quad (22a)$$

$$\frac{\partial \bar{s}_D}{\partial r_D} = \frac{-2}{p(l_D - d_D)} + \frac{C_D p \bar{s}_D}{l_D - d_D} - \beta \frac{\partial^2 \bar{s}_D}{\partial z_D^2}, r_D = 1 \quad (22b)$$

$$\lim_{r_D \rightarrow \infty} \bar{s}_D = 0 \quad (22c)$$

and

$$\frac{\partial \bar{s}_D}{\partial z_D} = 0, z_D = 0, 1 \quad (22d)$$

where the overbar represents the Laplace domain and p is the Laplace parameter.

To eliminate the variable z_D , the finite cos-Fourier transform is applied (Povstenko, 2015); that gives

$$p \tilde{\tilde{s}}_D = \frac{1}{r_D} \frac{d}{dr_D} \left(r_D \frac{d \tilde{\tilde{s}}_D}{dr_D} \right) - \xi_n^2 \tilde{\tilde{s}}_D \quad (23a)$$

$$\frac{\partial \tilde{\tilde{s}}_D}{\partial r_D} = -2 \frac{\sin(\xi_n l_D) - \sin(\xi_n d_D)}{(l_D - d_D) \xi_n p} + \frac{C_D p}{l_D - d_D} \tilde{\tilde{s}}_D + \xi_n^2 \beta \tilde{\tilde{s}}_D, r_D = 1 \quad (23b)$$

and

$$\lim_{r_D \rightarrow \infty} \tilde{\tilde{s}}_D = 0 \quad (23c)$$

where the overtilde represents the Fourier domain and ξ_n is Fourier parameter equal to $n\pi$.

The solution in Laplace–Fourier domain can then be derived as:

$$\tilde{s}_D = \frac{2(\sin(d_D \xi_n) - \sin(l_D \xi_n))K_0(r_D \lambda)}{\xi_n p(d_D - l_D) \left((C_D p / (l_D - d_D) + \beta \xi_n^2) K_0(\lambda) + \lambda K_1(\lambda) \right)} \quad (24)$$

where $\lambda = \sqrt{\kappa \xi_n^2 + p}$ and K_0 and K_1 are modified Bessel function of second type with orders 0 and 1, respectively.

To invert from the Fourier domain to the space domain, the finite cos-Fourier inversion can be adopted by using the following relation:

$$F^{-1}\{\tilde{s}_D(z_D, \xi_n)\} = \tilde{s}_D(z_D, \xi_0) + 2 \sum_{n=1}^{\infty} \tilde{s}_D(z_D, \xi_n) \cos(z_D \xi_n) \quad (25a)$$

in which

$$\tilde{s}_D(z_D, \xi_0) = \tilde{s}_D = \frac{2K_0(\sqrt{pr})}{C_D p^2 K_0(\sqrt{p}) + p^{3/2} K_1(\sqrt{p})} \quad (25b)$$

and $\tilde{s}_D(z_D, \xi_n)$ is equal to Eq. (24). The infinite sum can be approximated as a finite sum by truncating it when the terms in the summation converge. Finally, the time-domain solution can be obtained using a numerical Laplace inversion scheme. Many studies have shown that the Stehfest (1970) method works well for diffusion problems like groundwater flow (e.g., Hassanzadeh and Pooladi-Darvish, 2007). Therefore, the Stehfest method is employed here to invert Laplace solutions to time-domain solutions due to its straightforward implementation and accuracy. This method contains an integer parameter representing the optimal number of summation terms, which must be an even integer. Previous studies have determined that this value typically ranges from 8 to 24 (Wang and Zhan, 2015, 2017). In this study, a value of 10 is adopted because it provides sufficient accuracy for our solution while maintaining reasonable computation time

