Groundwater Hydraulics

6

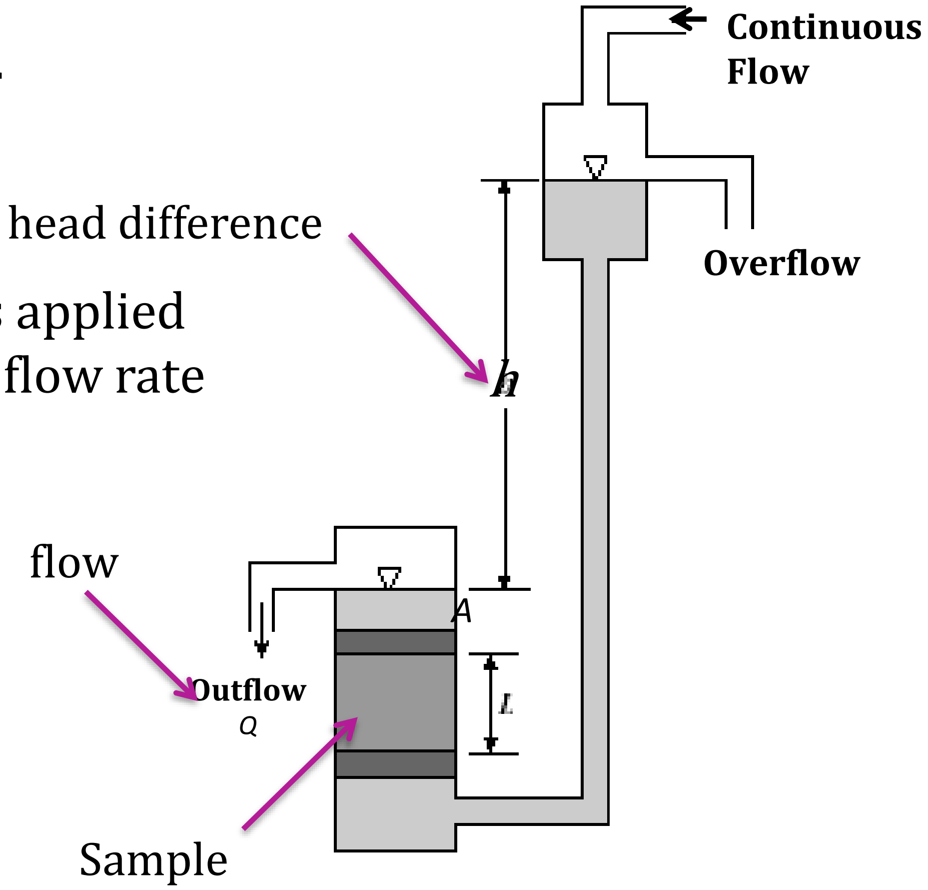
MEASURING CONDUCTIVITY - K

CONSTANT HEAD PERMEAMETER

- **Flow is steady**
- Sample: Right circular cylinder
  - Length, L
  - Area, A
- Constant head difference (h) is applied across the sample producing a flow rate Q
- Darcy's Law

$$Q = KA\frac{h}{L}$$

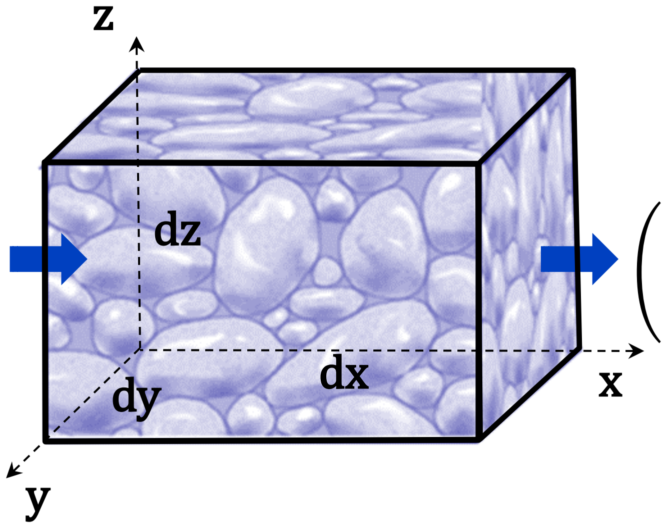
$$K = \frac{QL}{Ah}$$



CONTINUITY EQUATION– CONFINED AQUIFER - steady flow

Assumptions
-Saturated aquifer
-Darcy eq. is valid
-Balance of mass
-Steady flow

$(\rho v_x) dydz$



$\left((\rho v_x) + \frac{\partial(\rho v_x)}{\partial x} dx \right) dz\, dy$

Mass inflow rate - mass outflow rate = 0

Flow in ....dy.dz:

$$(\rho v_x) dydz$$

Flow out ....dy.dz:

$$\left((\rho v_x) + \frac{\partial(\rho v_x)}{\partial x} dx \right) dz\, dy$$

Flow in = Flow out (Continuity equation)

$$(\rho v_x) dy\, dz - \left((\rho v_x) + \frac{\partial(\rho v_x)}{\partial x} dx \right) dz\, dy = 0 \quad \Rightarrow \quad -\left(\frac{\partial(\rho v_x)}{\partial x} \right) dx\, dz\, dy$$

CONTINUITY EQUATION– **CONFINED AQUIFER -** steady flow

Balance of mass for x, y, z :

$$-\left(\frac{\partial(\rho v_x)}{\partial x}\right) dx\, dy\, dz - \left(\frac{\partial(\rho v_y)}{\partial y}\right) dx\, dy\, dz - \left(\frac{\partial(\rho v_z)}{\partial z}\right) dx\, dy\, dz \;\; = 0 \ldots \ldots / \frac{1}{dx\, dy\, dz}$$

$$-\left(\frac{\partial(\rho v_x)}{\partial x}\right) - \left(\frac{\partial(\rho v_y)}{\partial y}\right) - \left(\frac{\partial(\rho v_z)}{\partial z}\right) \;\; = 0$$

Incompressible liquid ρ = const.

CONTINUTY EQUATION FOR STEADY FLOW

$$\boldsymbol{\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0}$$

LAPLACE EQUATION

Darcy eq. for anisotropic porous media

$$v_x = -K_x \frac{\partial h}{\partial x} \quad v_y = -K_y \frac{\partial h}{\partial y} \quad v_z = -K_z \frac{\partial h}{\partial z}$$

From continuity equation

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

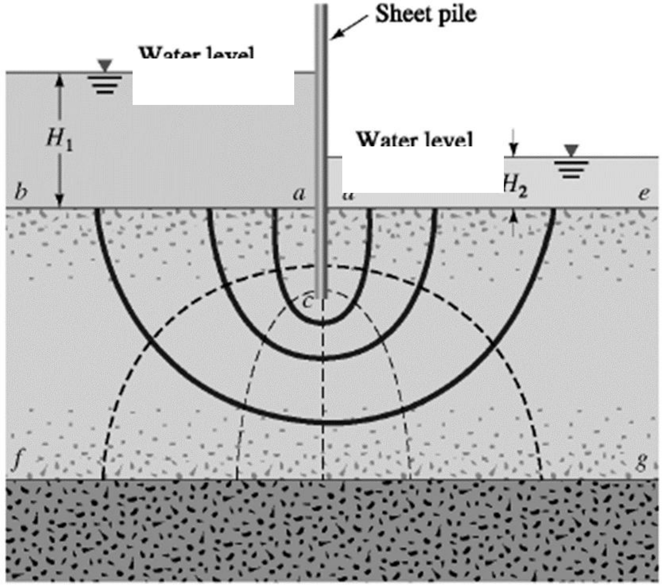
With Darcy eq.:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = 0$$

If soil is isotropic ....$K_x = K_y = K_z = K$

$$\boldsymbol{\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0}$$

$\nabla^2 h = 0$ ... **LAPLACE EQUATION**



DUPUIT´S ASSUMPTIONS

For unconfined ground water flow **Dupuit** developed a theory that allows for a simple solution based on the following assumptions:

1) The **water table** or free surface is only **slightly inclined**

2) **Streamlines** may be considered **horizontal** and **equipotential lines - vertical**

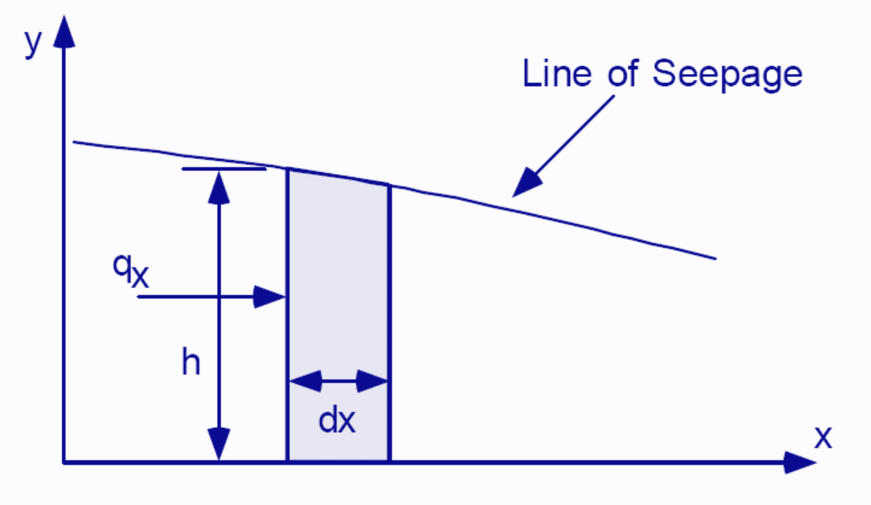
3) **Slopes** of the ***free surface*** and ***hydraulic gradient*** are **equal**

Velocities are horizontal !!!

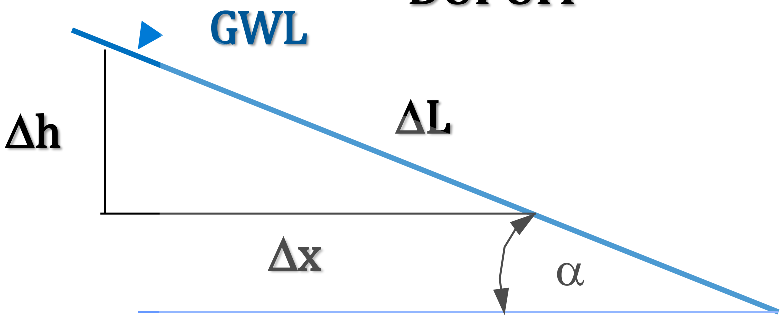
DUPUIT´S ASSUMPTIONS

Comparison:

α	sin(α)	tan(α)
0º	0	0
5º	0.087	0.087
10º	0.174	0.176
20º	0.342	0.346
30º	0.500	0.577
40º	0.643	0.839
50º	0.766	1.192

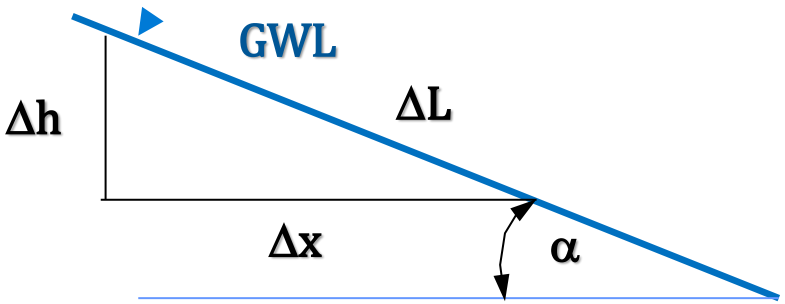


DUPUIT



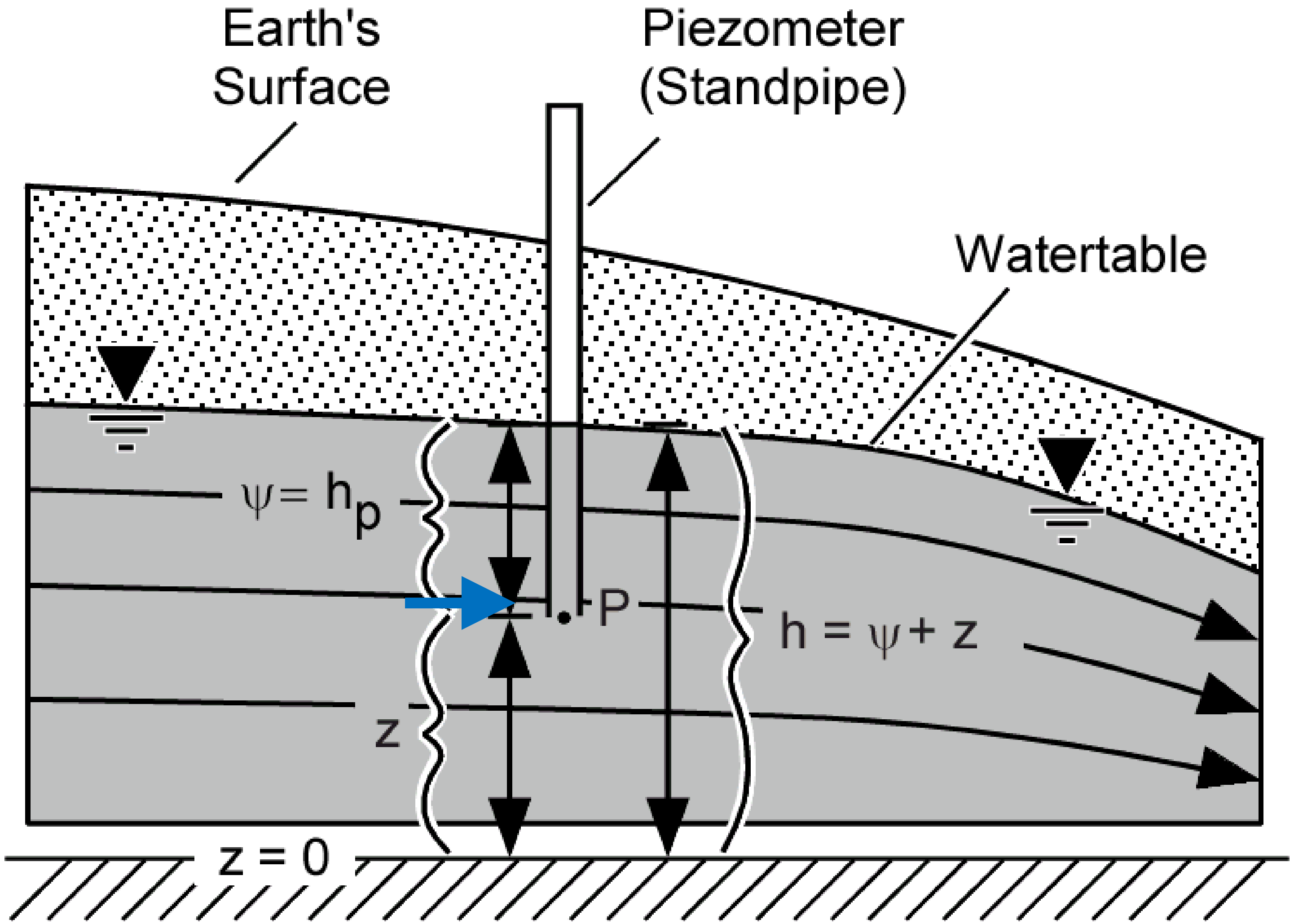
$$I = \frac{\Delta h}{\Delta x} = \tan\alpha$$

