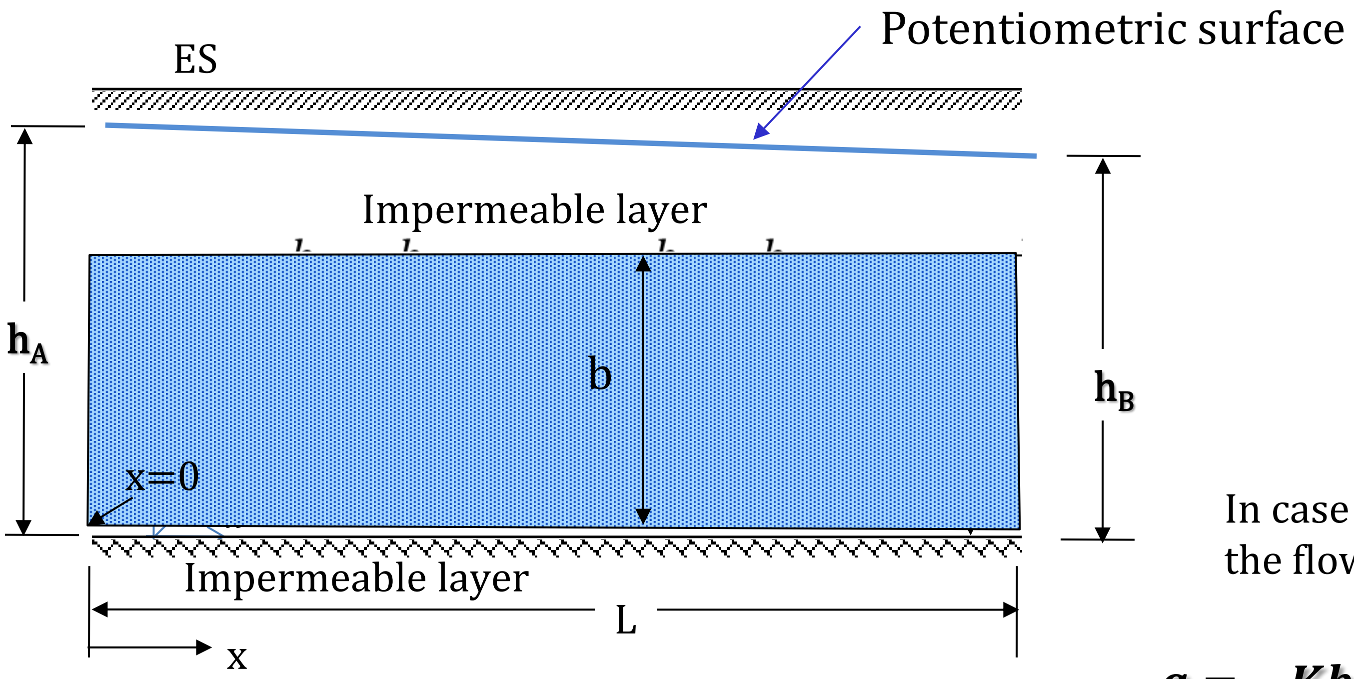

Groundwater Hydraulics
7

SEEPAGE – 1D

GROUNDWATER FLOW IN A CONFINED AQUIFER STEADY FLOW



Darcy´s equation

$$\boldsymbol{Q = K.A\frac{\Delta h}{L} = K.A.I}$$

Hydraulic gradient

$$I = \frac{dh}{dx}$$

In case of 1-D GW flow in an **isotropic, homogeneous** aquifer, the flow rate per unit width (q) is

$$\boldsymbol{q = -Kb\frac{dh}{dx}}$$

Eq. can be rewritten in the form
Eq. can be rewritten in the form

$$\boldsymbol{dh = \frac{q}{bK}\,dx}$$

$$\int_{h_A}^{h_B} dh = -\frac{q}{bK}\int_0^L dx$$

Boundary conditions

x = 0 ……… x=L
$h = h_A$……… $h = h_B$

After integration

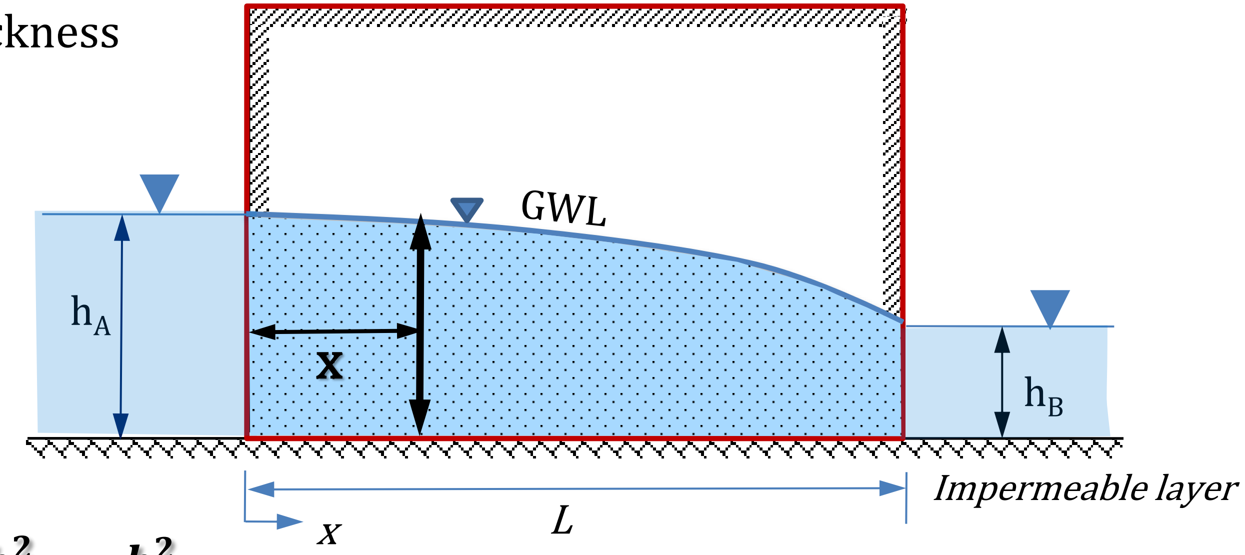
$$h_B - h_A = -\frac{q}{bK}\,L$$

The flow rate per unit width is

$$\boldsymbol{q = K\,b\,\frac{h_A - h_B}{L}}$$

GROUNDWATER FLOW IN AN UNCONFINED AQUIFER

With the **Dupuit asumptions**, the flow per unit thickness



$$\boldsymbol{q} = vh = (-\boldsymbol{K}\frac{\boldsymbol{dh}}{\boldsymbol{dx}})\boldsymbol{h}$$

Po úpravě

$$\boldsymbol{qdx = -Khdh}$$

Integration - **x = 0 h=h_A x=x h(x)=h**

$$q \int_{x=0}^{x} dx = -K \int_{h=h_A}^{h} h\,dh \Longrightarrow \boldsymbol{q\,x = K\frac{h_A^2 - h^2}{2}}$$

Then for GWL

$$\boldsymbol{h^2 = h_A^2 - \frac{2qx}{K}} \Longrightarrow \boldsymbol{h} = \sqrt{h_A^2 - \frac{2qx}{K}}$$

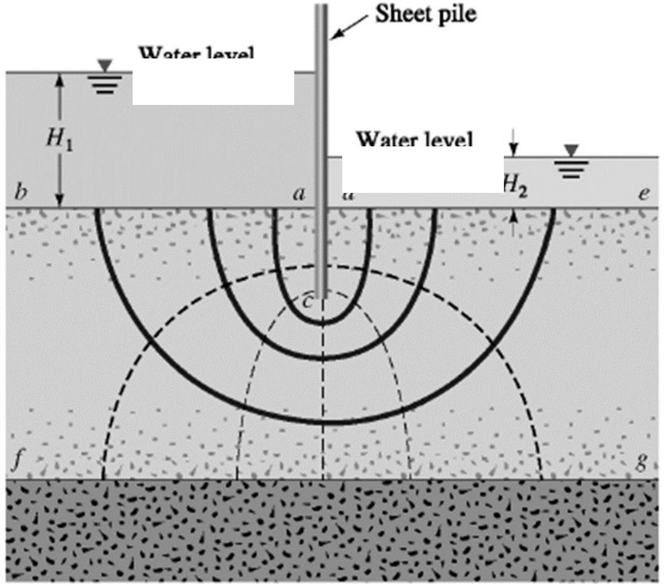
And specific discharge **x = L h = h_B**

$$q = K\frac{h_A^2 - h_B^2}{2L}$$

LAPLACE EQUATION

Darcy eq. for anisotropic porous media

$$v_x = -K_x \frac{\partial h}{\partial x} \quad v_y = -K_y \frac{\partial h}{\partial y} \quad v_z = -K_z \frac{\partial h}{\partial z}$$



From continuity equation

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} + \frac{\partial v_z}{\partial z} = 0$$

With Darcy eq.:

$$\frac{\partial}{\partial x}\left(K_x \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y \frac{\partial h}{\partial y}\right) + \frac{\partial}{\partial z}\left(K_z \frac{\partial h}{\partial z}\right) = 0$$

If soil is isotropic ....$K_x = K_y = K_z = K$

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

$\nabla^2 h = 0$ **... LAPLACE EQUATION**

2-D SEEPAGE – LAPLACE EQUATION

Laplace eguation is a very important equation in engineering
It represents loss of fluid flow through porous medium