2.5. Large-time solutions

The asymptotic solution for s_D at large times ($t_D \rightarrow \infty$) can be derived by considering the limiting behavior of certain functions. As $p \rightarrow 0$, the approximation $\sqrt{p} K_1(\sqrt{p}) \approx 1$ holds true, and $K_0(\sqrt{pr_D})$ approximates $-\ln(\sqrt{pr_D}/2) - \gamma$, where γ is the Euler–Mascheroni constant, approximately 0.577216. With these approximations and the Laplace transform formula (Abramowitz and Stegun, 1968), the large-time solution can be expressed as:

$$s_D = \ln\left(\frac{4t_D}{r_D^2} e^{-\gamma}\right) + 2 \sum_{n=1}^{\infty} G(z_D, \xi_n) \cos(z_D \xi_n) \quad (26)$$

with

$$G(z_D, \xi_n) = \frac{2(\sin(d_D \xi_n) - \sin(l_D \xi_n))K_0(r_D \lambda_0) (\beta \xi_n^2 K_0(\lambda_0) - \lambda_0 K_1(\lambda_0))}{\xi_n^3 (d_D - l_D) \left(\beta^2 \xi_n^2 K_0(\lambda_0)^2 - \kappa K_1(\lambda_0)^2 \right)} \quad (27)$$

where $\lambda_0 = \xi_n \sqrt{\kappa}$. As indicated in the equation, the large-time solution includes a logarithmic approximation of the (Theis, 1935) solution (Chen and Chang, 2006), along with a component related to the partial penetration effect (Hantush, 1961).

2.6. Sensitivity analysis

Sensitivity analysis is a powerful tool to evaluate the effects of various parameters on model output. The normalized sensitivity coefficient is defined as the partial derivative of the model-dependent variable with respect to a specific parameter (Kabala, 2001; Dell'Oca et al., 2020; Knabe et al., 2021):

$$X_k = P_k \frac{\partial s_D(P_k)}{\partial P_k} \quad (28)$$

Table 1

Default parameter values modified from (Lin and Yeh, 2017).

Parameter	Symbol	Value	Unit
Radial hydraulic conductivity	K_r	10^{-5}	m/s
Vertical hydraulic conductivity	K_z	10^{-5}	m/s
Specific storage	S_s	10^{-6}	m ⁻¹
Well radius	r_w	0.05	m
Aquifer thickness	b	50	m
Depth to bottom of well screen	d	0	m
Depth to top of well screen	l	45	m
Pumping rate	Q	0.1	m ³ /s

where X_k indicates the sensitivity of s_D to the k th input parameter P_k . Eq. (28) may be approximated by a finite-difference formula as suggested by Liou and Yeh (1997):

$$X_k \approx P_k \frac{s_D(P_k + \Delta P_k) - s_D(P_k)}{\Delta P_k} \quad (29)$$

where ΔP_k is chosen as $10^{-3} P_k$.

Borgonovo and Apostolakis (2001) introduced a measure of sensitivity analysis known as the differential importance measure, which is widely applied in the area of risk management for decision making (Borgonovo et al., 2003; Vinod et al., 2003; Marseguerra and Zio, 2004). This measure is defined as:

$$DIM_k = \frac{P_k \frac{\partial s_D(P_k)}{\partial P_k}}{\sum_{j=1}^N P_j \frac{\partial s_D(P_j)}{\partial P_j}} \quad (30)$$

where DIM_k is the differential importance measure of parameter P_k and N is the total number of model parameters. According to Eq. (28), Eq. (30) can be rewritten as:

$$DIM_k = \frac{X_k}{\sum_{j=1}^N X_j} \quad (31)$$

Since X_k may be negative, the absolute value is taken for X_k and Eq. (32) is then expressed as (Lin and Yeh, 2020):

$$DIM_k = \frac{|X_k|}{\sum_{j=1}^N |X_j|} \quad (32)$$

Eq. (32) is used in the NSF model (Eq. (25)) to assess the effects of C_D , κ , l_D , and α and in the PSF model to evaluate C_D , κ , l_D , and β .

3. Results and discussion

3.1. Comparing to large-time solution

In this section, the default values of the parameters to evaluate the dimensionless drawdown value are listed in Table 1.