ACTUAL



$$I = \frac{\Delta h}{\Delta L} = \sin\alpha$$

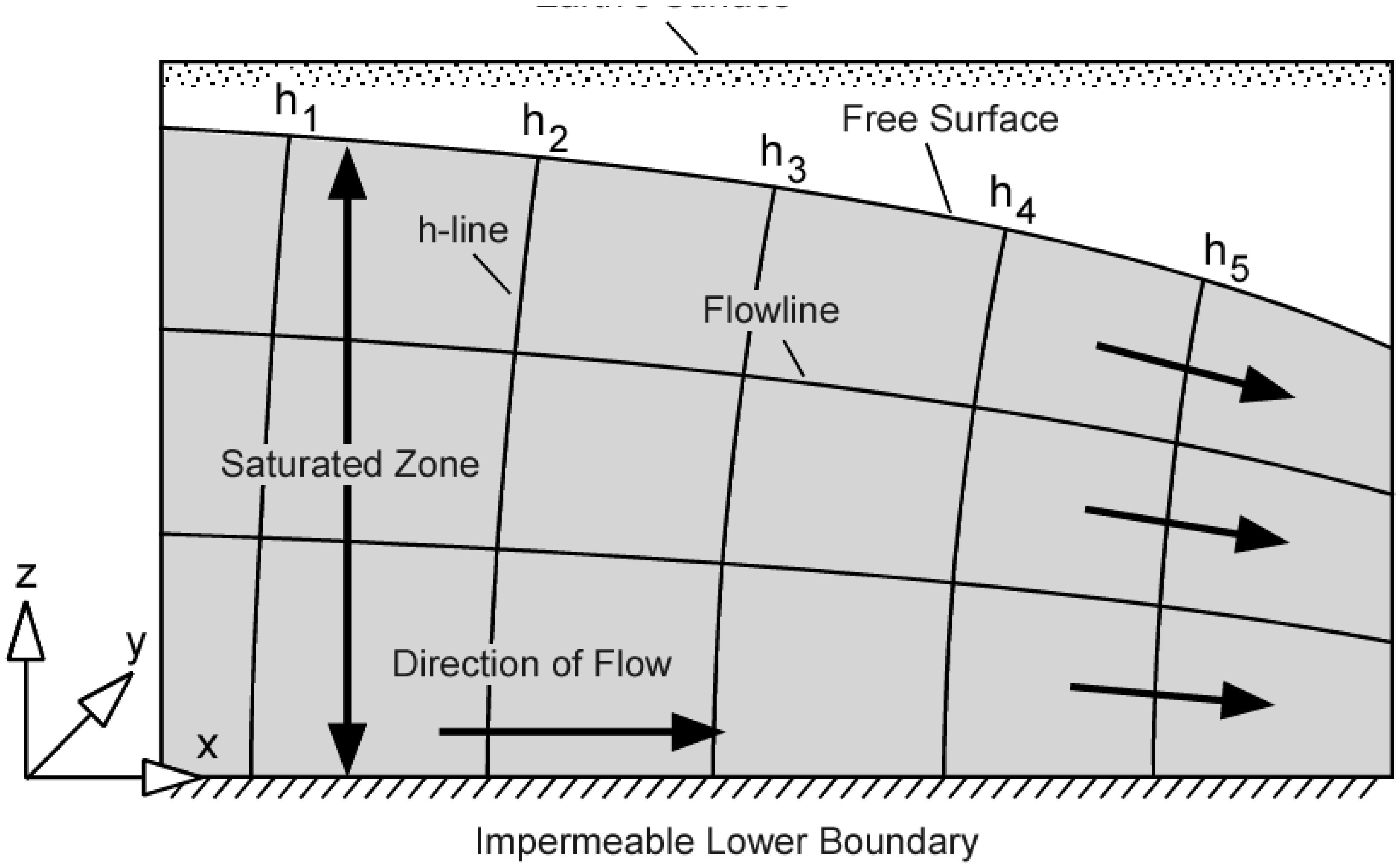
THE ELEVATION OF WATER IN THE PIEZOMETER PROVIDES A MEASURE OF h_p



A piezometer measures the hydraulic head at a point.

DUPUIT ASSUMPTIONS

POTENTIOMETRIC SURFACES (H-LINES, EQUIPOTETIAL LINES) FOR UNCONFINED FLOW.

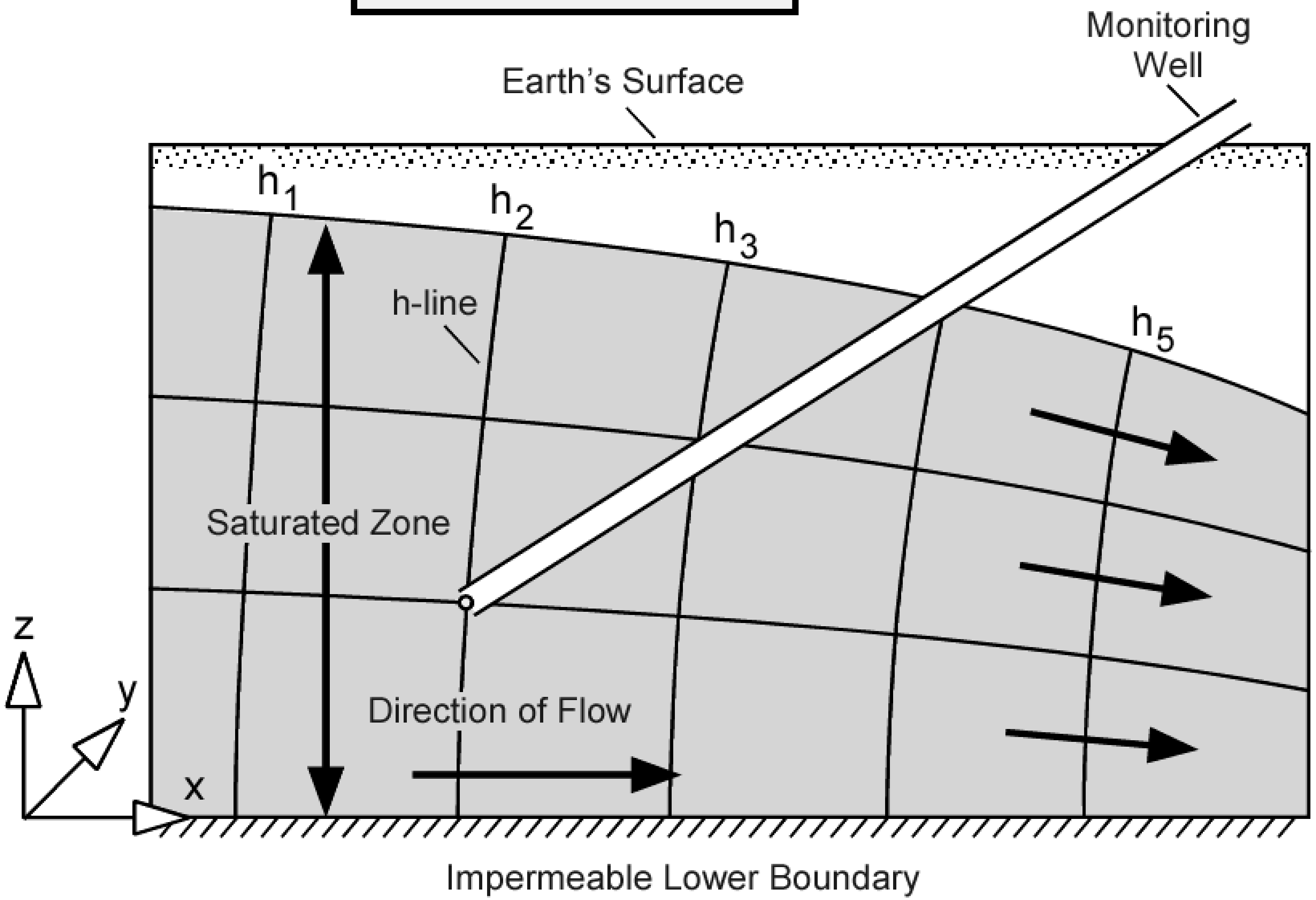


Flowline=streamline **h-line= equipotential line**

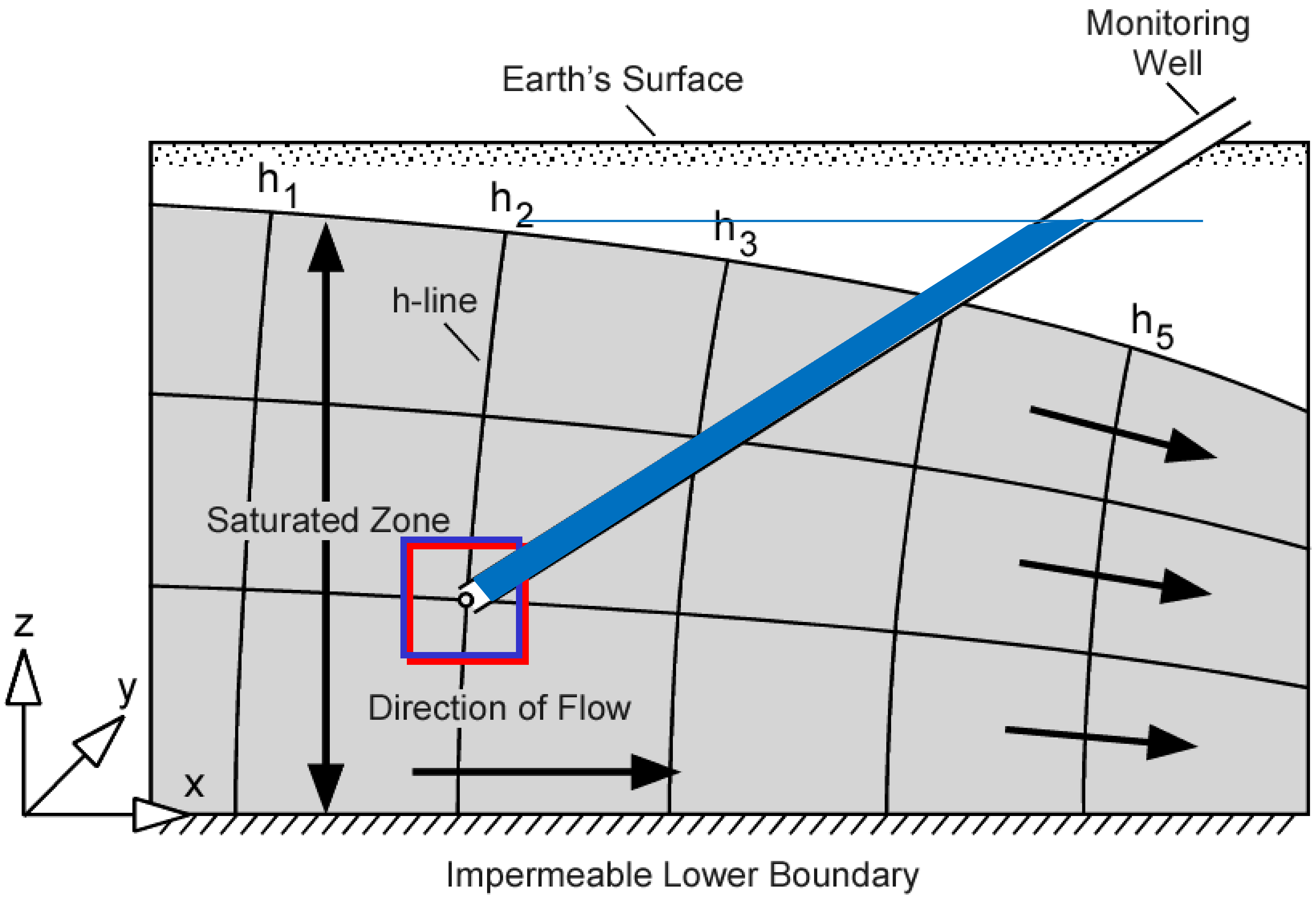
DUPUIT ASSUMPTIONS

APPLICATIONS

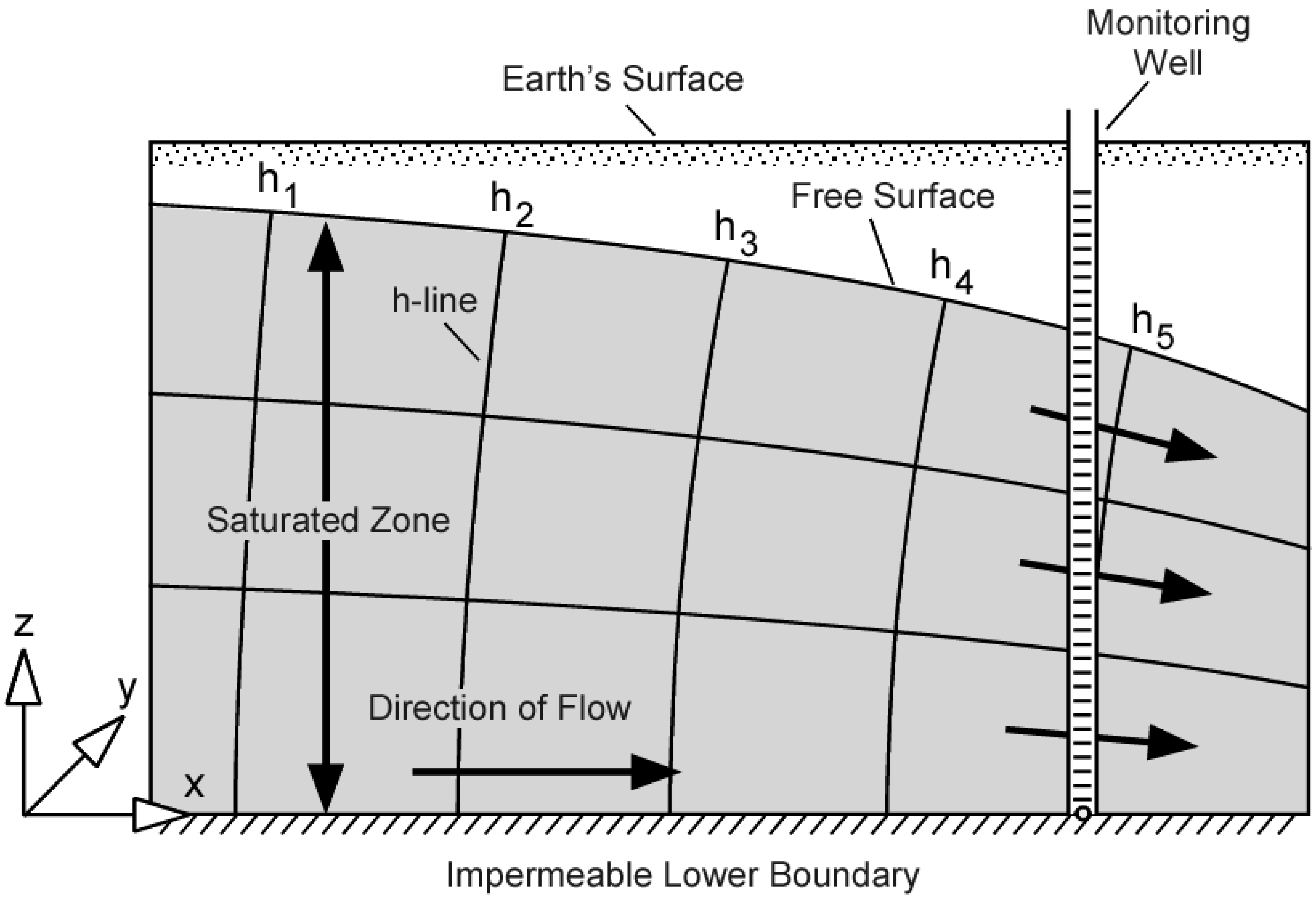
Piezometer:



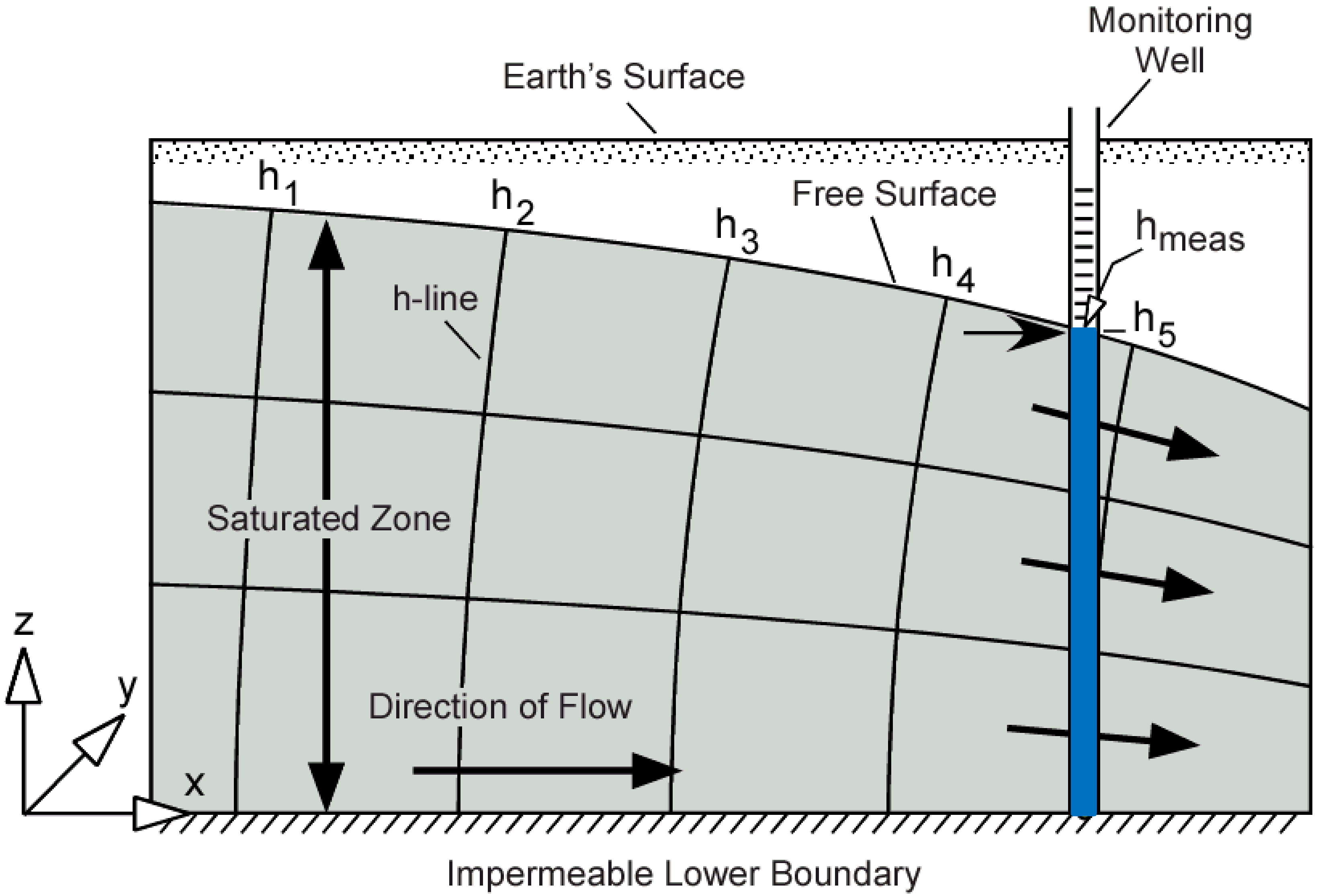
DUPUIT ASSUMPTIONS



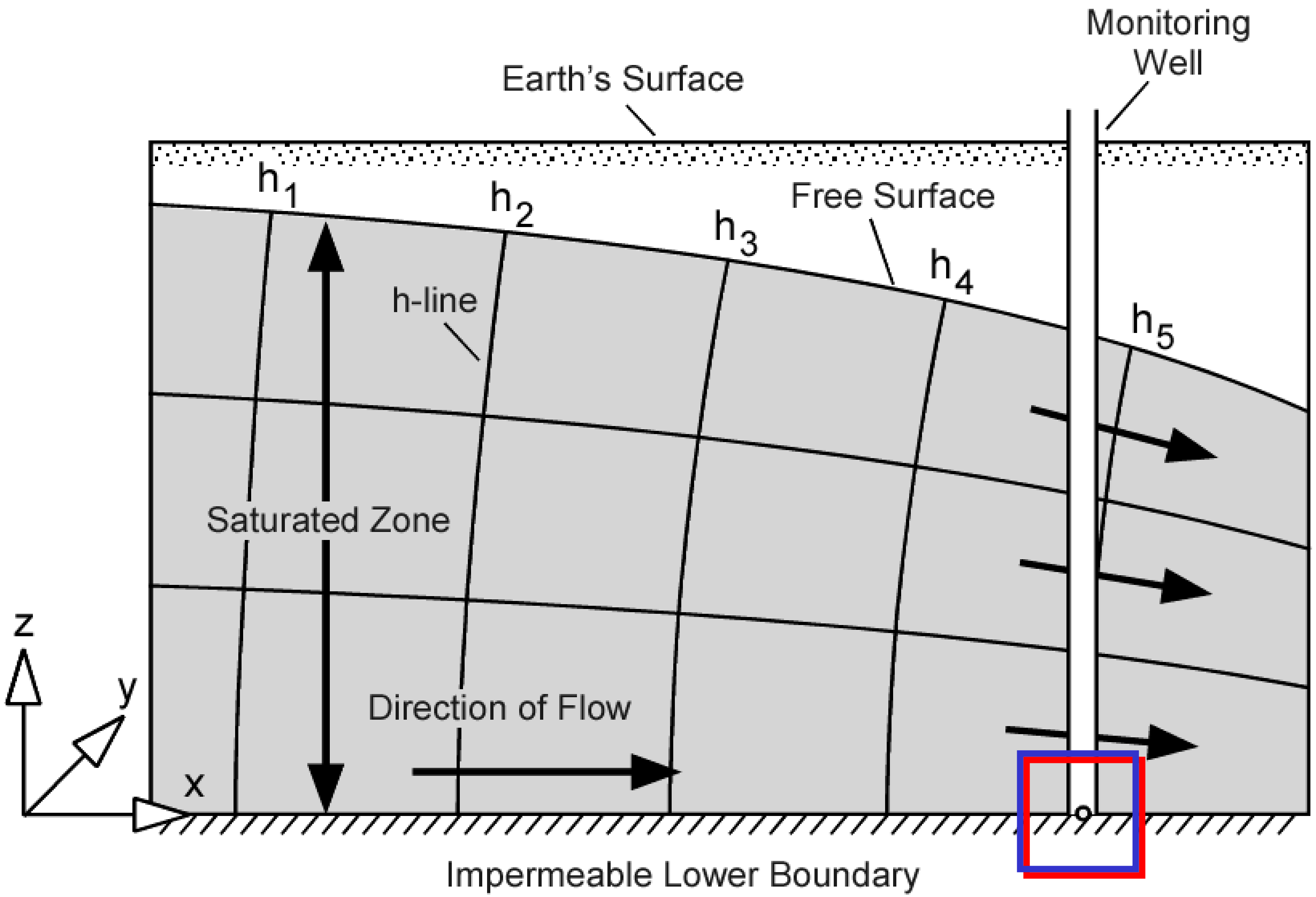
DUPUIT ASSUMPTIONS



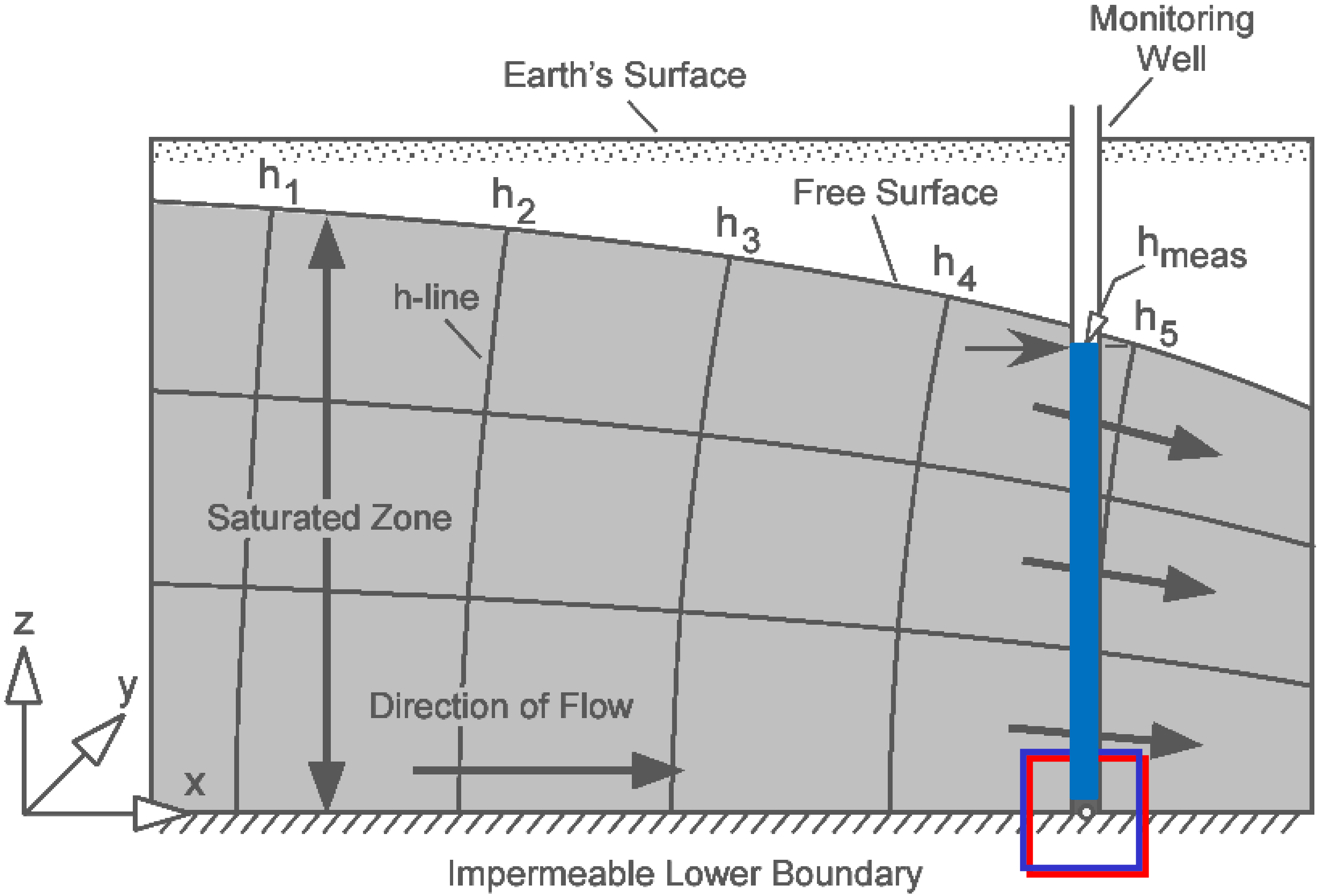
DUPUIT ASSUMPTIONS



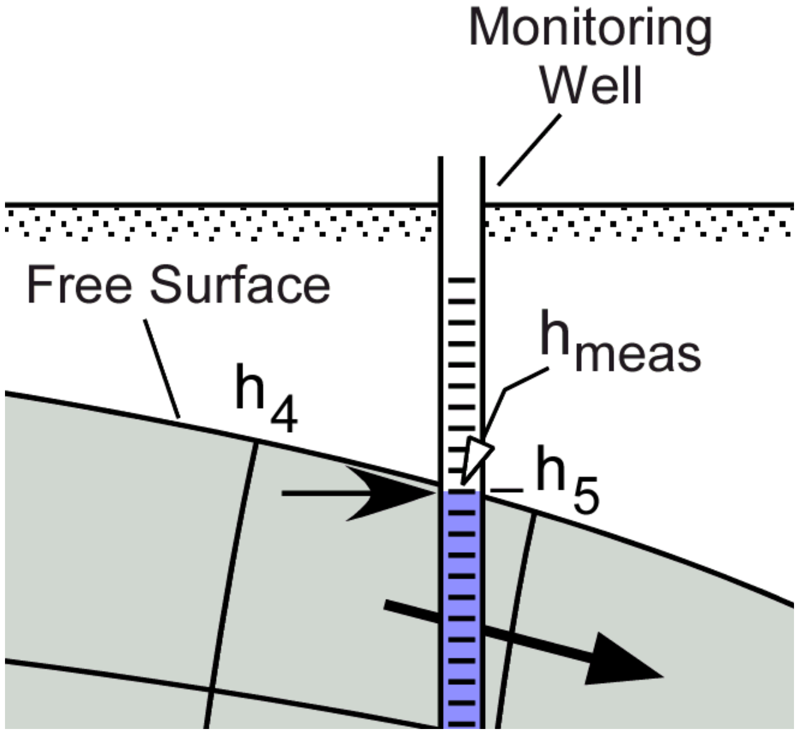
DUPUIT ASSUMPTIONS



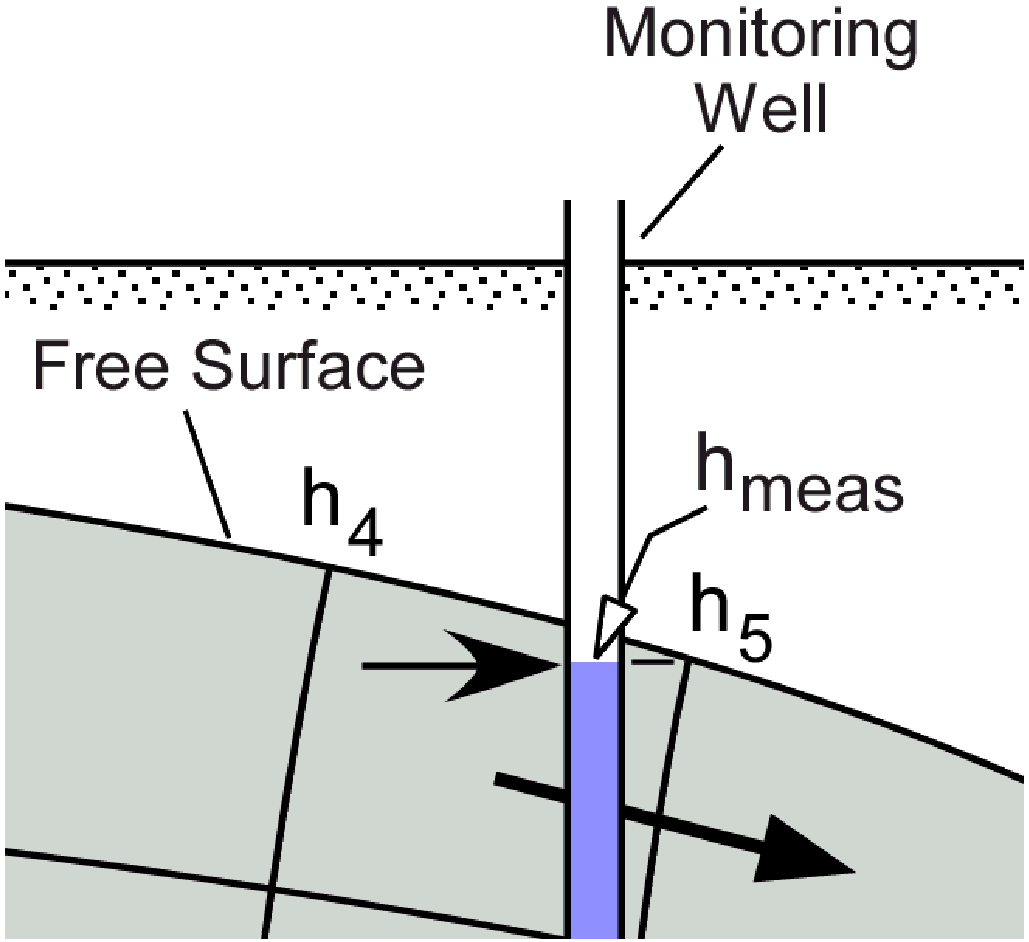
DUPUIT ASSUMPTIONS



DUPUIT ASSUMPTIONS

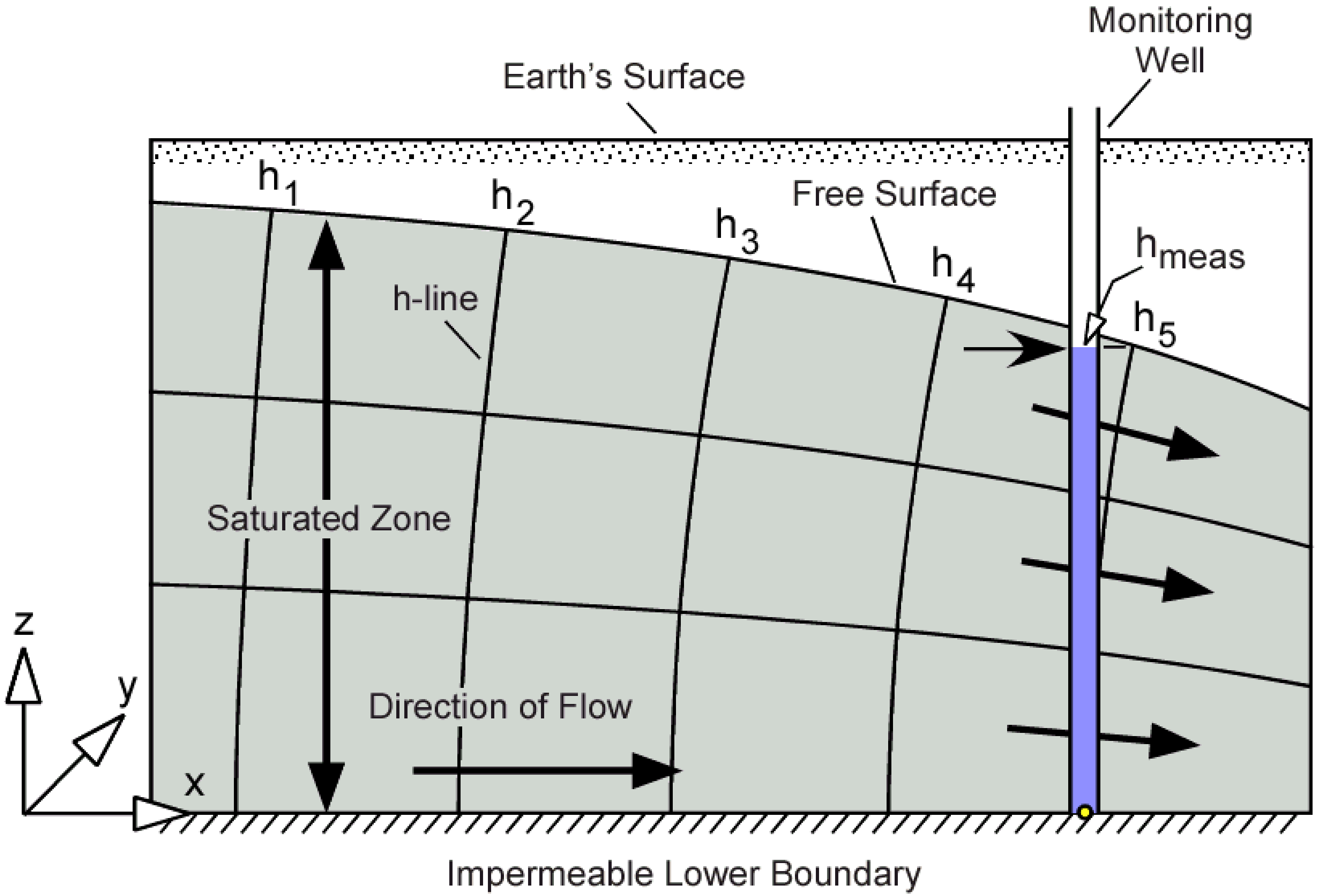


DUPUIT ASSUMPTIONS

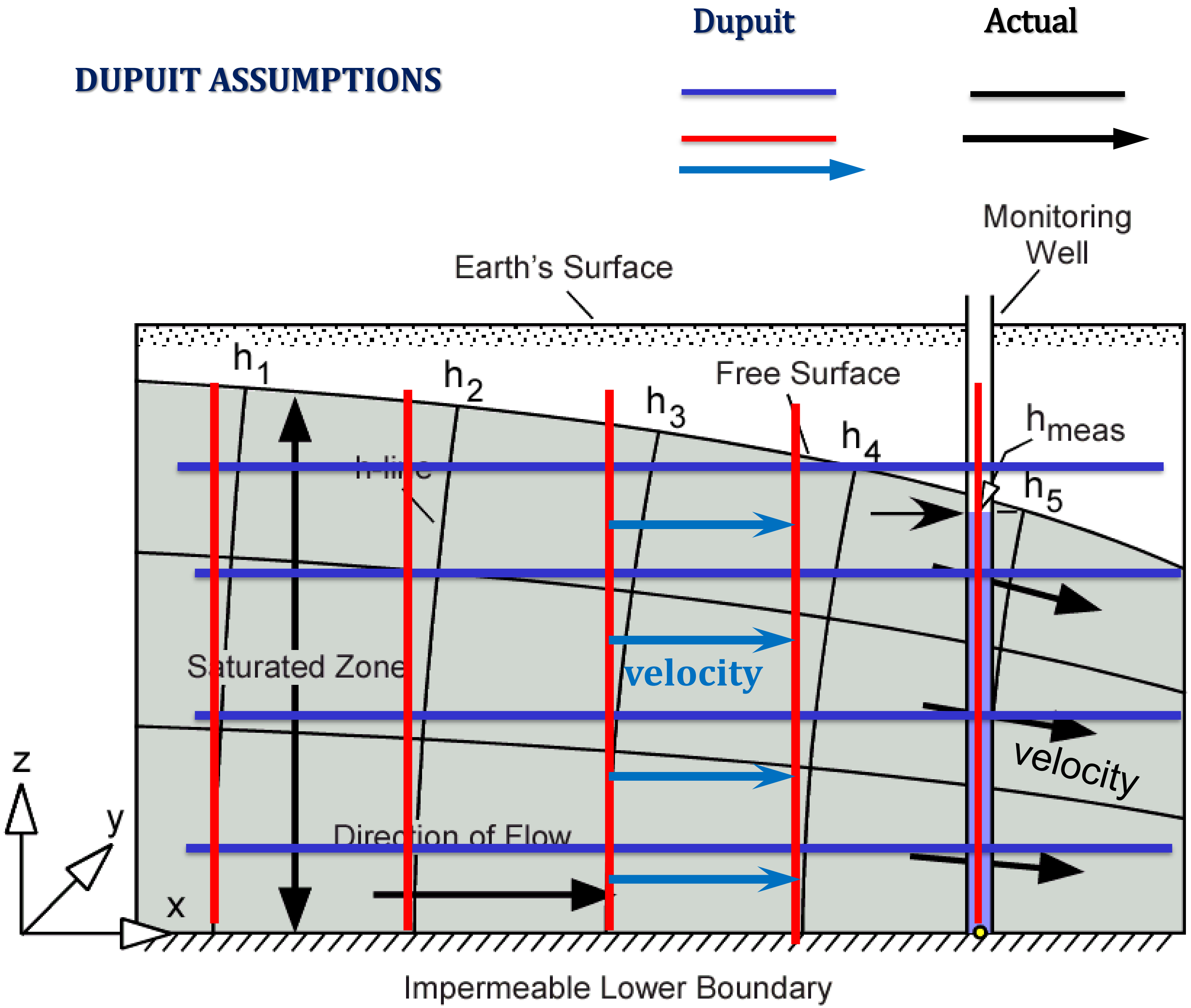


DUPUIT ASSUMPTIONS

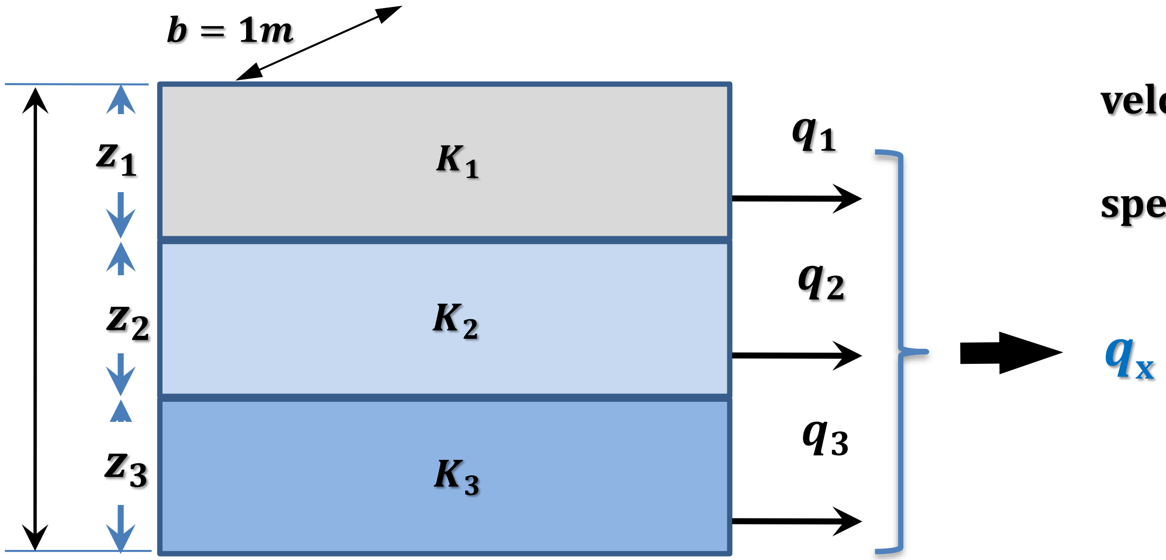
Actual



DUPUIT ASSUMPTIONS



LAYERED POROUS MEDIA (flow parallel to layers)



velocity (Darcy) ➡ $v = K\,i$

specific discharge (b=1m) ➡ $q = K\,i\,z$

$$q_1 = K_1 i\, z_1$$

$$q_2 = K_2 i\, z_2$$

$$q_3 = K_3 i\, z_3$$

Total horizontal specific discharge, **q**:

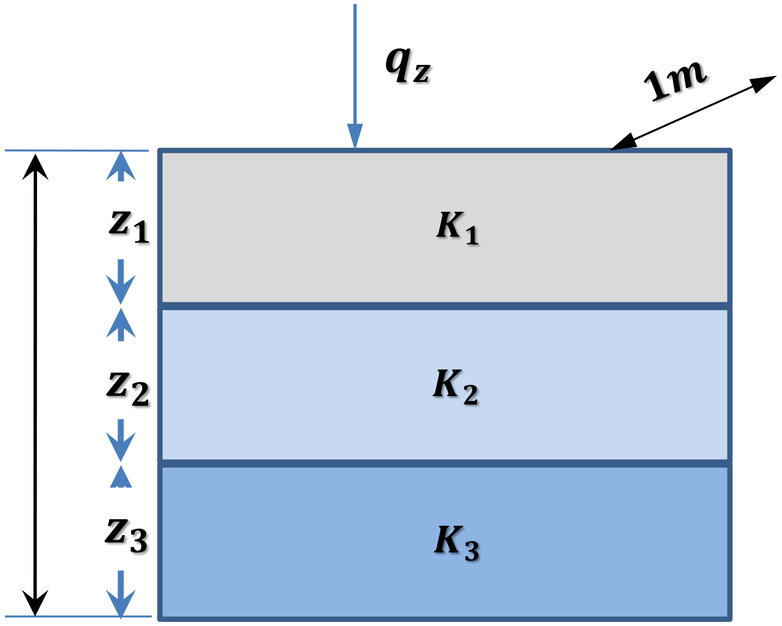
$$q_x = q_1 + q_2 + q_3 = i(K_1 z_1 + K_2 z_2 + K_3 z_3)$$

Homogeneous aquifer