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial z^2} = 0$$

Exat solution of Laplace´s equation for 2-D seepage can be obtained for cases with **simple boundary conditions**
For most practical geotechnical problems, it is simpler to solve this equation graphically by drawing **FLOW NETS.**

It is assumed:

- the soil is **homogeneous** and **isotropic** with respect to permeability
- The pore fluid is incompressible

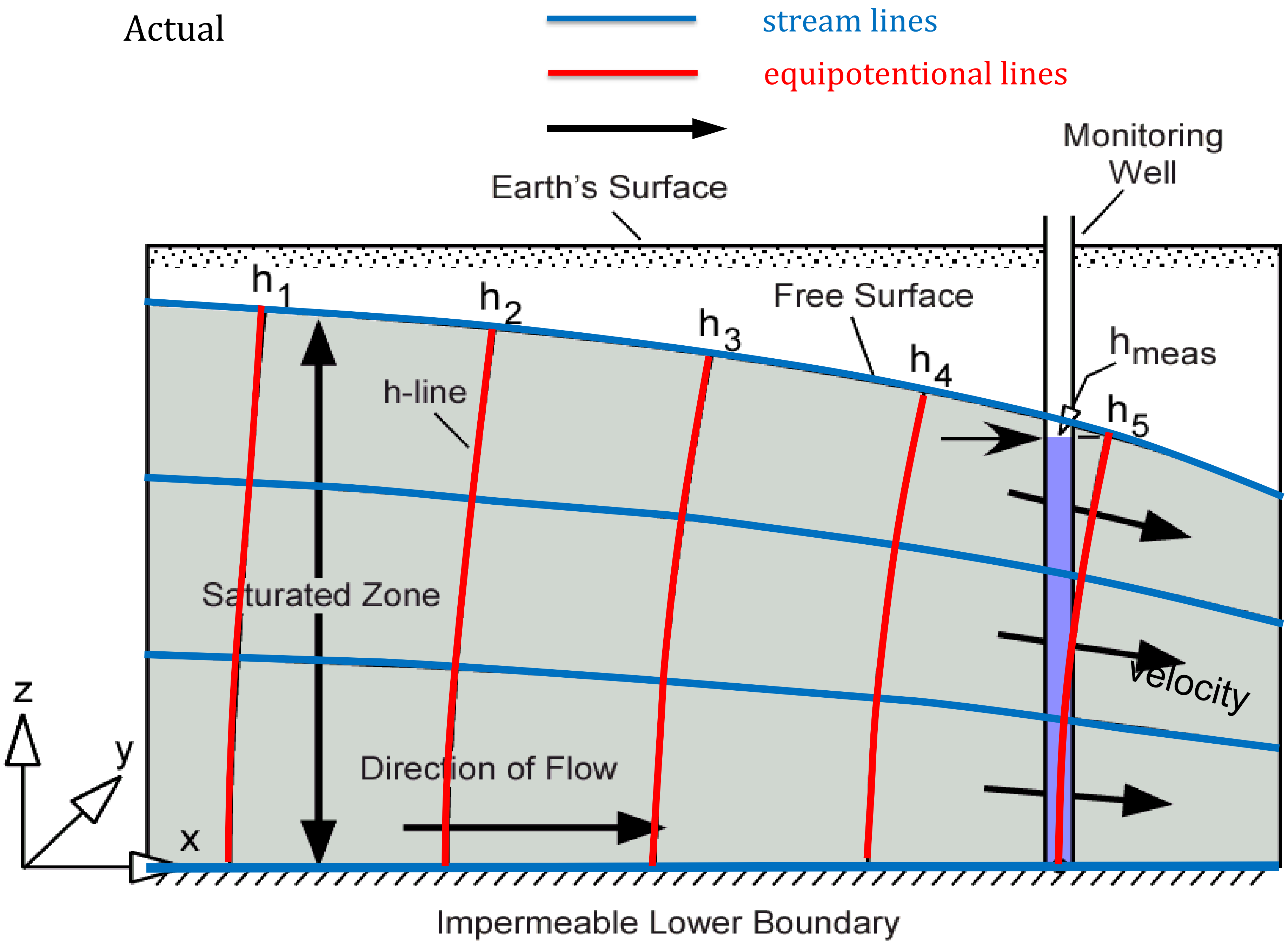
SEEPAGE TERMINOLOGY

- **FLOW NET** consists of two sets of curves – equipotentials and flow lines (stream lines) – that intersect each other at 90°
- Along an equipotentional, the total head is constant
- A pair of adjacent **stream lines** define a **FLOW CHANNEL** through which the rate of flow of pore fluid is constant
- The loss of head between two successive equipotentials is called the **EQUIPOTENTIAL DROP**

Stream line is simply the path of a water particle (molecule).

From upstream to downstream, **total head steadily decreases** along the stream line.

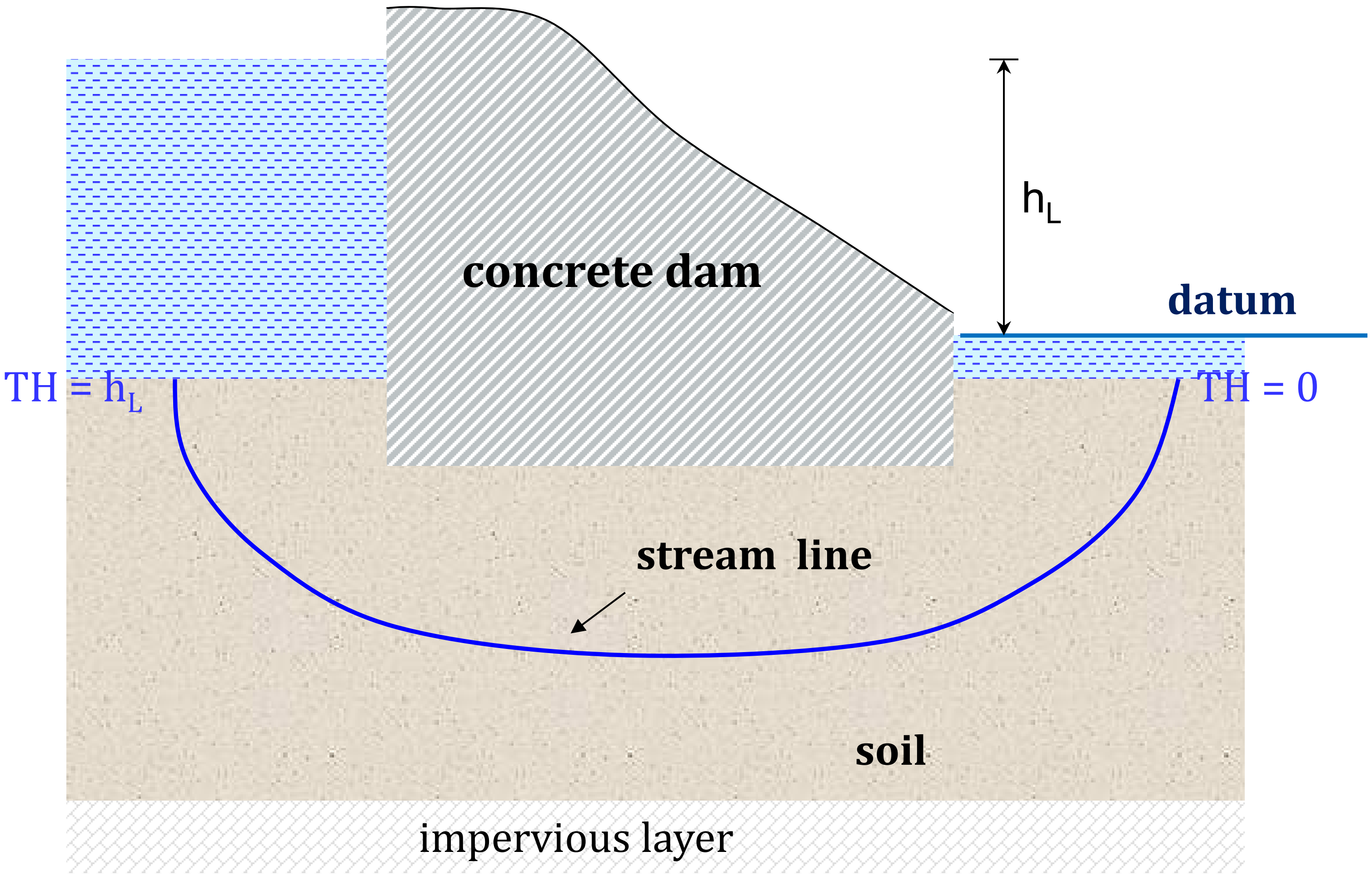
Actual



stream lines
equipotentional lines
Monitoring Well
Earth's Surface
h1
h2
h3
h4
h5
hmeas
Free Surface
h-line
Saturated Zone
Direction of Flow
velocity
z
y
x
Impermeable Lower Boundary