In the large-time solution (Eq. (26)), the wellbore storage effect is negligible ($C_D = 0$) because it primarily influences aquifer flow in the early stages. Consequently, the effect of wellbore storage on aquifer drawdown can be considered negligible. Fig. 3 illustrates temporal drawdown curves predicted by (a) the proposed NSF solution (Eq. (25)) and (b) the large-time solution (Eq. (26)) across different β values at $r_D = 1$ and $z_D = 0.5$. The large-time solution in the figure shows straight lines because it neglects wellbore storage effects. This result is consistent with the pattern predicted by Theis (1935) solution. Furthermore, the drawdown curves predicted by both solutions align well across different β values when $t_D > 10^6$, or when $t > S_s r_w^2 / K_r \times 10^6 \approx 2,500$ s (41.6 min) in our case. This suggests that the wellbore storage effect can be omitted for $t_D > 10^6$. The long-term alignment of these curves confirms the robust formulation of the large-time solution, making it a reliable alternative for assessing the late-time behavior of the NSF solution.

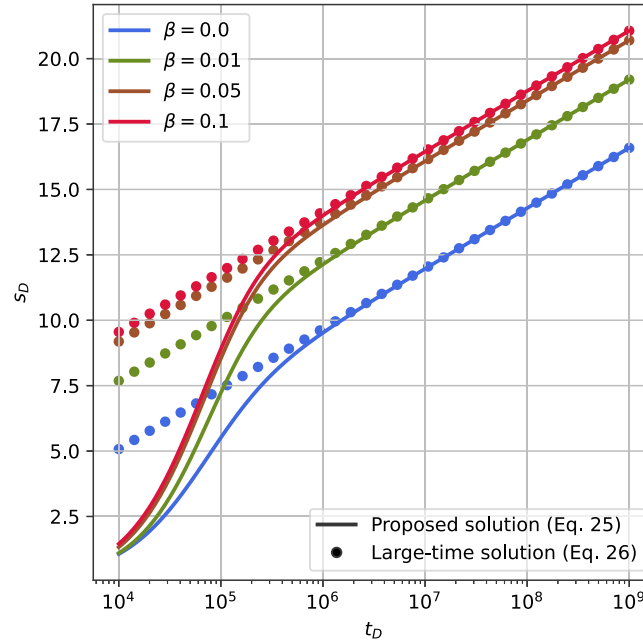


Fig. 3. Temporal drawdown curves predicted by the proposed NSF solution (Eq. (25)) and corresponding large-time solution (Eq. (26)) observed at $r_D = 1$ and $z_D = 0.5$.

The drawdown curves predicted by the large-time solution suggest that the dimensionless drawdown s_D can be expressed as:

$$s_D = m \log t_D + c \quad (33)$$

where m is the slope and c is the intercept. Using a least-squares fitting method, the slope m is found to be 1, while the intercept c varies with the value of β . Applying a saturating growth model to find the relationship between c and β , it is approximated as:

$$c \approx -4.1 + 4.65(1 - e^{74.7\beta}) \quad (34)$$

Thus, the relationship between s_D and t_D in this specific case can be expressed as:

$$s_D = \log t_D - 4.1 + 4.65(1 - e^{74.7\beta}) \quad (35)$$

As observed in Eq. (35), when β is very small, drawdown changes rapidly. However, as β continues to increase, the change gradually slows down and approaches a stable value.

3.2. Comparing the skin-factor models for positive and negative skins

Fig. 4 illustrates the temporal drawdown distributions predicted by the NSF model and the PSF model with varying corresponding skin factors β and α , measured at $(r_D, z_D) = (1, 0.5)$. As indicated in the figure, increases in α and β result in higher drawdown values due to increased head loss in the skin zone. However, the drawdown curves influenced by changes in α and β exhibit different patterns. For instance, α primarily affects the drawdown curves at early times; conversely, β significantly impacts drawdown throughout the entire period. This difference may be attributed to the fact that α operates in conjunction with wellbore storage, primarily influencing the drawdown response in early times. In contrast, the new factor β depends on the vertical diffusion near the wellbore, which is not time-dependent, allowing it to continuously affect the flow system.