$$q_x = K_x\, i(z_1 + z_2 + z_3) \;\Rightarrow\; K_x = \frac{q_x}{i\,(z_1 + z_2 + z_3)} \;\Rightarrow\; K_x = \frac{i(K_1 z_1 + K_2 z_2 + K_3 z_3)}{i(z_1 + z_2 + z_3)}$$

$$K_{eq} = \frac{\sum_{i=1}^{n} K_i\, z_i}{\sum_{i=1}^{n} z_i}$$

LAYERED POROUS MEDIA (flow perpendicular to layers)



$$q_z = K_1 \frac{dh_1}{z_1}$$

$\mathbf{dh_1}$ – pressure loss in the 1st layer

$$dh_1 = \frac{z_1}{K_1} q_z$$

From continuity eq. – $\mathbf{q_z}$ = **const.**

$$dh_1 + dh_2 + dh_3 = \left[\frac{z_1}{K_1} + \frac{z_2}{K_2} + \frac{z_2}{K_2}\right] q_z$$

Homogeneous porous media

$$dh_1 + dh_2 + dh_3 = \left[\frac{z_1 + z_2 + z_2}{K_z}\right] q_z$$

$$q_z = K_z \left[\frac{dh_1 + dh_2 + dh_3}{z_1 + z_2 + z_2}\right] \Rightarrow K_z = q_z \left[\frac{z_1 + z_2 + z_2}{dh_1 + dh_2 + dh_3}\right] \Rightarrow K_z = \frac{z_1 + z_2 + z_3}{\frac{z_1}{K_1} + \frac{z_2}{K_2} + \frac{z_3}{K_3}}$$