SEEPAGE – FLOW NETS

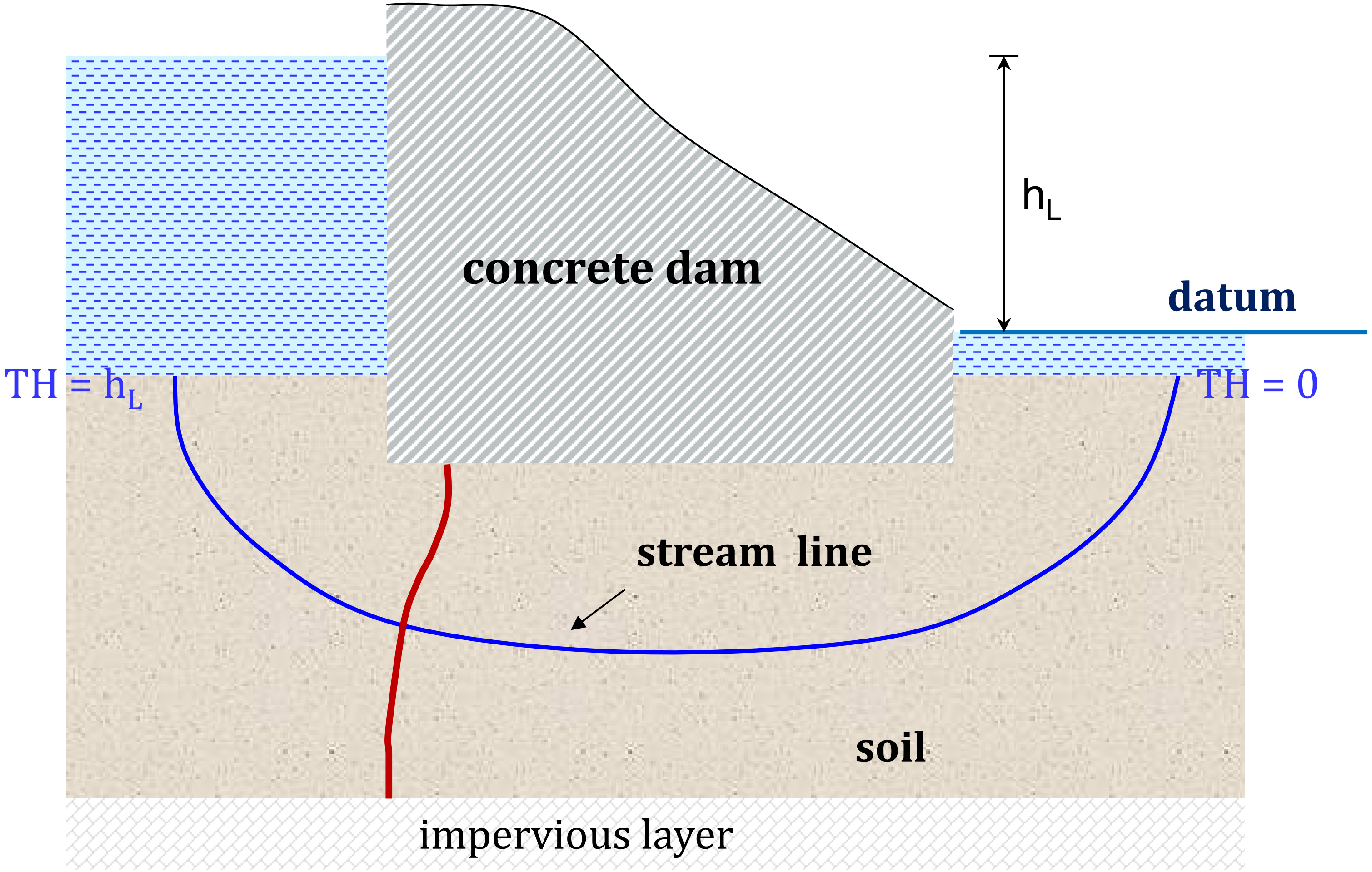
STREAM LINE



SEEPAGE – FLOW NETS

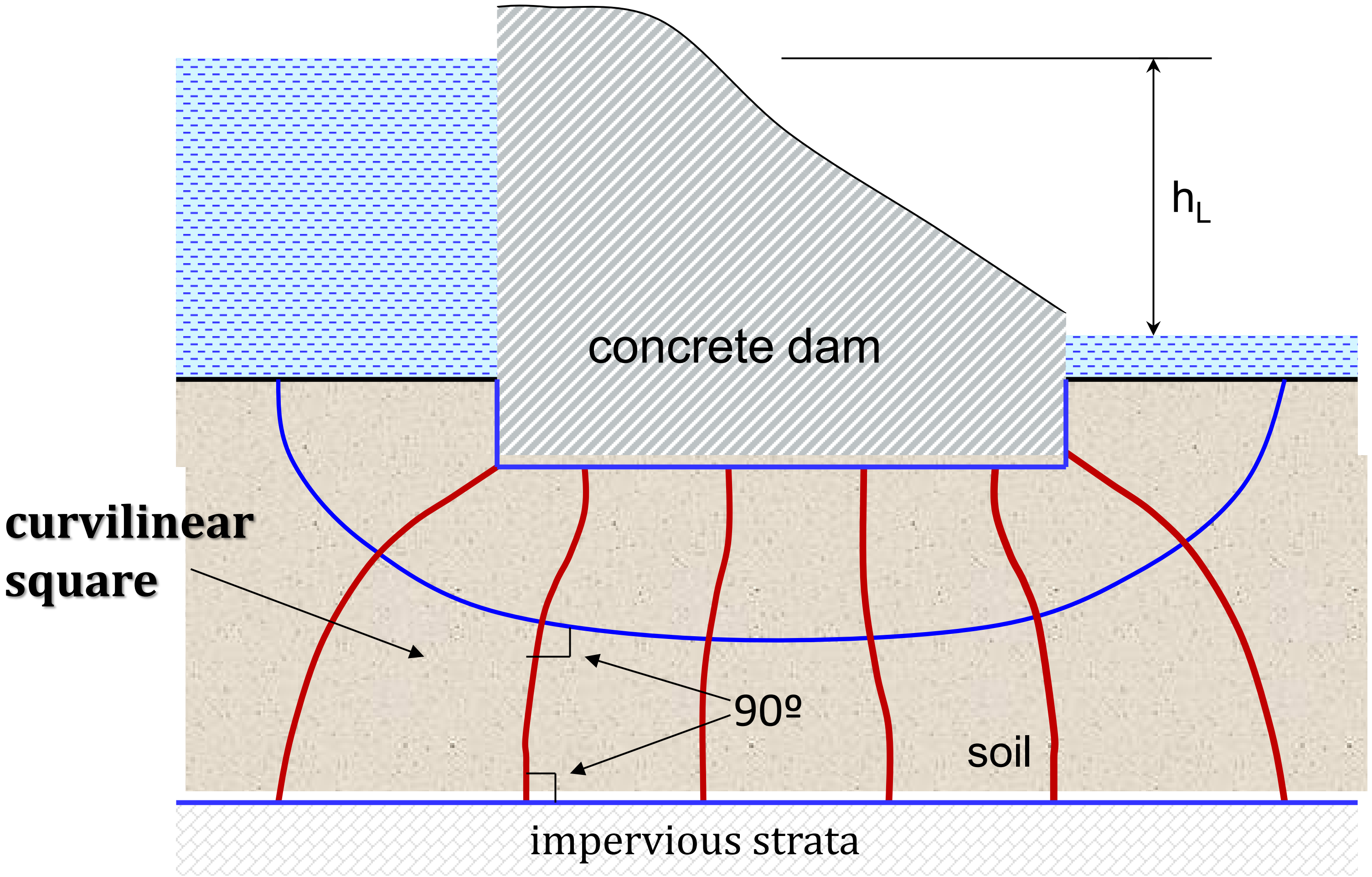
STREAM LINE

EQUIPOTENTIAL LINE is simply a contour of constant total head.



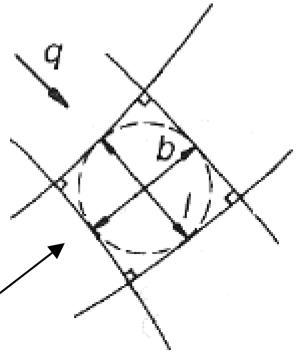
FLOWNET - a network of selected **stream lines** and **equipotential lines.**

A **flownet** is a grid obtained by drawing a series of streamlines and equipotential lines