Fig. 5 presents the fluxes due to the skin factors α and β along the screen portion of the well ($z_D \in [d_D, l_D]$). The flux attributed to β , denoted by q_β , is defined as $\beta \partial^2 s_D / \partial z_D^2$ at $r_D = 1$, while the flux due to α , denoted by q_α , is defined by $\alpha \partial s_D / \partial r_D$ at $r_D = 1$. The figure reveals that increases in both α and β lead to higher skin-induced fluxes. The spatial distribution of q_α displays a uniform distribution with depth;

however, the spatial distribution of q_β exhibits an increasing trend and slopes with depth. This suggests that the skin portion in the casing section of the pumping well may also exert effects on the skin-induced flux, namely, the head loss caused by the high-permeability skin can be amplified near the casing portion.

3.3. Comparing to two-zone solution

The two-zone model for considering the wellbore skin effect on aquifer drawdown induced by constant-rate pumping is applied for comparison to the proposed model. Pasandi et al. (2008) proposed a semi-analytical solution of the two-zone model for unconfined flow using (Moench, 1997) free-surface approach. For comparison, the specific yield in Pasandi et al. (2008) model is assumed to be zero, reducing their model to confined flow. To ensure comparability, the dimensionless parameters defined in Section 2.4 are imposed on the two-zone model. Additionally, the ratio of hydraulic conductivities of the wellbore skin and the aquifer is set to 10 (Yeh and Yang, 2006), and the skin zone thickness is assumed to be 0.001 m (Novakowski, 1989; Houben et al., 2023). For simplicity, the effect of wellbore storage is neglected (i.e., $C_D = 0$).

To verify the assumption used in the NSF model, the drawdown curves predicted by Eq. (25) are compared to the two-zone solution modified from (Pasandi et al., 2008) (Fig. 6). As shown in Fig. 6, the temporal drawdown curves predicted by both solutions match very well.

3.4. Findings from sensitivity analysis

In the sensitivity analysis, four parameters are considered: C_D , κ , l_D , and α or β for different models. The value of d_D is fixed at zero, and thus, l_D represents the well penetration ratio. Fig. 7 displays the temporal distribution curves of the DIM for (a) the NSF model and (b) the PSF model, observed at $(r_D, z_D) = (1, 0.5)$. Fig. 7(a) reveals that the effect of changes in C_D is most pronounced during early times, while the impact of changes in κ is negligible. The effects of l_D and β become significant at intermediate and later times, with the influence of β slightly surpassing that of l_D . Conversely, Fig. 7(b) indicates that changes in C_D and l_D are most significant during early and late times, respectively. κ exhibits some effects at late times but these are minor.

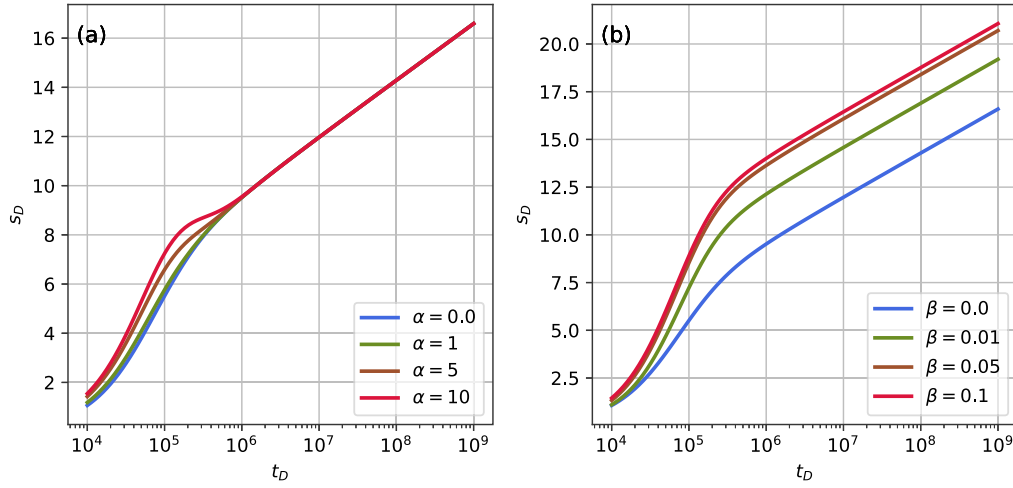


Fig. 4. Temporal dimensionless drawdown distribution predicted by (a) the PSF model and (b) the NSF model (Eq. (25)) with varying corresponding skin factors observed at $r_D = 1$ and $z_D = 0.5$.