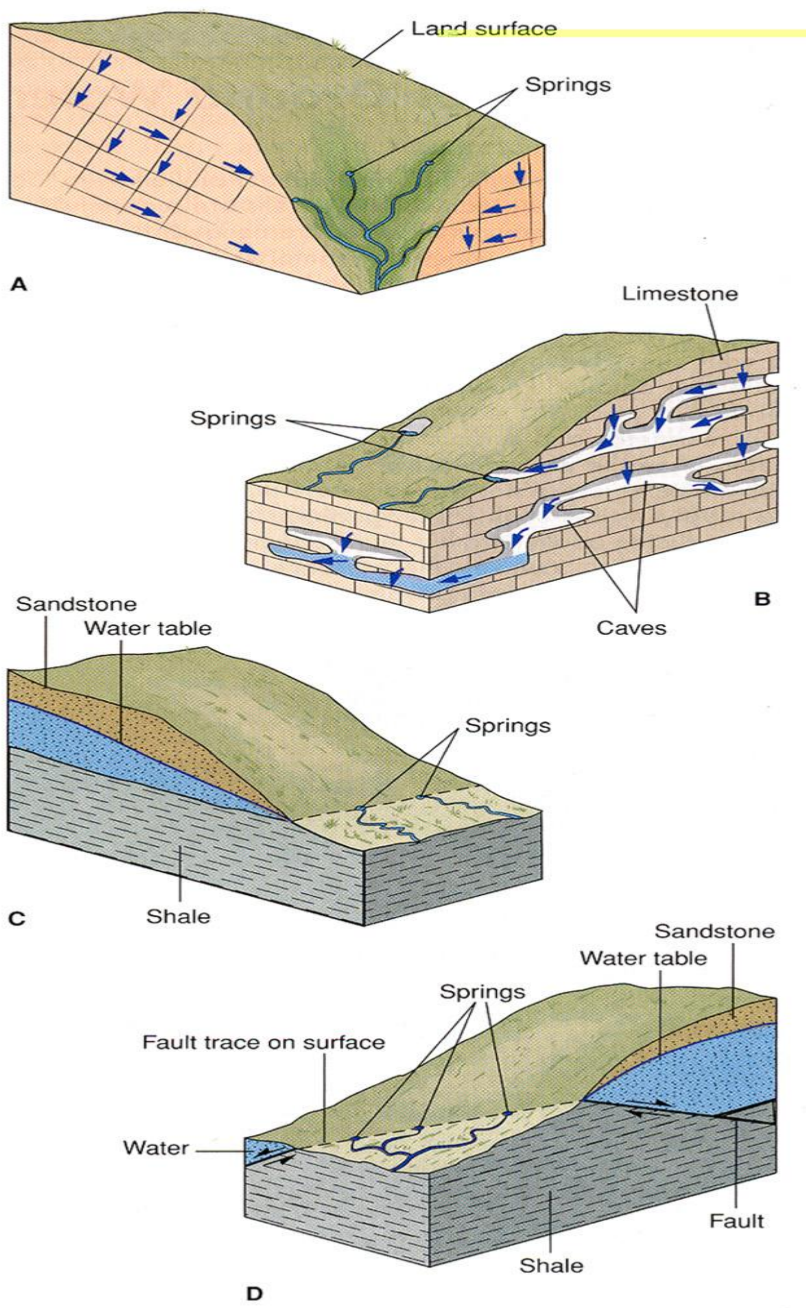
$$K_{eq} = \frac{\sum_{i=1}^{n} z_i}{\sum_{i=1}^{n} \frac{z_i}{K_i}}$$

HYDRAULICS OF WELLS

SPRINGS

- *Spring* - a place where water flows naturally from rock or sediment onto the ground surface

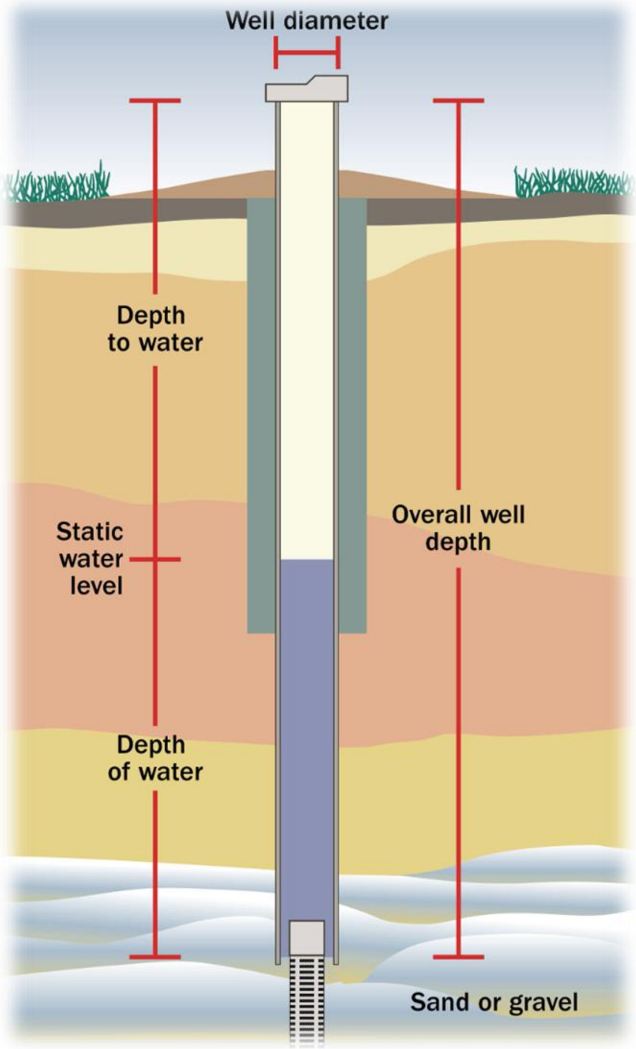
A spring is a natural exit point at which **groundwater** emerges out of the aquifer and flows onto the top of the **Earth´s crust** (pedosphere) to become surface water. It is a component of the **hydrosphere** as well as a part of the **water cycle**. Springs have long been important for **humans** as a source of **fresh water**, especially in *arid* regions which have relatively little annual **rainfall**. Springs are driven out onto the surface by various natural forces, such as **gravity** and hydrostatic pressure. The **yield** of spring water varies widely from a **volumetric flow rate** of **nearly zero** to more than **14 m^3 s^{-1}** for the biggest springs.



WELL HYDRAULICS

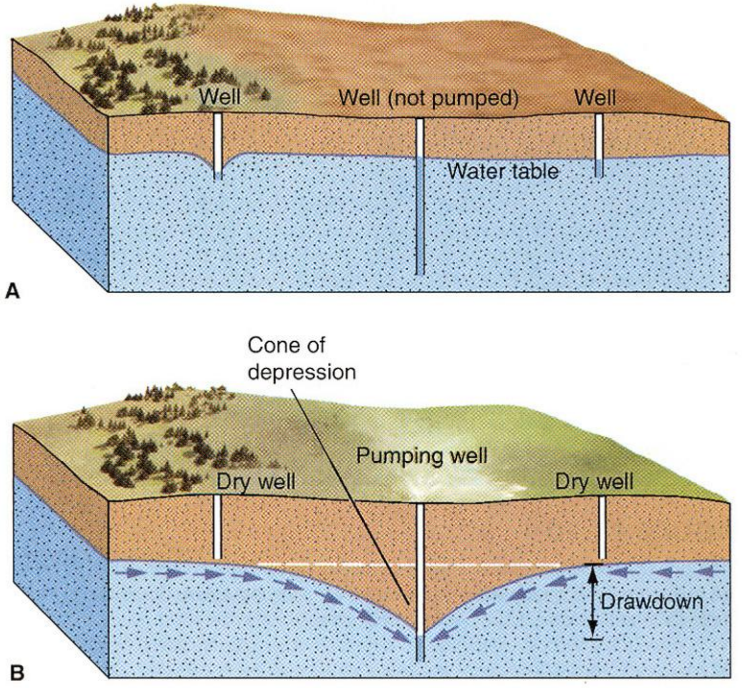
- A **water well** is a hydraulic structure that is designed and constructed to permit economic withdrawal of water from an aquifer

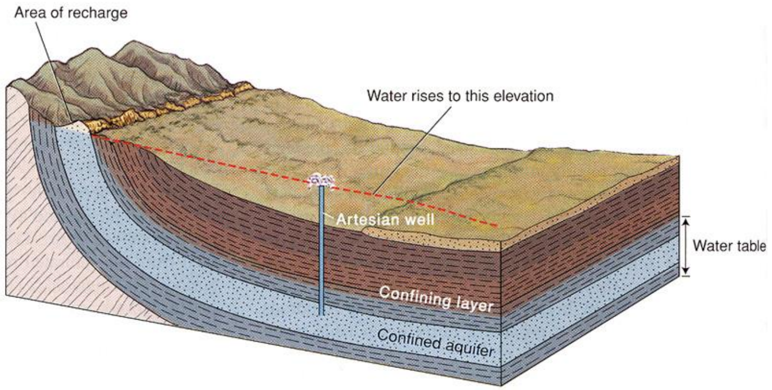
- **Water well construction includes**:
  - **Selection** of appropriate **drilling methods**
  - **Selection** of appropriate **completion materials**
  - Analysis and interpretation of well and aquifer performance



WELLS

- ***Well*** - a deep hole dug or drilled into the ground to obtain water from an aquifer
  - For wells in **unconfined aquifers**, water level before pumping is the **water table**
  - Water table can be lowered by pumping, a process known as ***drawdown***
  - Water may rise to a level above the top of a confined aquifer, producing an ***artesian well***





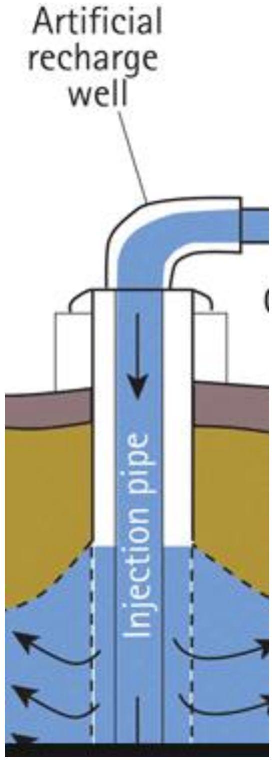
WELL

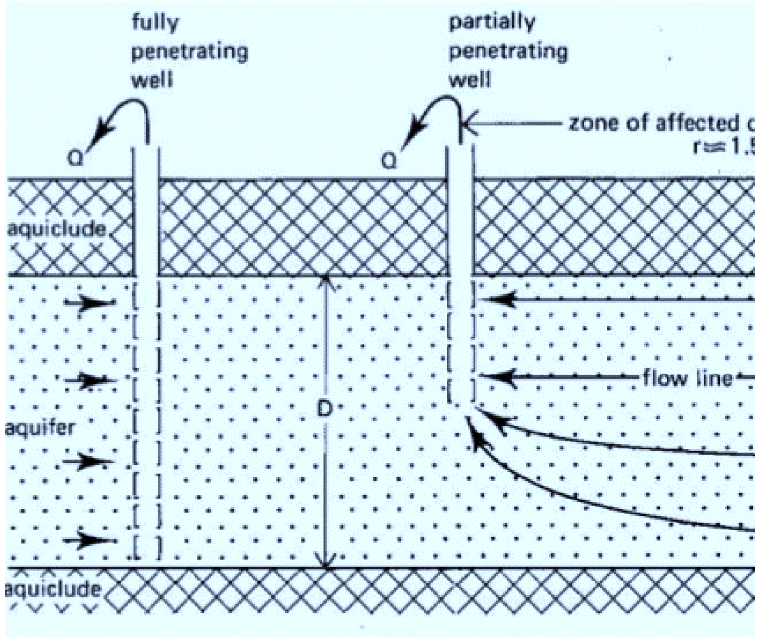
A) - Unconfined aquifer
- Confined aguifer
- (Artesian aquifer)

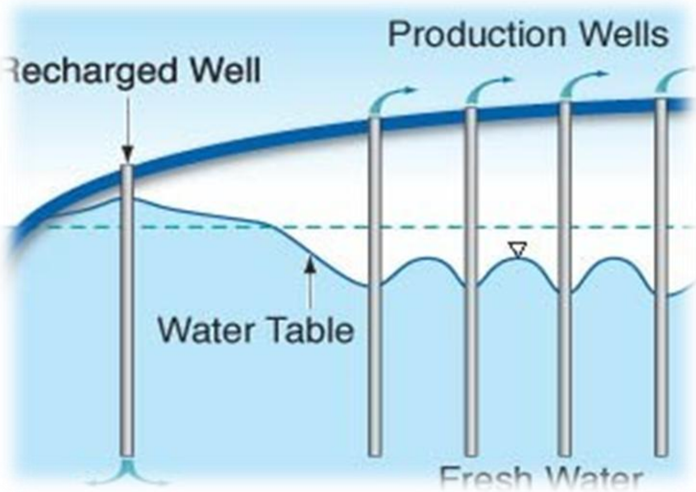
B) – Fully penetrating
- Partially penetrated

C) – Dug
- Drilled

D) – Discharge
(Production, pumped)
- Recharge

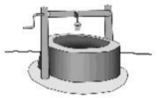






ADVANTAGES AND DISADVANTAGES

Dug Well



Advantages:

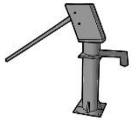
- High degree of involvement of the local community during the whole process
- Under supervision, **no skilled workers are required**
- Simple equipment sufficient for both construction and maintenance
- **Low cost** for construction
- Involvement of private sector possible (local well diggers)
- **Yield can be increased** after construction
- **Reservoir included** (large diameter)

Disadvantages:

- **Long construction phase**
- Dangerous excavation
- Application restricted to regions with rather **soft geological formation** and relatively **high groundwater levels**
- Alteration of groundwater level can adversely affect the surrounding environment
- People (i.e. children) can fall in if the well is uncovered

ADVANTAGES AND DISADVANTAGES

Drilled Well

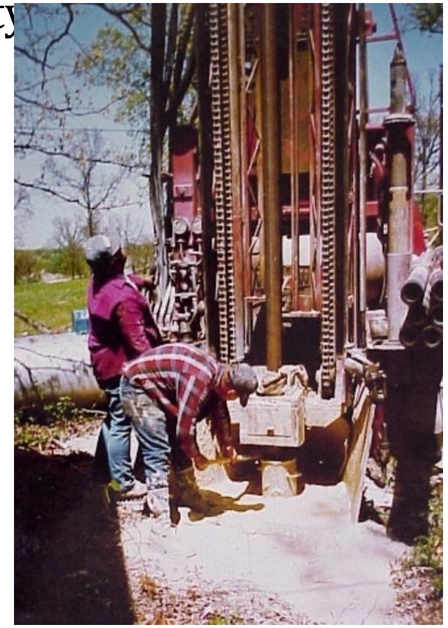


Advantages:

- **Quicker** and cheaper to sink than hand-dug wells
- Less susceptible to contamination
- **No dewatering** during sinking required
- Safer in construction and use
- The **well** itself **needs barely maintenance**
- Many simple drilling techniques available suiting many geological conditions

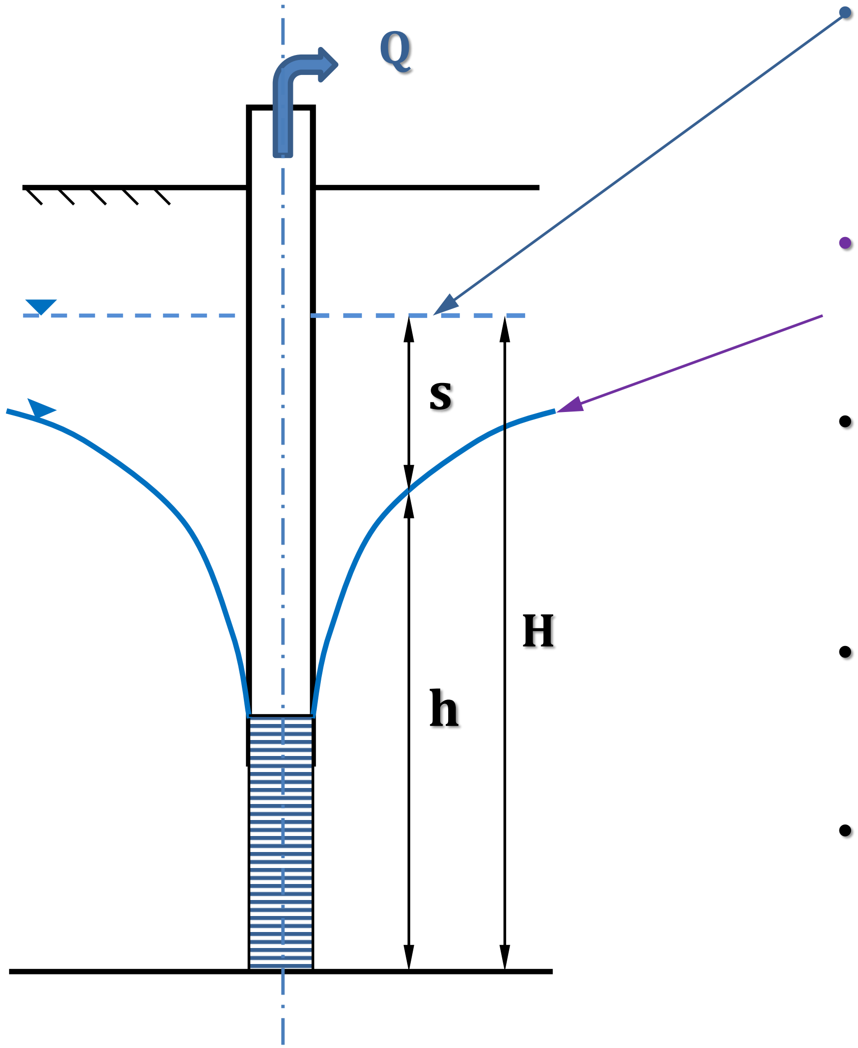
Disadvantages:

- Skilled staff and experts required for drilling
- **Pump required**
- Lower yield than hand-dug wells (smaller diameter)
- **Overexploitation** may lead to adverse effects on the environment
- More technical equipment and skills necessary for construction
- **No integrated storage** capacity

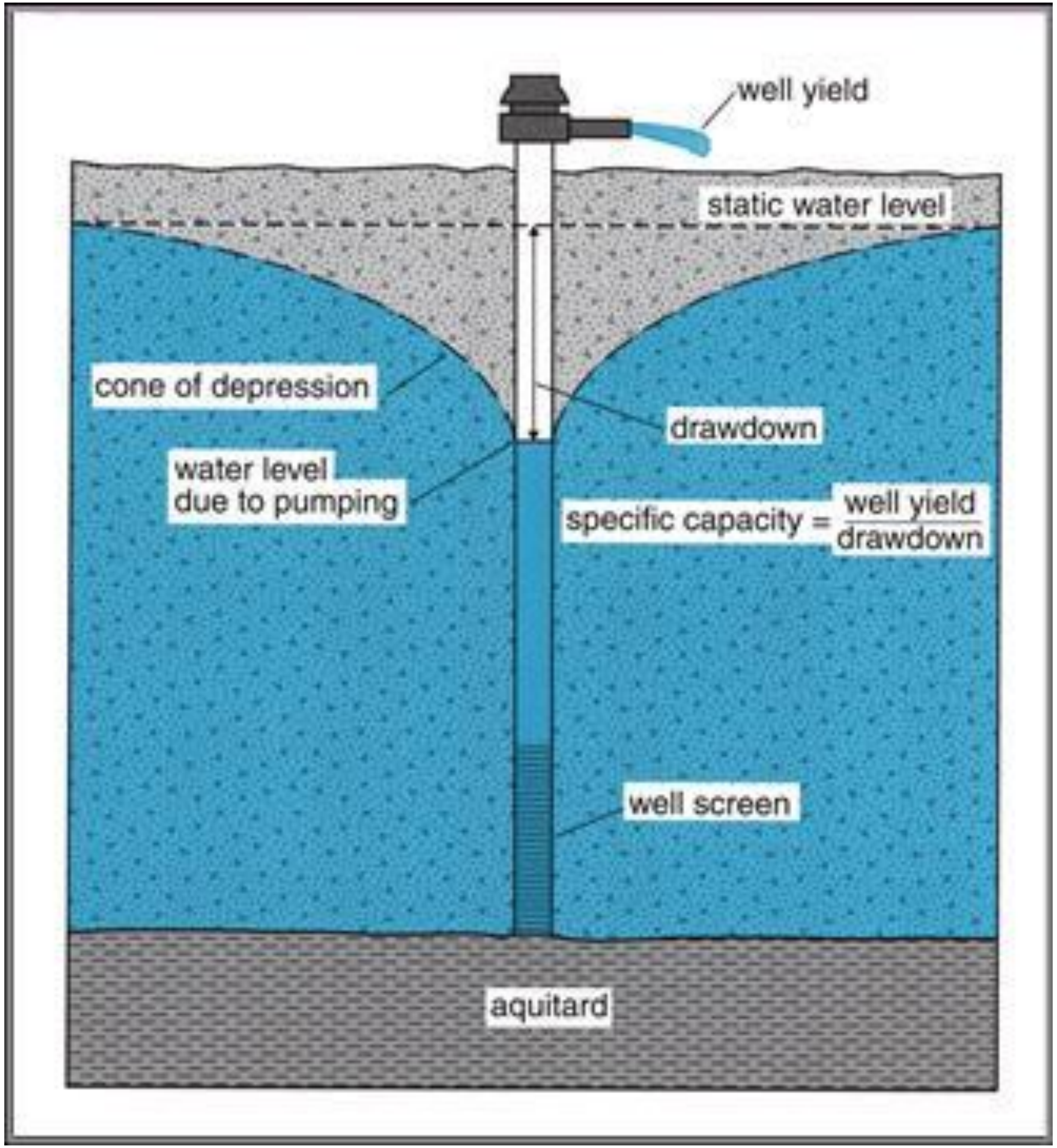


Film

PUMPING WELL TERMINOLOGY

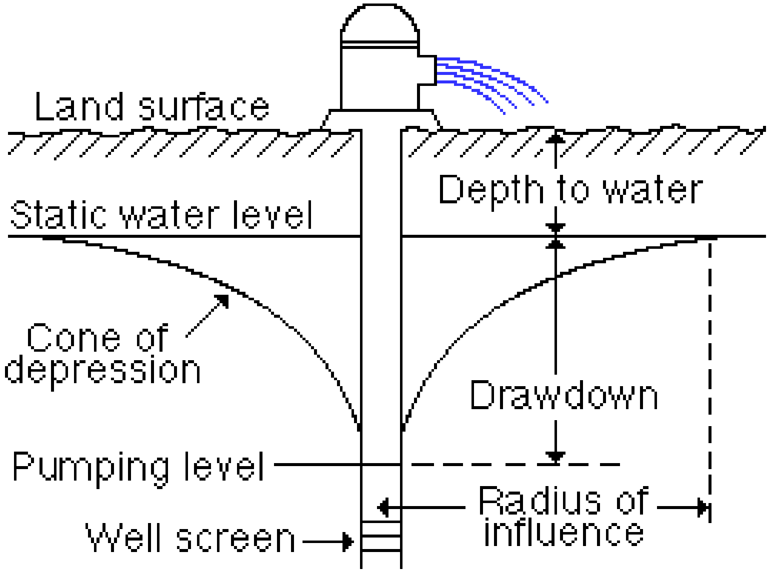


- **Static Water Level** [SWL] (H) is the equilibrium water level before pumping commences
- **Pumping Water Level** [PWL] (h) is the water level during pumping
- **Drawdown** (s = H - h) is the difference between SWL and PWL
- Well Yield (Q) is the volume of water pumped per unit time
- Specific Capacity (Q/s) is the yield per unit drawdown

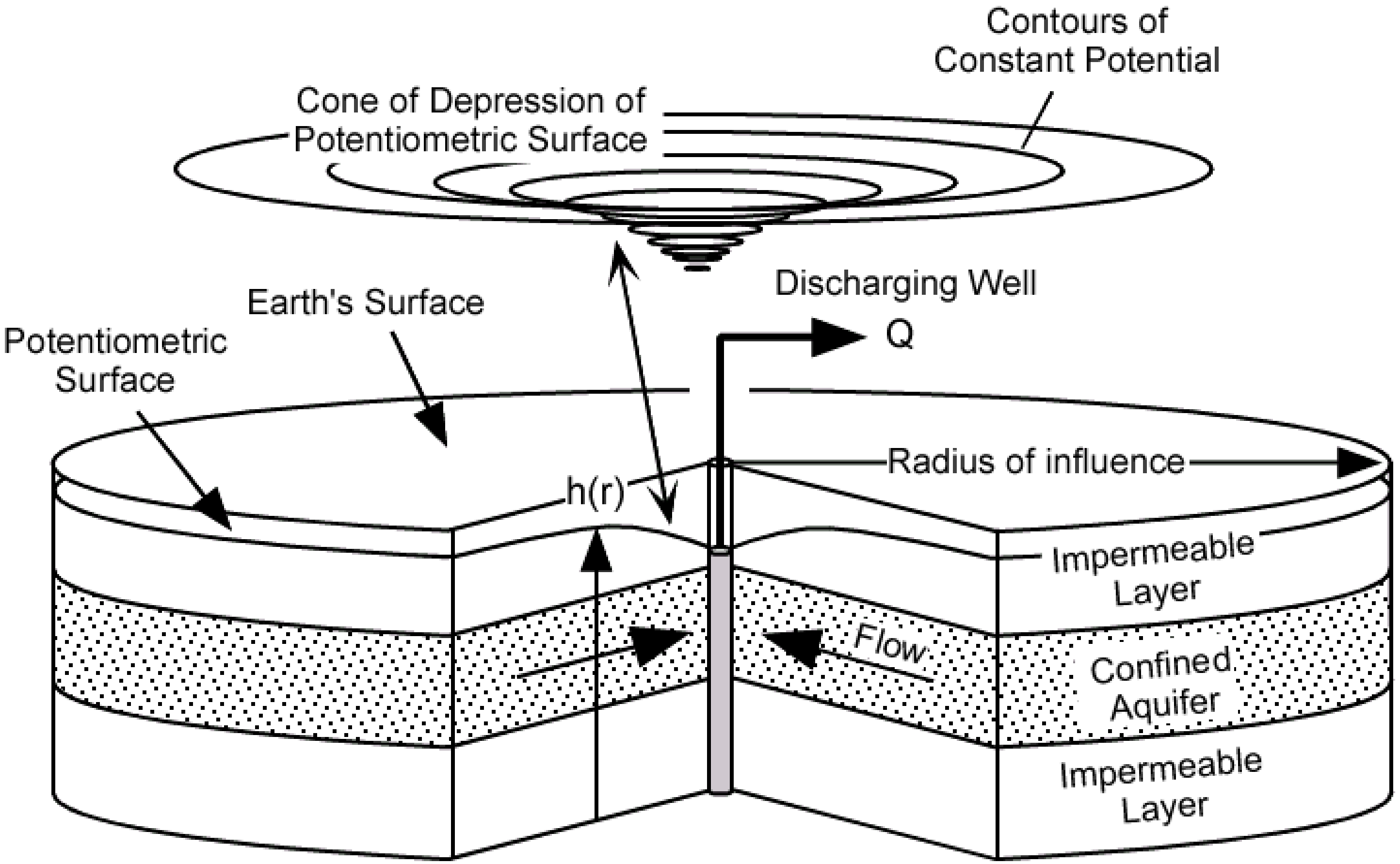


Some useful terms to know:

- ✓ **Cone of depression**
- ✓ **Drawdown, s**
- ✓ **Radius of influence, R**
- ✓ **Specific capacity, q**



Details on the geometry of drawdown and the "cone of depression".



SCHEMATIC OF A TYPICAL WELL INSTALLATION

Well screen
perforated pipe or slotted pipe

- **Well screen** (holds back sediments while allowing water to infiltrate the well)

Filter pack (sand / gravel)
extends at least 0.5 m above well screen

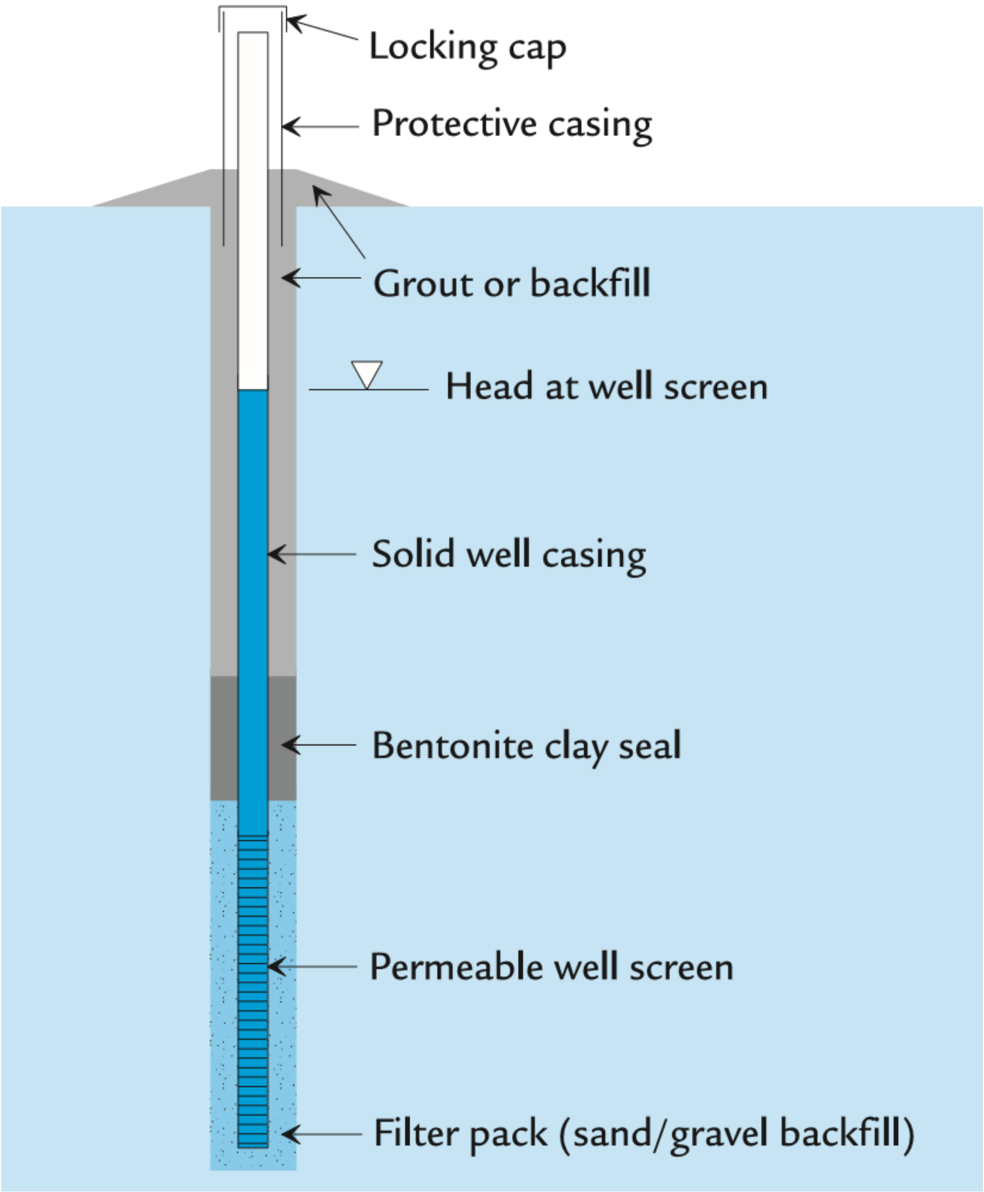
- (prevents the well screen from becoming clogged)