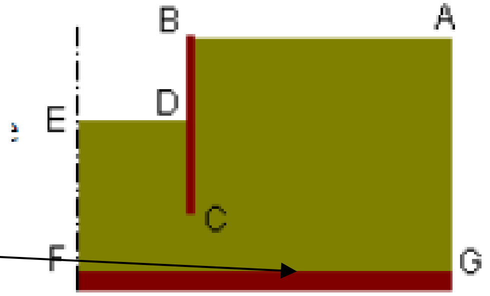


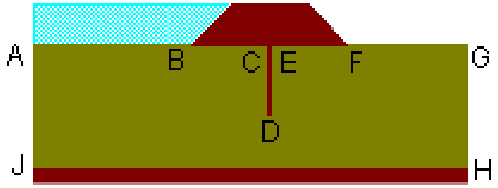
SKETCHING RULES

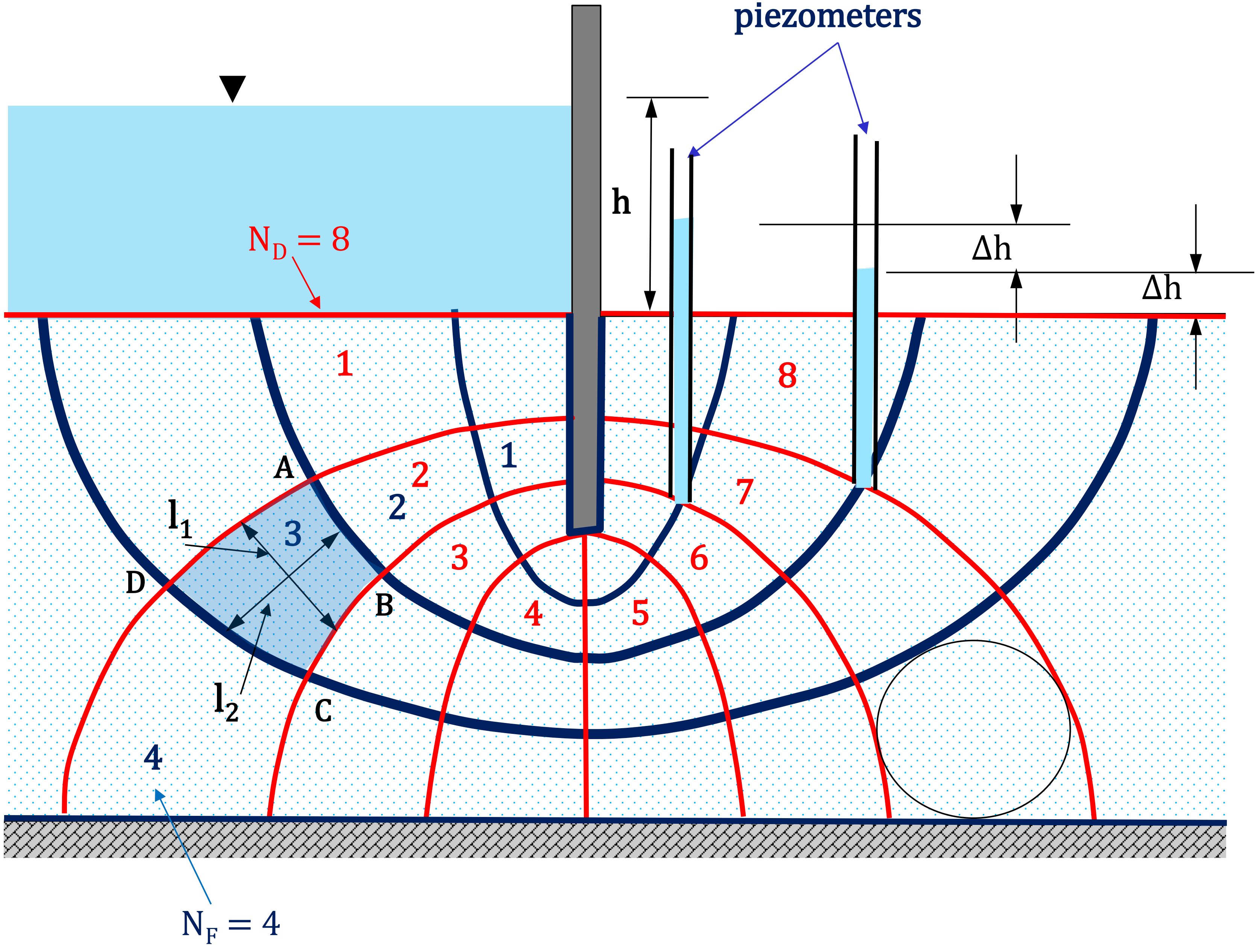
- **Flow lines** cross the **equipotentials at right angles** (by definition, there is no flow along an equipotentials and therehore, all of flow must be at 90° to it
- **An equipotentials cannot cross other equipotentials** (one point cannot have two different values of total head)
- Although an infinite number of flow lines could be sketched, **the flow net must be constructed so that each element is as curvilinear square** (sides may be curved…. **Curvilinear square** is as broad as it is long, so that a **circle** may be **inscribed** within it that touches all four of its sides



- Impermeable boudaries and lines of symetry are flow lines (**FG** is flow line)
- Bodies of water (such as reservoir) behind a dam, are equipotentials (**AB**)







Each interval between two equipotential corresponds to a head loss **Δh** equal to 1/N_D

$$\Delta h = \frac{h}{N_D}$$

N_D – total number of **equipotentials drops**

N_f – number of flow tubes

QUANTITY OF SEEPAGE (Q)

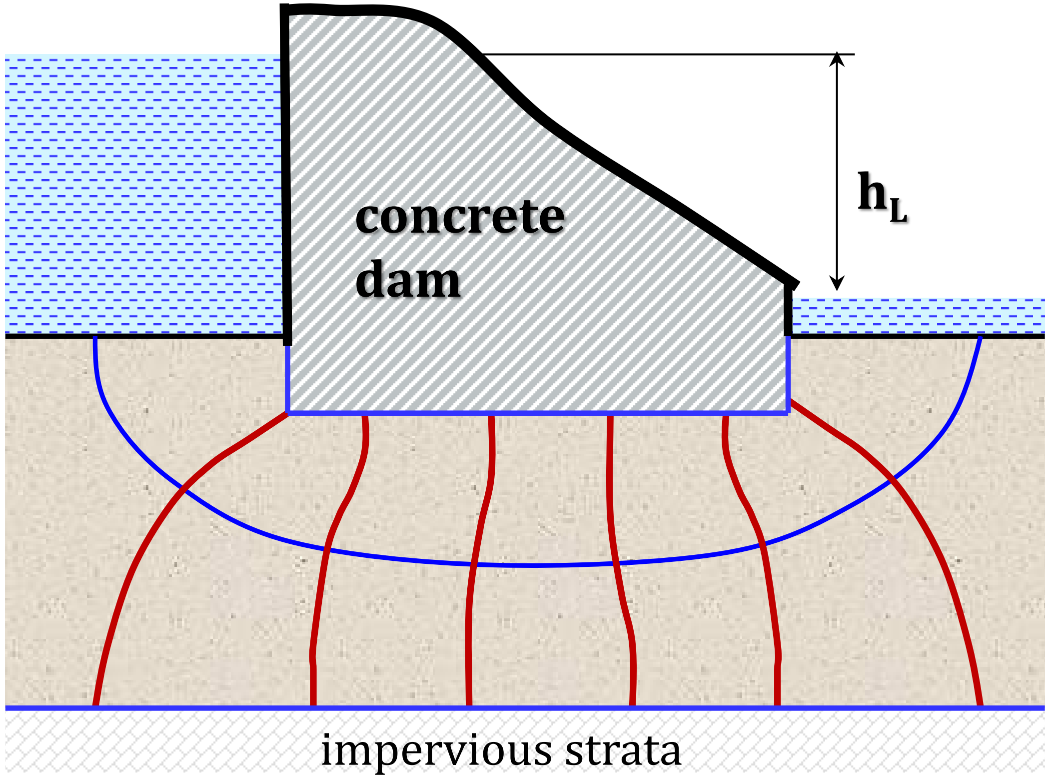
$$Q = k\boldsymbol{h_L}\frac{\boldsymbol{N_f}}{\boldsymbol{N_d}}$$