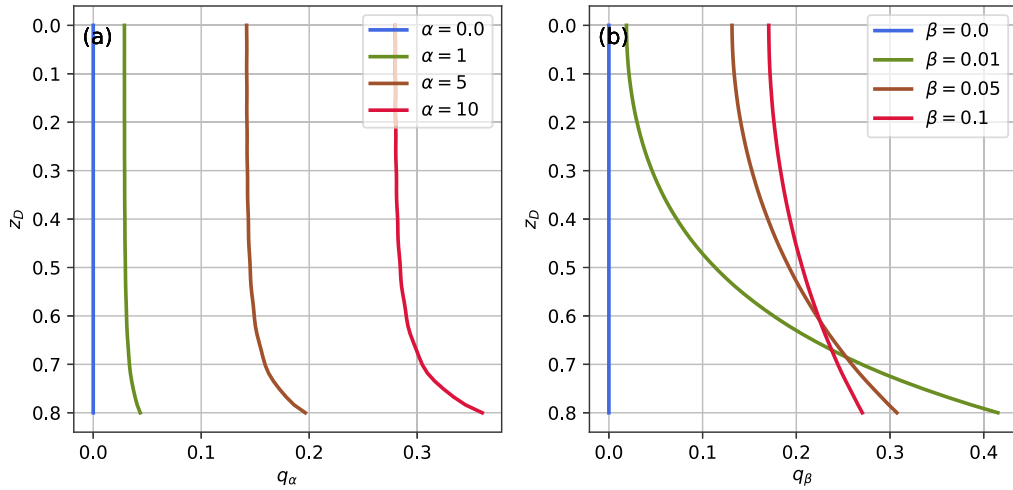


Fig. 5. Temporal dimensionless flux induced by the skin predicted by (a) the PSF model and (b) the NSF model (Eq. (25)).

The skin factor α affects the early time, with a peak around $t_D = 10^6$. As indicated in Section 3.1, $t_D = 10^6$ is the time when the wellbore storage effect becomes negligible. Beyond this time, the DIM curves for C_D and α merge due to both effects being encompassed in the wellbore drawdown term. It is noteworthy that results clearly demonstrate that when considering a high-permeability skin zone, its impact on late-time drawdown is significant and cannot be ignored. This is because water flow in the skin layer, facilitated by a higher hydraulic conductivity, leads to vertical water flow distributing within the skin zone region. In contrast, the existing PSF model exhibits skin zone effects only during a brief initial period. As the pumping duration extends, these effects can be disregarded since the groundwater flow is dominated by the aquifer flow.

4. Conclusions

This study introduces a new skin-factor approach to describe the negative skin, addressing limitations of past models derived based on positive skin assumption. The new skin factor and associated condition diverge from the traditional reliance on discontinuous drawdown between the wellbore and the aquifer. Instead, the proposed skin-factor approach depends solely on the discontinuous flux at the interface

between the wellbore and the aquifer, enhancing the model's realism. The newly developed skin-factor approach has been integrated into a groundwater flow model to describe pump-induced drawdown distribution in a confined aquifer.

Utilizing the solution to this model through drawdown prediction and sensitivity analysis, it was found that the new skin factor predominantly influences the system at intermediate and late times, whereas the existing skin factor impacts the system primarily at early times due to its combined effect with the wellbore storage effect. Additionally, the existing skin factor does not account for the water redistribution effects in the skin zone, which may have a prolonged impact on the drawdown response.