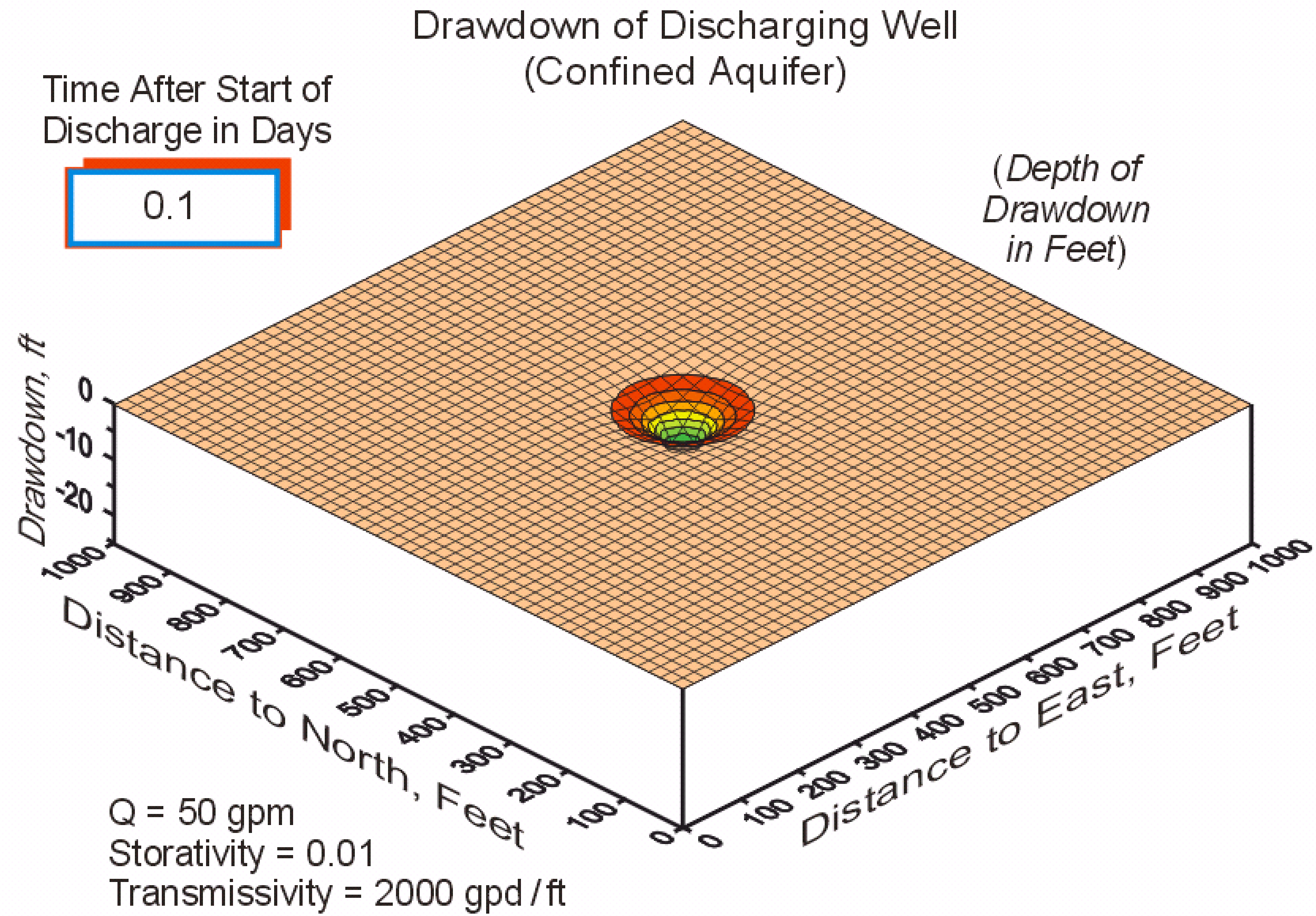
Casing
PVC or Steel

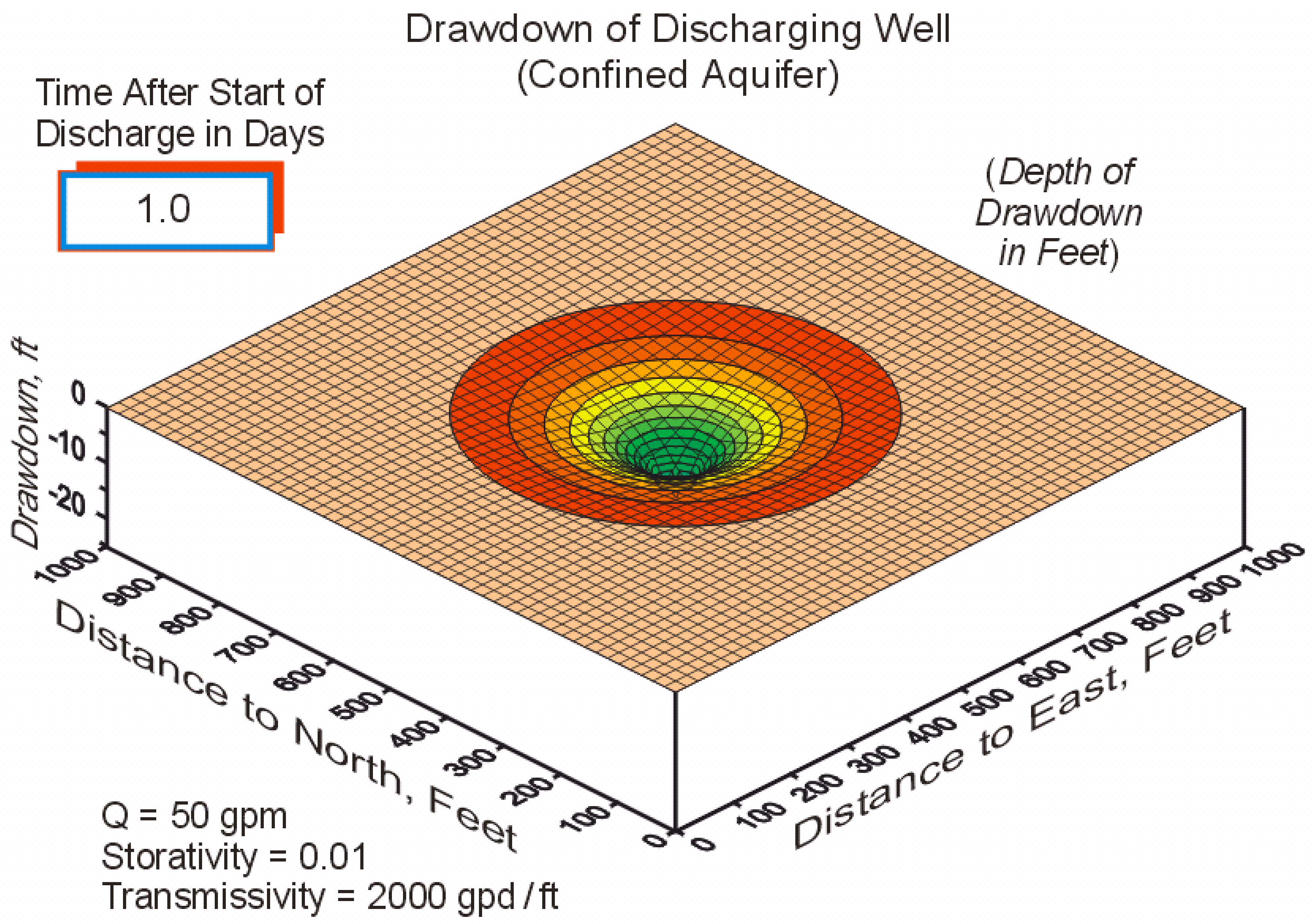
(prevents the well from collapse)

Seal
Grout, bentonite, cement

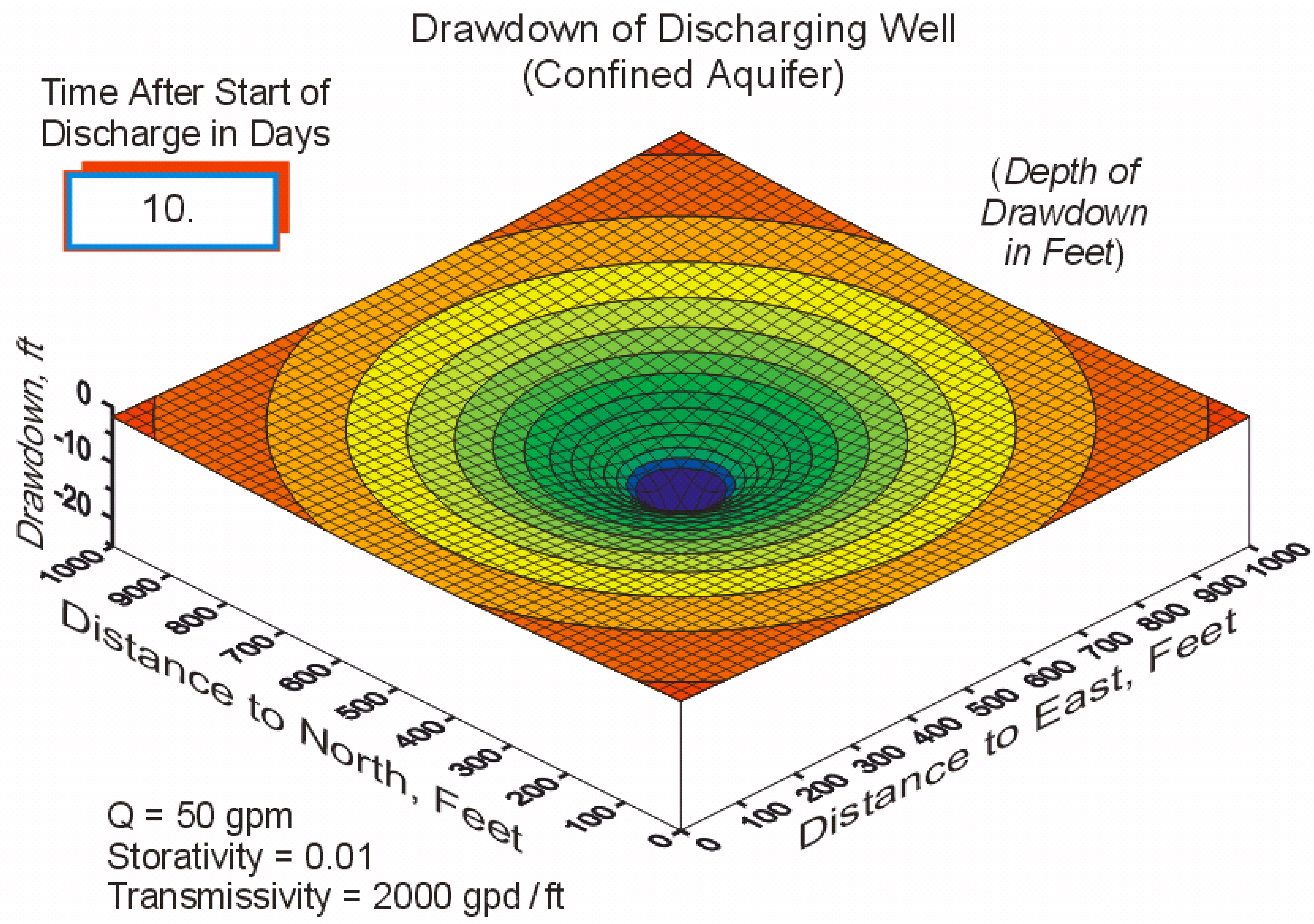
Well development
purge and surge
pump till clean




Drawdown of Discharging Well
(Confined Aquifer)
Time After Start of Discharge in Days
0.1
(Depth of Drawdown in Feet)
Drawdown, ft
0
-10
-20
Distance to North, Feet
1000 900 800 700 600 500 400 300 200 100 0
0 100 200 300 400 500 600 700 800 900 1000
Distance to East, Feet
Q = 50 gpm
Storativity = 0.01
Transmissivity = 2000 gpd / ft



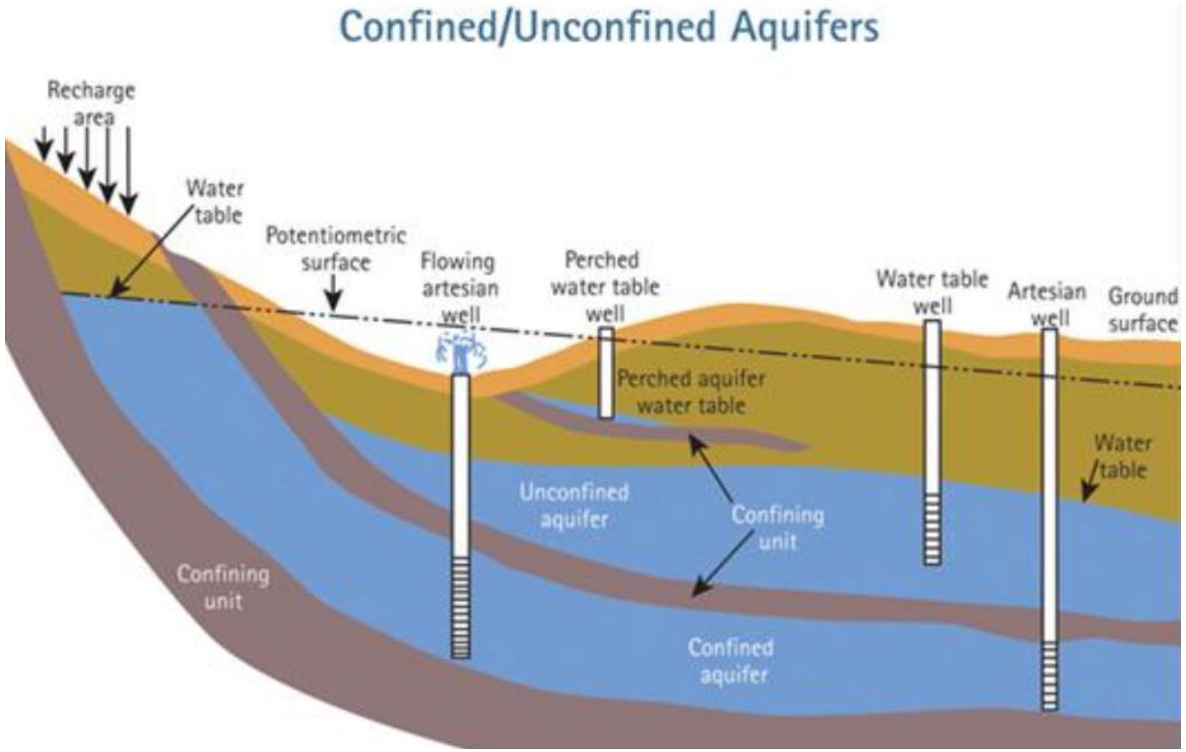
Drawdown of Discharging Well
(Confined Aquifer)
Time After Start of Discharge in Days
1.0
(Depth of Drawdown in Feet)
Drawdown, ft
0
-10
-20
Distance to North, Feet
1000 900 800 700 600 500 400 300 200 100 0
Distance to East, Feet
0 100 200 300 400 500 600 700 800 900 1000
Q = 50 gpm
Storativity = 0.01
Transmissivity = 2000 gpd / ft


Drawdown of Discharging Well
(Confined Aquifer)
Time After Start of
Discharge in Days
10.
(Depth of
Drawdown
in Feet)
Drawdown, ft
0
-10
-20
Distance to North, Feet
0
100
200
300
400
500
600
700
800
900
1000
Distance to East, Feet
0
100
200
300
400
500
600
700
800
900
1000
Q = 50 gpm
Storativity = 0.01
Transmissivity = 2000 gpd / ft