number of flow channels

….per unit length normal to the plane

number of equipotential drops

head loss from upstream to downstream

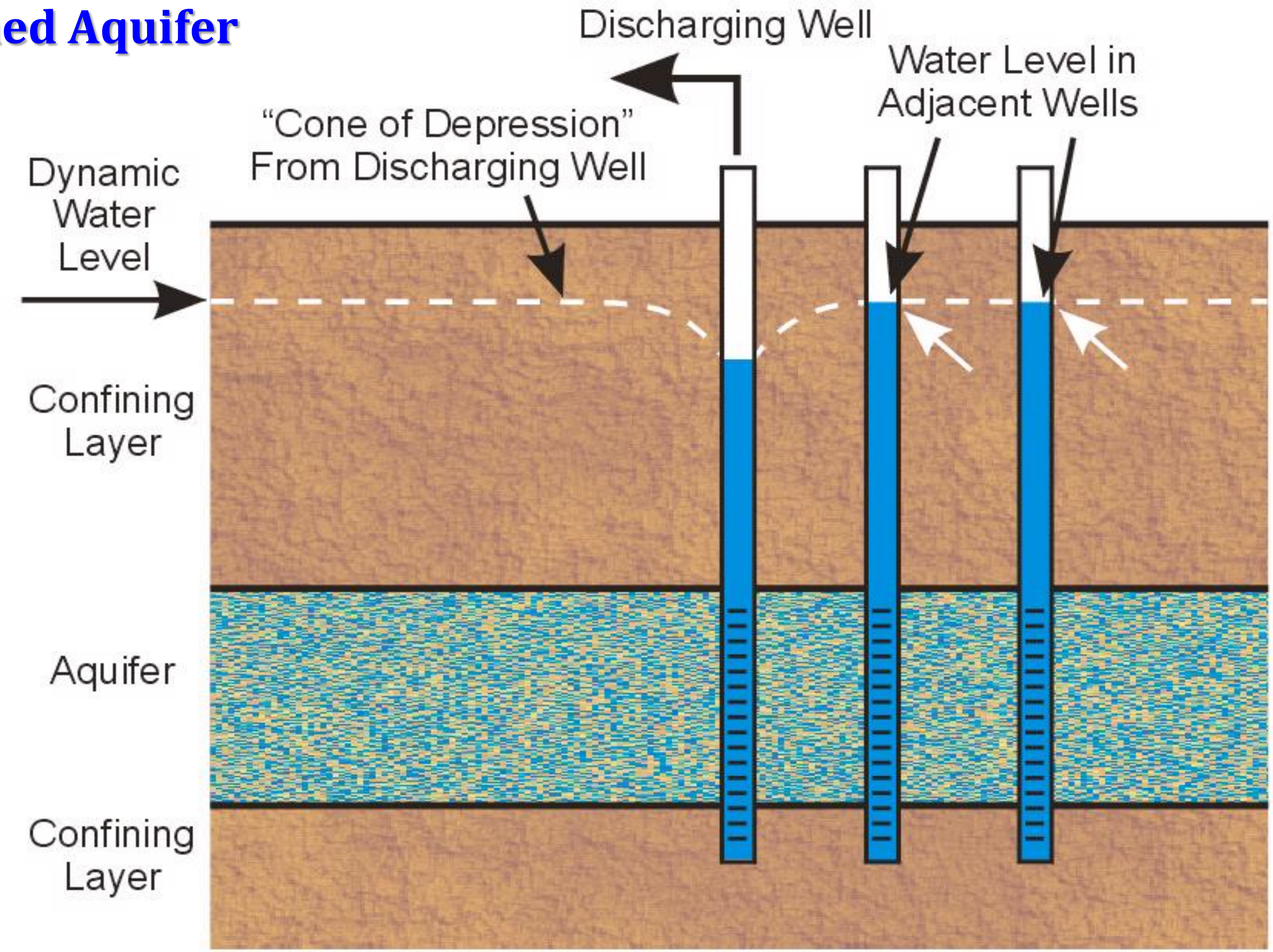


UNSTEADY FLOW TO A WELL IN A CONFINED AQUIFER

Theis method

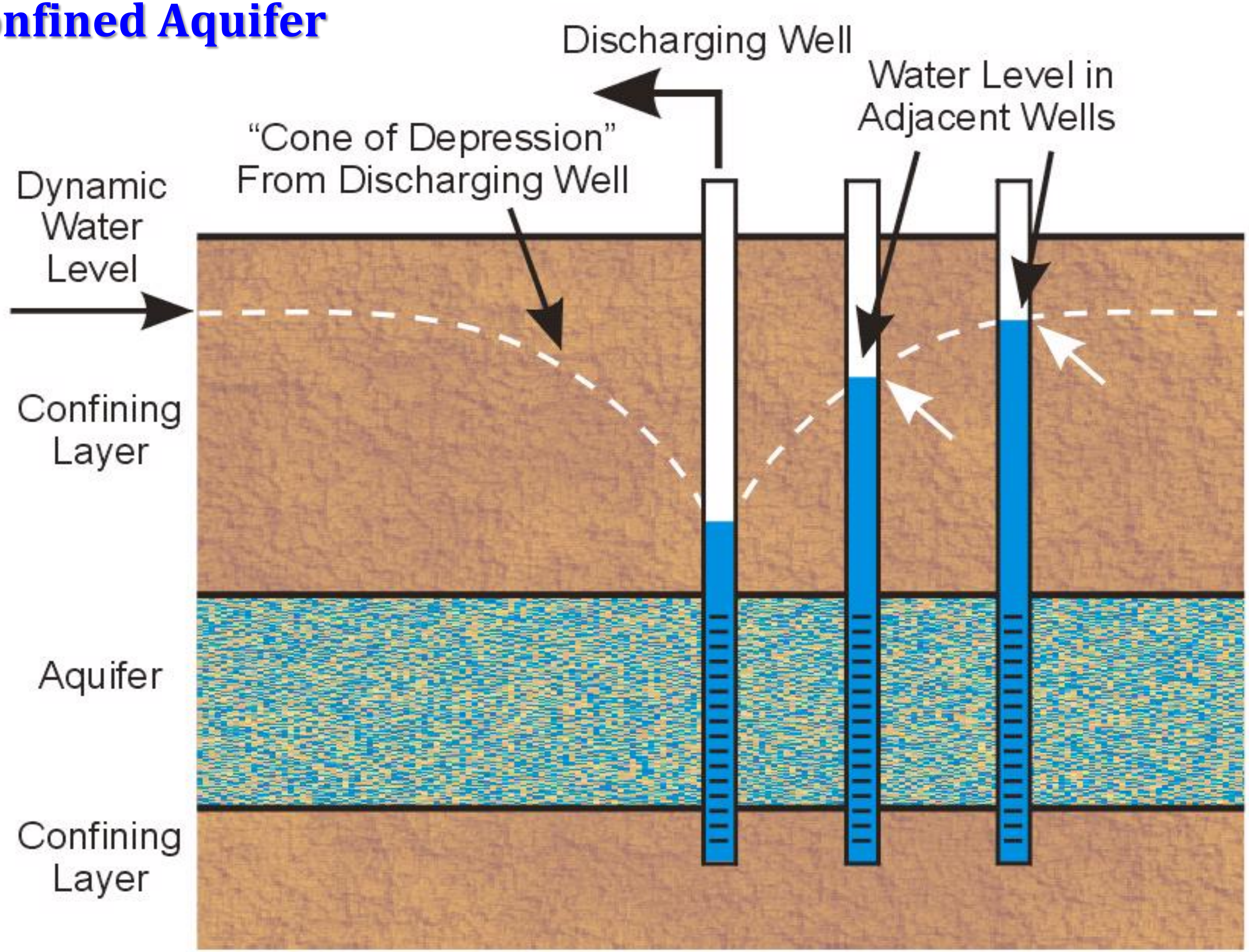
Cooper-Jacob method

Confined Aquifer



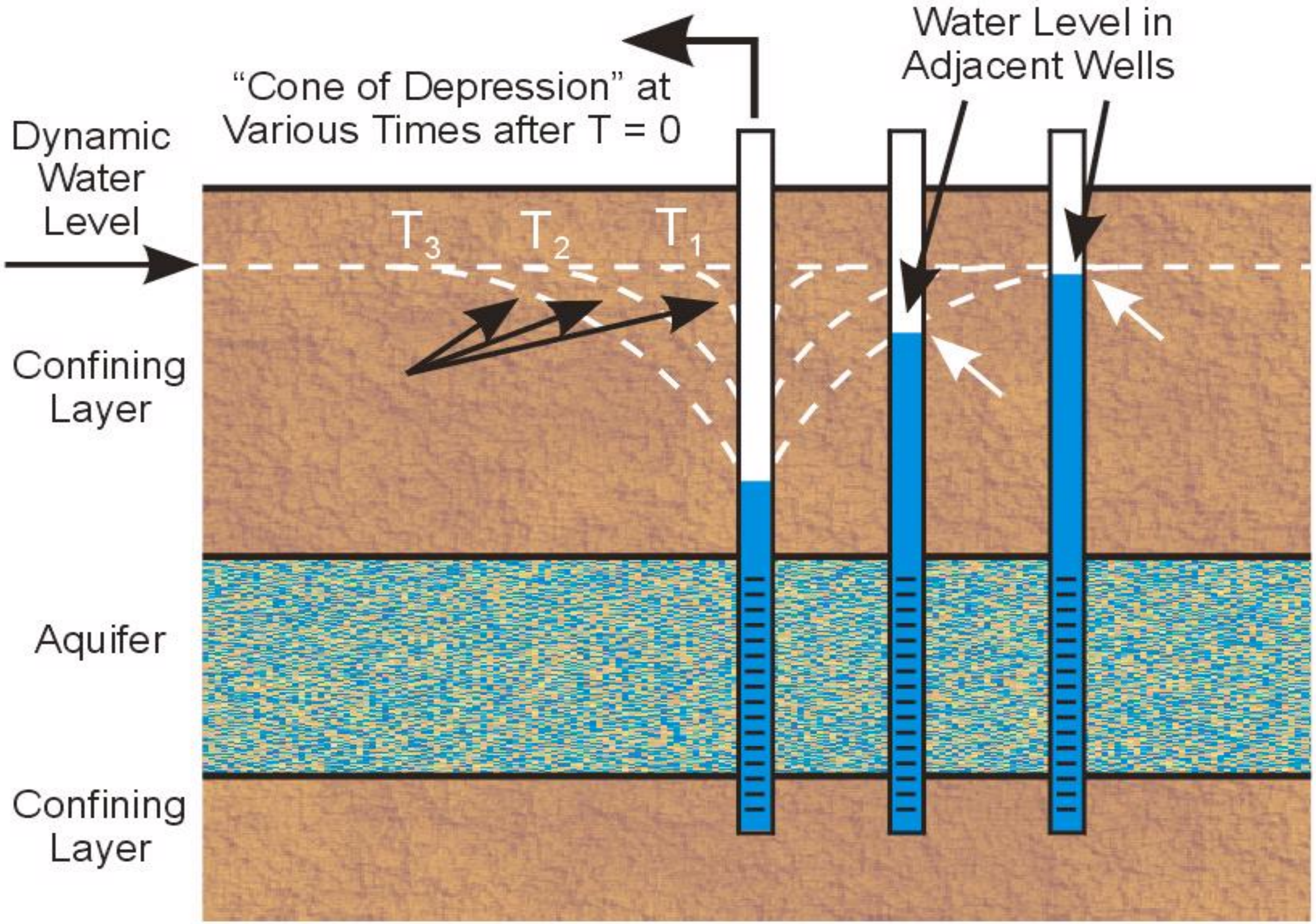
Water Levels in Confined Aquifer Adjacent to Discharging Well