The proposed NSF approach offers a robust alternative for developing groundwater flow models in well hydraulic modeling, particularly in areas with high skin zone permeability. This method is versatile, applicable not only to constant-rate pumping tests but also to constant-drawdown tests and slug tests through boundary condition adjustments, enhancing its utility in hydrogeological investigations. A key limitation of this study is the lack of field data for analysis, which prevents direct demonstration of the approach's application to real-world pumping tests. However, the method holds potential for estimating aquifer parameters at field sites during such tests. This can be achieved by

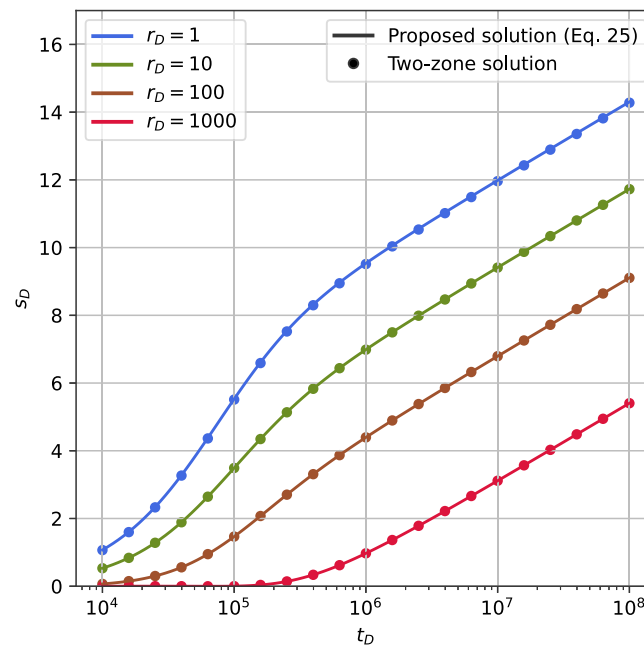


Fig. 6. Temporal dimensionless drawdown predicted by (a) the NSF model (Eq. (25)) and (b) two-zone model (Pasandi et al., 2008).

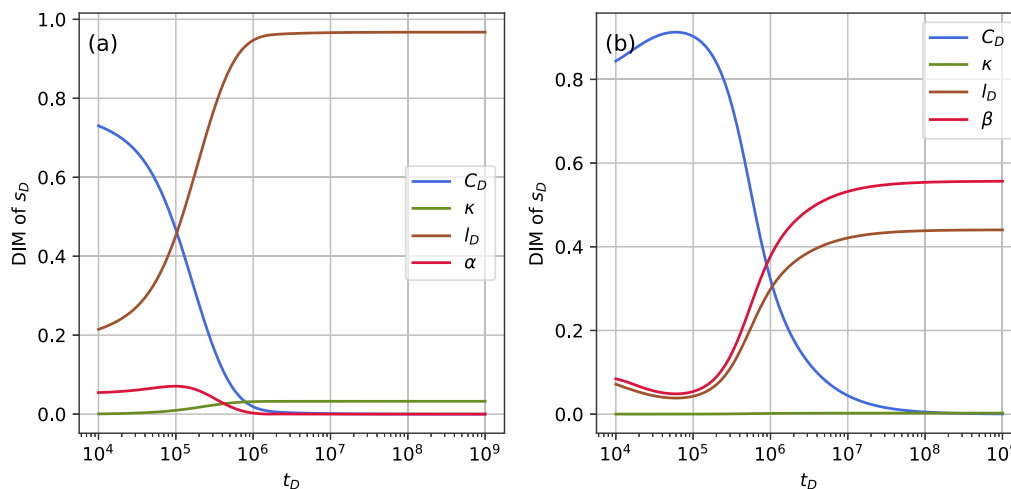


Fig. 7. Temporal DIM curves for each of four parameters observed at $(r_D, z_D) = (1, 0.5)$ for (a) the PSF model and (b) NSF model (Eq. (25)).

combining Eq. (25) with optimization techniques to analyze drawdown data when available. Future research incorporating field data would be valuable to validate and refine the approach under real-world conditions.

CRediT authorship contribution statement

Jun-Hong Lin: Writing – review & editing, Validation, Methodology, Formal analysis. **Ying-Fan Lin:** Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Methodology, Formal analysis.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Data availability

No data was used for the research described in the article.

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