AQUIFER CHARACTERISTICS

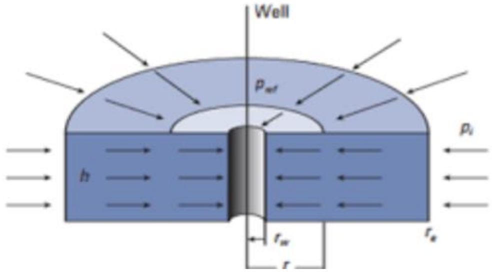
PUMP TESTS allow estimation of transmission and storage characteristics of aquifers

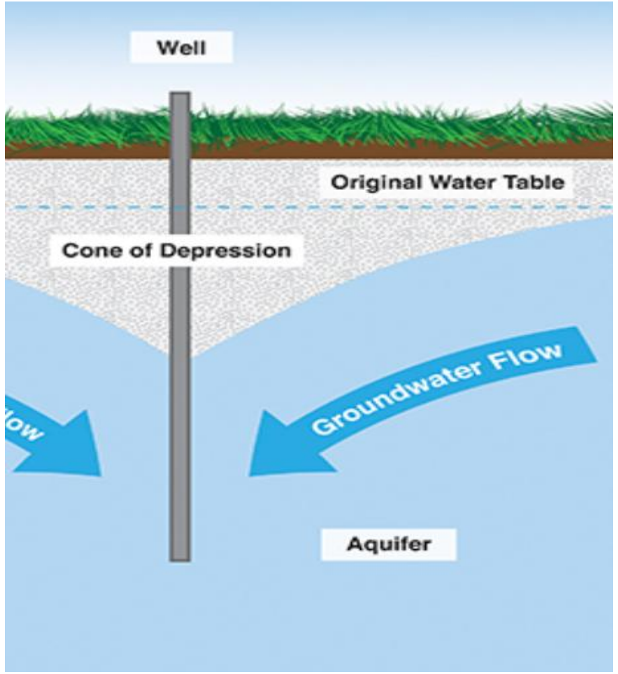
- **Transmissivity** (**T = Kb**) is the rate of flow through a vertical strip of aquifer (thickness b) of unit width under a unit hydraulic gradient
- **Storage Coefficient** (**S** = $S_y + S_s b$) is storage change per unit volume of aquifer per unit change in head
- **Radius of Influence** (**R**) for a well is the maximum horizontal extent of **the cone of depression** when the well is in equilibrium with inflows



BASIC ASSUMPTIONS

- The aquifer is **homogeneous** and **isotropic**.
- All **flow is radial** toward the well.
- Ground water flow is **horizontal.**
- **Darcy's law** is valid.
- Ground water has a **constant density** and **viscosity.**
- The pumping well and the observation wells are **fully penetrating** aquifer.
- The pumping well has an **infinitesimal diameter** and is 100% efficient.



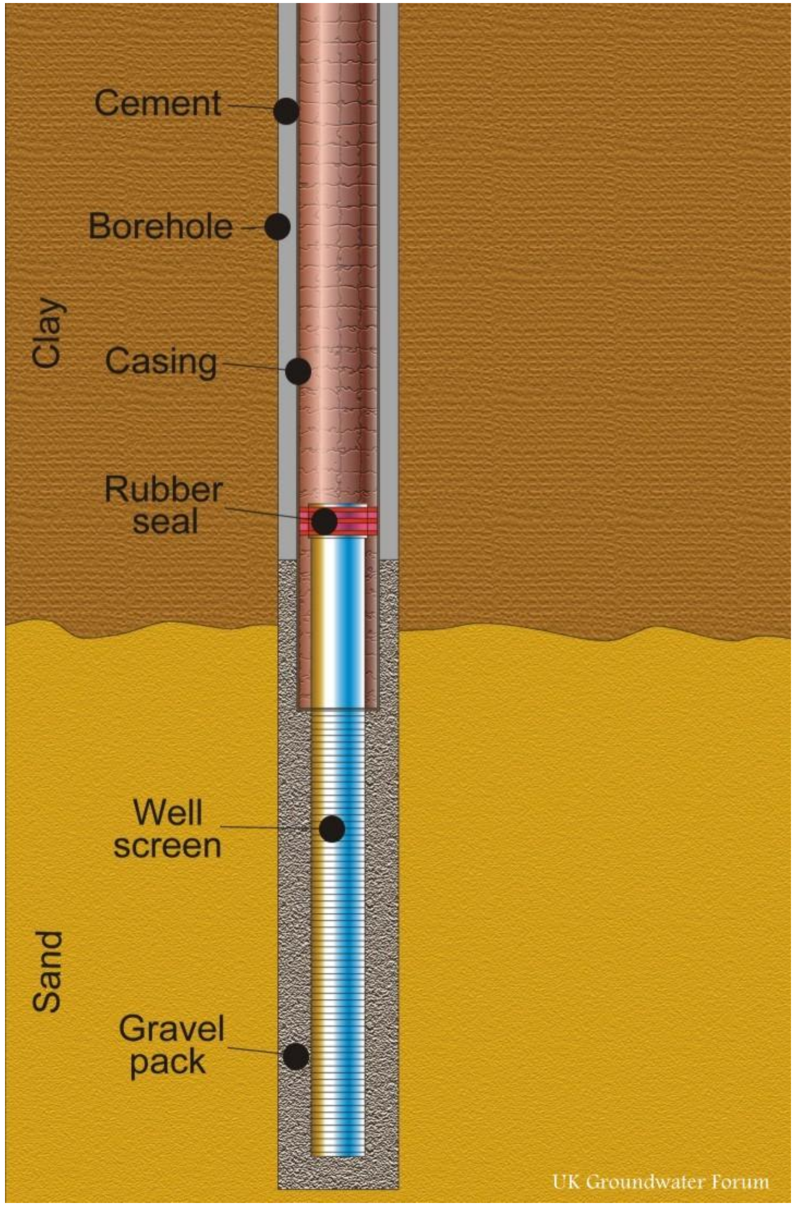


WELL COMPONENTS

Borehole **diameter**
Depth and length **of screen**
Filter pack
Seal necessary

BOREHOLE DIAMETERS

Piezometers: 2.5 – 5 cm
Monitoring wells: 5 – 20 cm
Domestic supply: 10 – 40 cm
Public water supply: 20 cm



STEADY FLOW - WELL

PUMP TEST PLANNING

- Pump tests will not produce satisfactory estimates of either aquifer properties or well performance unless the data collection system is carefully is addressed in the design.
- Several preliminary estimates are needed to design a successful test:
  - Estimate the maximum drawdown at the pumped well
  - Estimate the maximum pumping rate
  - Evaluate the best method to measure the pumped volumes
  - Plan discharge of pumped volumes distant from the well
  - Estimate drawdowns at observation wells
  - Simulate the test before it is conducted
  - Measure all initial heads several times to ensure that steady-conditions prevail
  - Survey elevations of all well measurement reference points

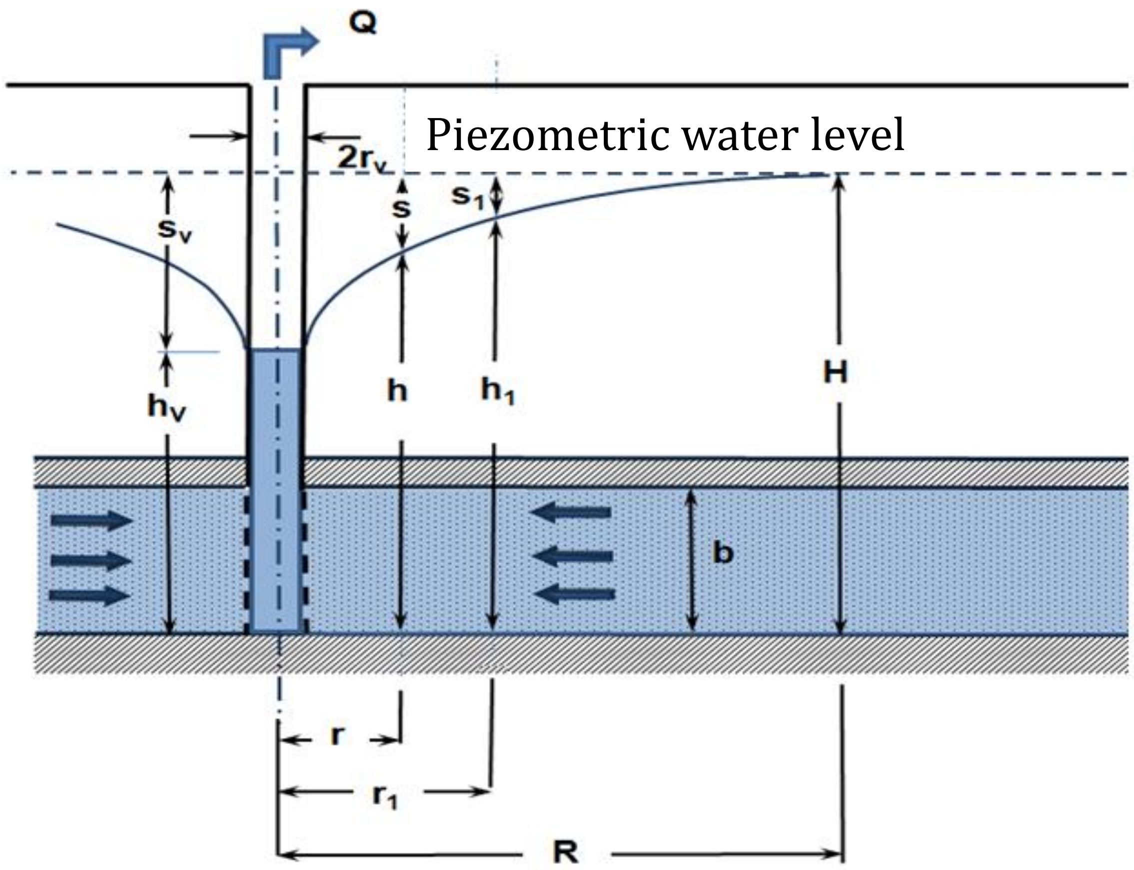
ADVANTAGES OF PUMPING TESTS

- Measure **parameters *in situ***.
- **Average parameters** over a large volume.
- **Measure T** and **S** simultaneously.

DISADVANTAGES OF PUMPING TESTS

- **High cost**.
- Non-uniqueness of T and S results.

STEADY RADIAL FLOW CONFINED FLOW



$$\int_{h_v}^{H} dh = \frac{Q}{2\pi Kb} \int_{r_v}^{R} \frac{dr}{r} \Rightarrow H - h_v = s_v = \frac{Q}{2\pi Kb} \ln \frac{R}{r_v}$$

Hydraulic head:

$\mathbf{h}$=z+p/γ

$$\boldsymbol{Q} = \boldsymbol{A}.\boldsymbol{v_r}$$

$$\boldsymbol{A} = \boldsymbol{2\pi r\ b} \qquad v_r = -K\frac{dh}{dr}$$

$$\boldsymbol{Q} = A.v_r = 2\pi rb\ K\frac{dh}{dr}$$

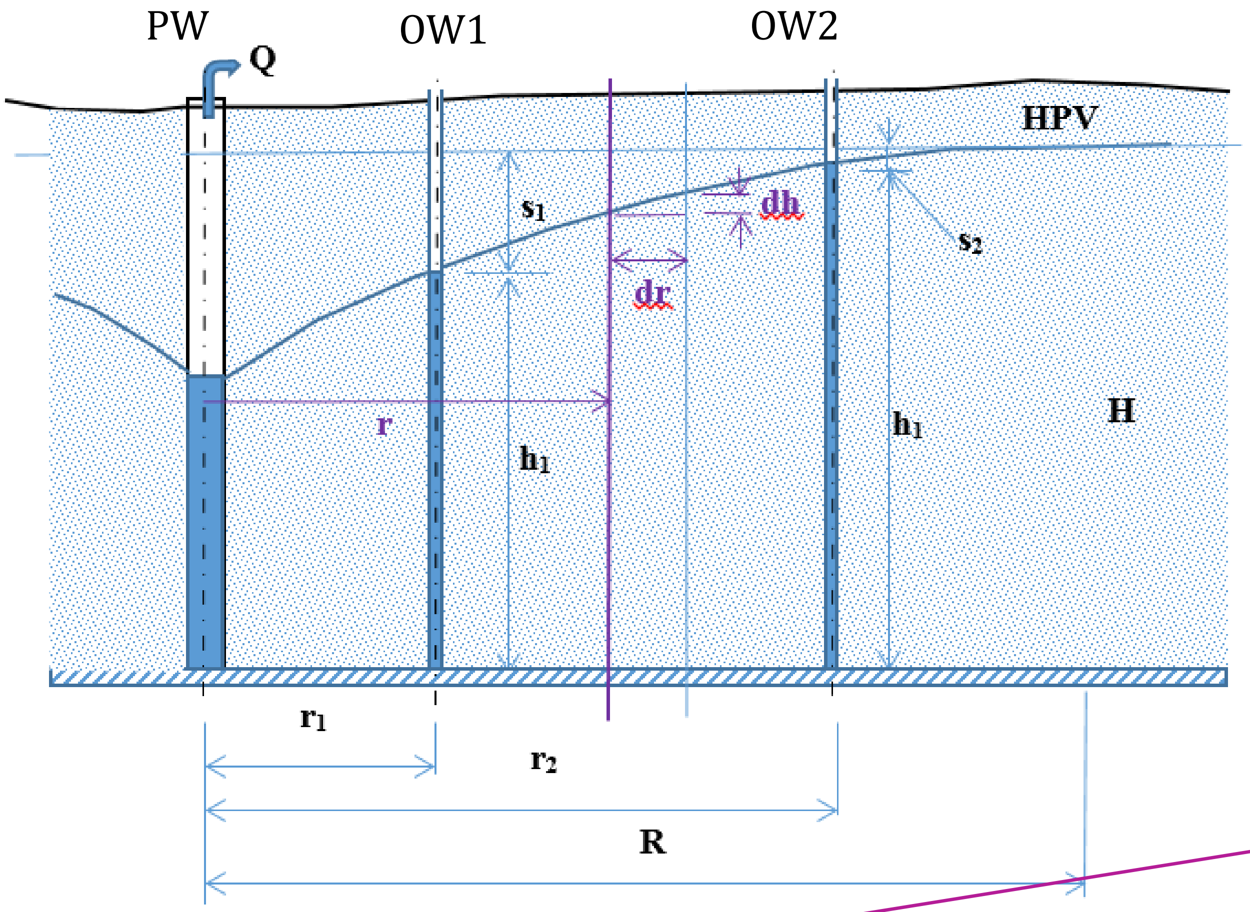
Boundary condition

$$r = r_v \quad ...... h = h_v$$

$$r = R \quad ...... h = H$$

This is the Thiem equation

STEADY UNCONFINED RADIAL FLOW



Continuity eq.

$$Q = \boldsymbol{A}.v_r$$

$$\boldsymbol{A} = 2.\pi rh$$

$$\boldsymbol{v_r} = -K.I = -\boldsymbol{K}.\frac{\boldsymbol{dh}}{\boldsymbol{dr}}$$

$$\boldsymbol{Q} = 2.\pi rh\ K\frac{dh}{dr}$$

$$\int_{h_V}^{H} hdh = \frac{Q}{2\pi K}\int_{r_V}^{R}\frac{dr}{r}$$

$$\left[\frac{h^2}{2}\right]_{h_V}^{H} = \frac{Q}{2\pi K}[\ln r]_{r_V}^{R}$$

$$H^2 - h_V^2 = \frac{Q}{\pi K}\ln\frac{R}{r_V}$$

This is the Thiem equation