Confined Aquifer



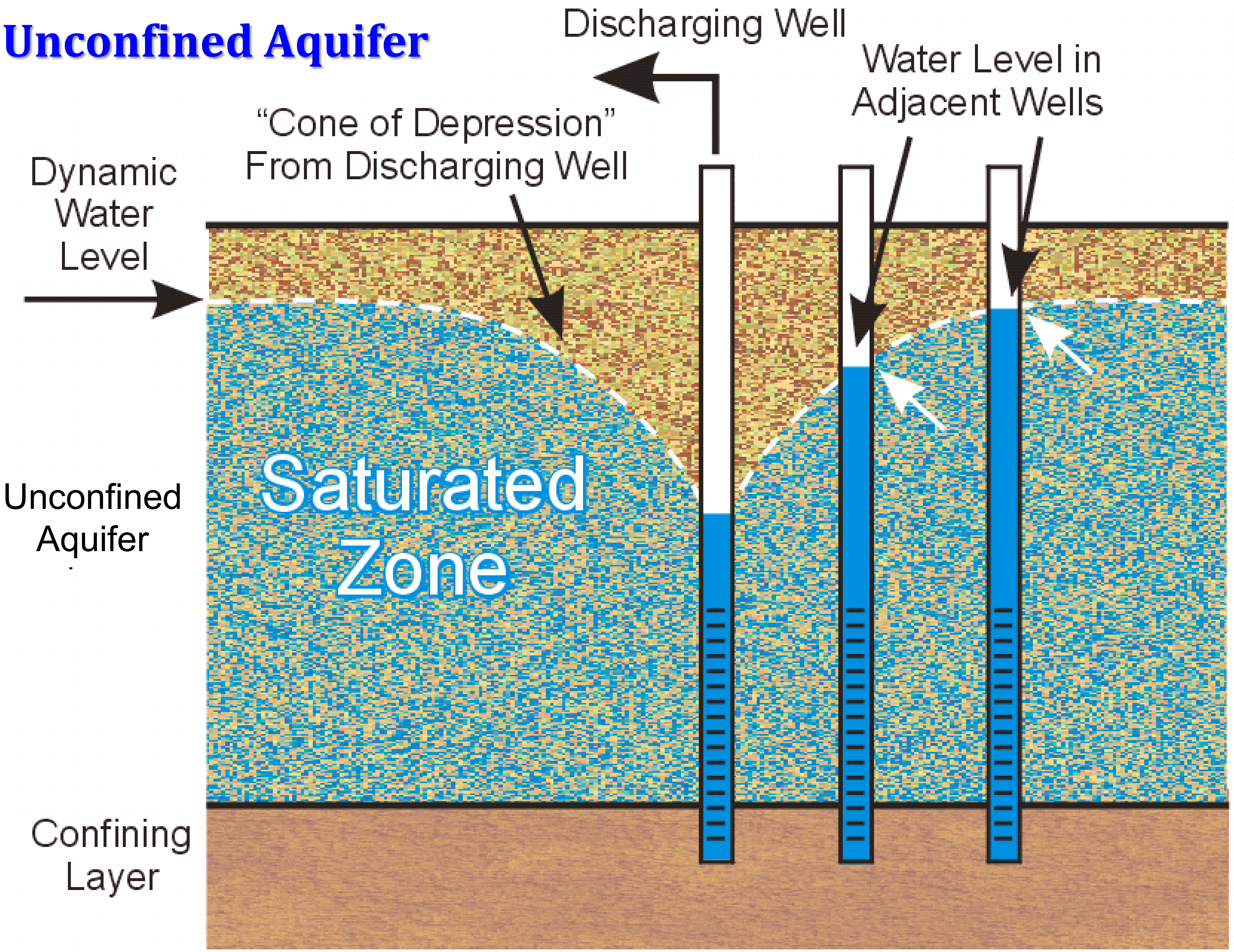
Water Levels in Confined Aquifer Adjacent to Discharging Well

Confined Aquifer



Water Levels in Confined Aquifer
Adjacent to Discharging Well

Unconfined Aquifer



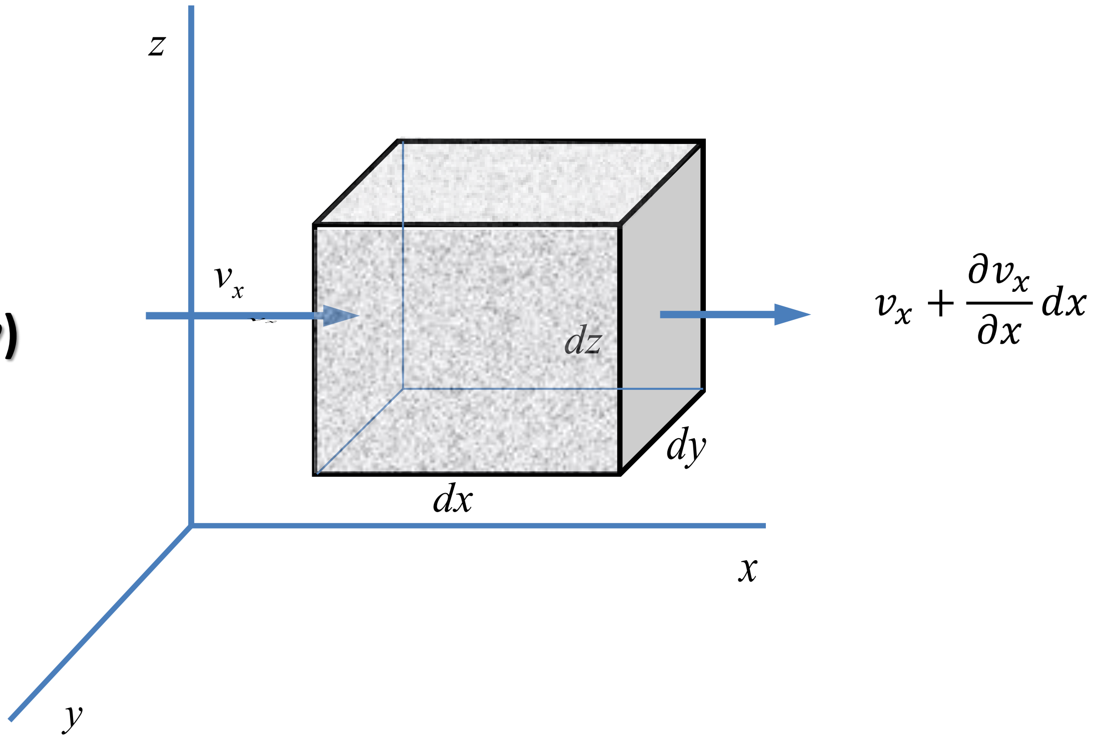
Effect of Discharging Water from a Well in an Unconfined Aquifer on the Adjacent Watertable

BASIC EQ. – UNSTEADY FLOW THROUGH POROUS MEDIA

Assumptions:

- **confined aquifer**
- **Darcy eq.**
- **balance of mass**
- **homogeneous, isotropic**

Input – output = 0 mass $(\boldsymbol{\rho v})$



UNSTEADY FLOW:

Inflow – outflow = change storage

BASIC EQ. – UNSTEADY FLOW CONFINED AQUIFER

For hydraulic head

$$\frac{\partial^2 h}{\partial x^2} + \frac{\partial^2 h}{\partial y^2} + \frac{\partial^2 h}{\partial z^2} = \frac{S}{T}\frac{\partial h}{\partial t}$$

For drawdown

$$\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} + \frac{\partial^2 s}{\partial z^2} = \frac{S}{T}\frac{\partial s}{\partial t}$$

RADIAL FLOW

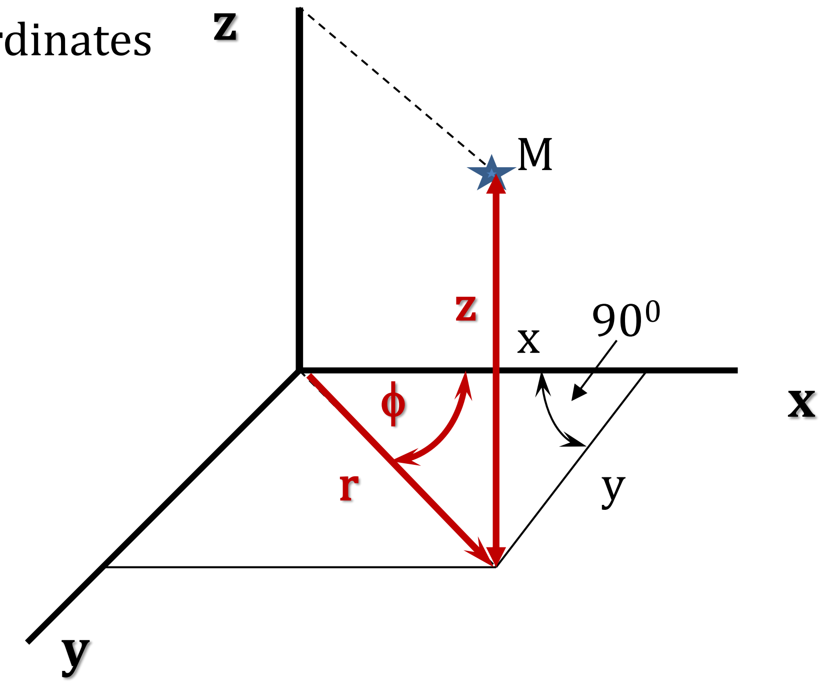
Polar coordinate system : r,ϕ,z

relationship between rectangular and cylindrical coordinates

$x = r.\cos\phi$ $\quad r = (x^2 + y^2)^{1/2}$

$y = r.\sin\phi$ $\quad tg\,\phi = (y/x)$

$z = z$ $\quad z = z$



2D Flow in a confined aquifer

$$\frac{\partial^2 s}{\partial x^2} + \frac{\partial^2 s}{\partial y^2} = \frac{S}{T}\frac{\partial s}{\partial t}$$

2D Flow in a confined aquifer in radial coordinates

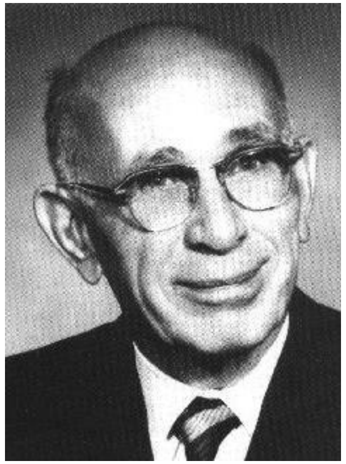
$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t}$$

RADIAL FLOW TO A WELL

The solution of the governing equation of unsteady radial flow was solved by C.V. Theis in 1935

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t}$$

s = drawdown [L]
H = initial head [L]
h = head at r at time t [L]
t = time since pumping began [T]
r = distance from pumping well [L]
Q = discharge rate [L^3/T]
T = transmissivity [L^2/T]
S = Storativity [-]



C.V. Theis

ASSUMPTIONS – THEIS SOLUTION:

- **confined aquifer**
- pumping rate **Q = const**.
- **Darcy´s law** is valid
- all **flow is radial** to well
- well is **fully penetrating** aquifer
- **flow is horizontal**
- piezometric heads -surface steady prior to pumping
- **homogeneous**, **isotropic**, infinite areal extent aquifer
- pumping well receives water from the entire thickness of the aquifer
- **transmissivity is constant** in space and time
- **storativity is constant** in space and time
- **additional resistances** at a well =**0** (ideal well)
- well has **infinitesimal diameter**
- water removed from storage is discharged instantaneously

Basic equation: $$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t}$$

Theis solution is given as:

$$s(r,t) = \frac{Q}{4\pi T}\left(-E_i(-u)\right) = \frac{Q}{4\pi T}\int_u^{\infty}\frac{e^{-u}}{u}du = \frac{Q}{4\pi T}W(u)$$

Where (-Ei(-u)) is written as W(u)

W(u) – Theis well function; **u – parameter of well function** (-)

$$u = \frac{r^2 S}{4Tt} \quad \Rightarrow \quad \frac{1}{u} = \frac{4Tt}{r^2 S}$$

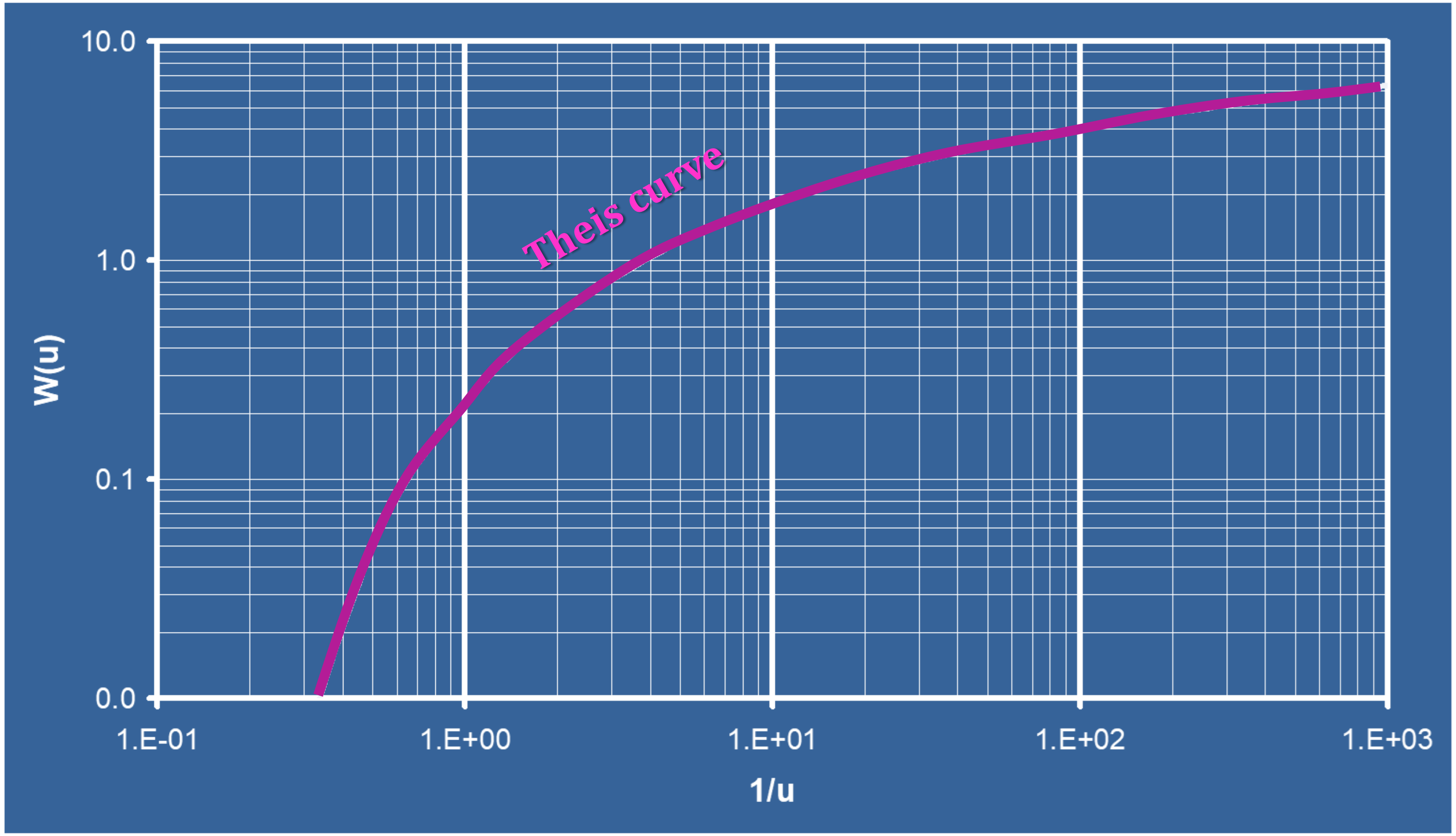
T – time [T]; r – radial distance [L];

$$W(u) = -\gamma - \ln u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \frac{u^4}{4.4!} + \ldots\ldots\ldots\ldots$$

γ – Euler´s number … 0,577216

$$s(r,t) = \frac{Q}{4\pi T}W(u)$$

Theis type curve method



W(u) versus 1/u on log-log paper

Theis type curve method

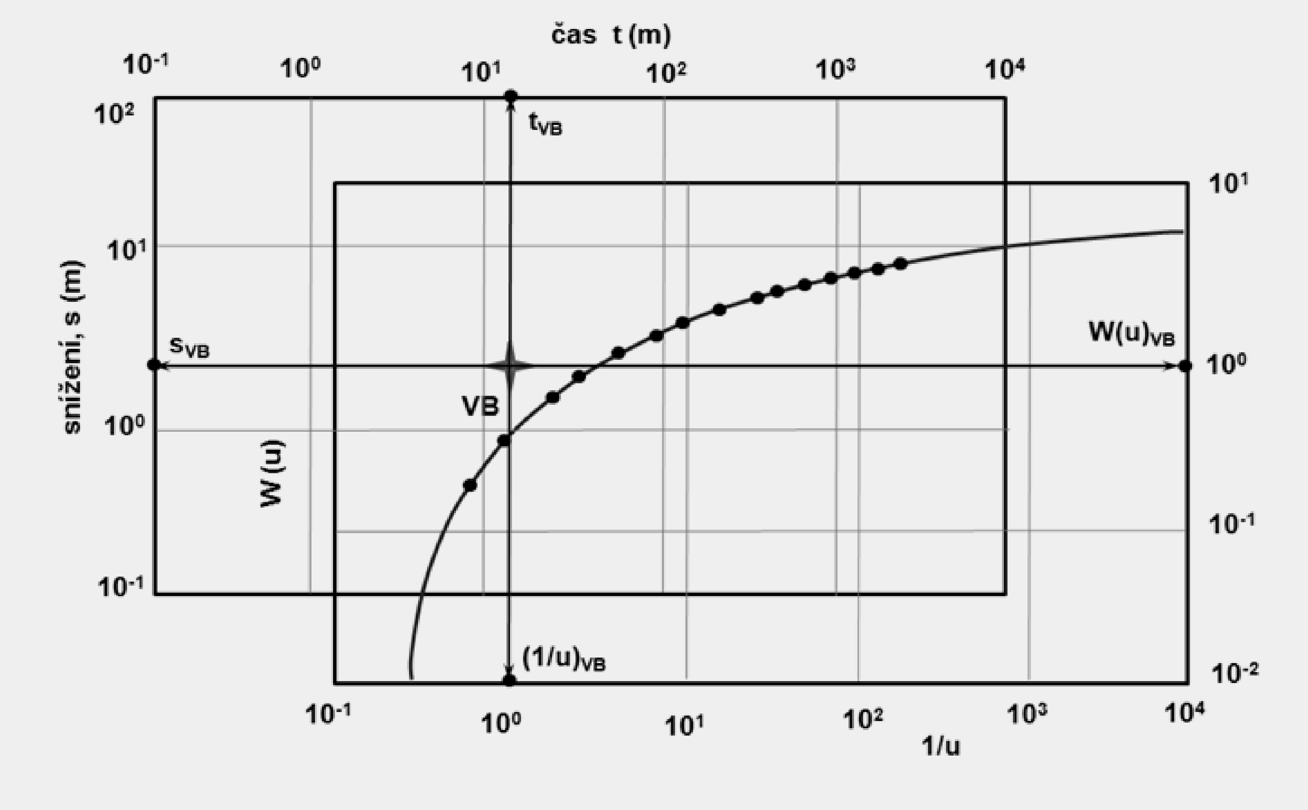
Consequently, we use curve matching techniques

type curve is **W(u)** vs **1/u**
plot **s** vs **t** for **field data**
type curve & field data must be plotted on same log-log paper
field curve is overlaid on **Type curve**
axes must be kept **parallel**
best match of curves is found
pick **any convenient point** - **VB**
read corresponding W(u), 1/u, S and t
use **Theis equation** for evaluation **T** & **S**

Theis type curve method

Plot drawdown versus time on log-log paper of same scale

Overlay the two plots and match the curves



Match point may be arbitrary point **VB**

Select match point and read **W(u)**, **1/u**, **s** and **t**

Use these values, plus Q and r from well to solve for T and S

$$T = \frac{Q}{4\pi s_{VB}} W(u)_{VB} \qquad S = \frac{4T u_{VB} t_{VB}}{r^2}$$

Cooper – Jacob – semilog method

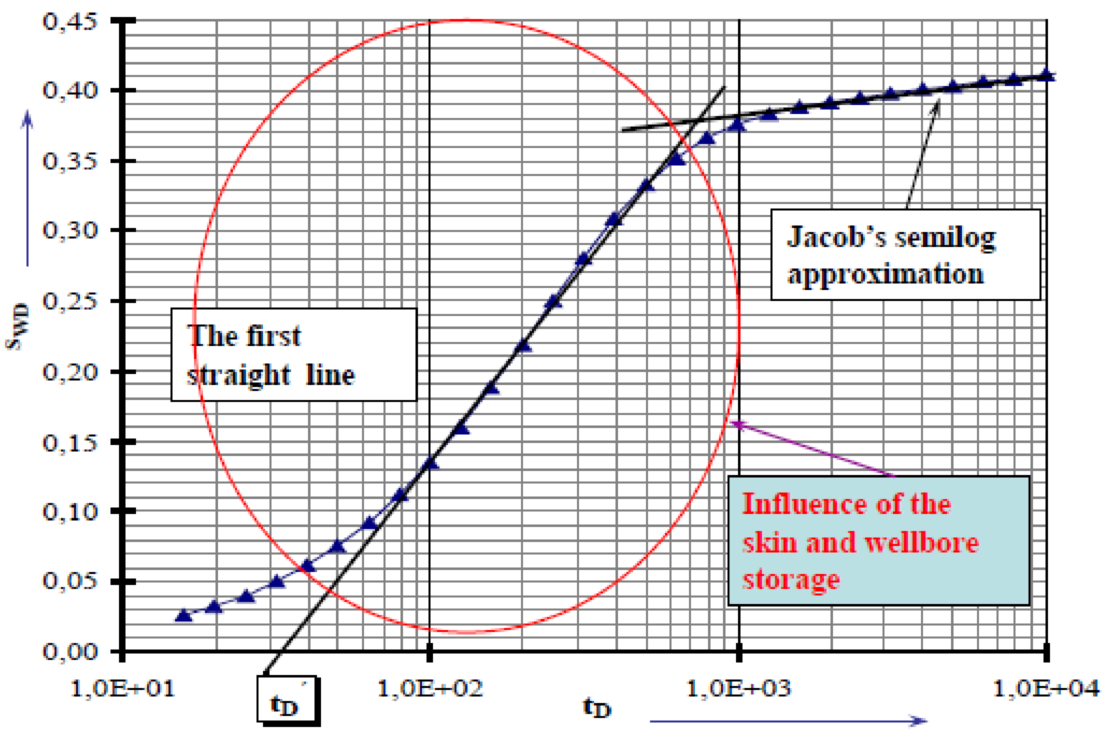
Theis

$$W(u) = -\gamma - \ln u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \frac{u^4}{4.4!} + \dots\dots\dots$$

$$s(r,t) = \frac{Q}{4\pi T} W(u)$$

$$s(r,t) = \frac{Q}{4\pi T} \ln \frac{2,246Tt}{r^2 S}$$

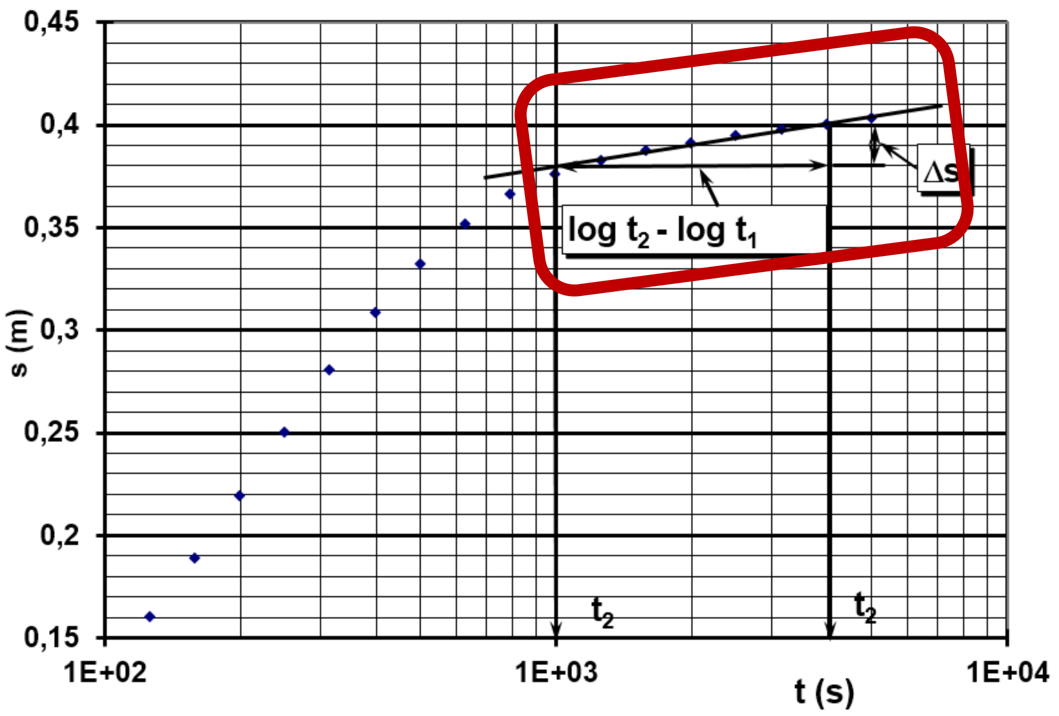
$$s(r,t) = \frac{2.3Q}{4\pi T}\log_{10}\left(\frac{2.25Tt}{r^2 S}\right) = \frac{0.183Q}{T}\log_{10}\left(\frac{2.25Tt}{r^2 S}\right)$$



Cooper – Jacob – semilog method (calculation of transmissivity)

$$s(r,t) = \frac{Q}{4\pi T}\ln\frac{2{,}246Tt}{r^2S}$$

$$\boldsymbol{s(r,t) = \frac{0,183Q}{T}\log\frac{2,246Tt}{r^2S}}$$



Transmissivity, T:

$$\boldsymbol{\Delta s} = s_2 - s_1 = \frac{0{,}183Q}{T}\log\frac{2{,}246Tt_2}{r^2S} - \frac{0{,}183Q}{T}\log\frac{2{,}246Tt_1}{r^2S}$$

$$\boldsymbol{i} = (s_2 - s_1)/(\log t_2 - \log t_1) \quad \Rightarrow \quad \boldsymbol{T = \frac{0,183Q}{i}}$$

For: $(\log t_2 - \log t_1) = 1 \quad \Rightarrow \quad \boldsymbol{T = \frac{0,183Q}{\Delta s}}$

Cooper – Jacob – semilog method (calculation of storativity)

$$s(r,t) = \frac{0,183Q}{T}\log\frac{2,246Tt}{r^2S}$$

for t_0 ……. $s_{OW} = 0$

Storativity – from observation well

$$0 = s_{OW}(r_{OW}, t_0) = \frac{0,183\,Q}{T}\,.\,log\frac{2,246\,T\,t_0}{r_{OW}^2 S}$$

≠0 =0

$$0 = s_{OW}(r_{OW}, t) = log\frac{2,246\,T\,t_0}{r_{OW}^2 S}$$

$$1 = \frac{2,246\,T\,t_0}{r_p^2 S}$$

and **storativity S**: →

$$S = \frac{2,246\,T\,t_0}{r_p^2}$$

Pumping test-observation well