Groundwater Hydraulics

8

UNSTEADY FLOW TO A WELL IN A CONFINED AQUIFER

Theis method

Cooper-Jacob method

Cooper – Jacob – semilog method

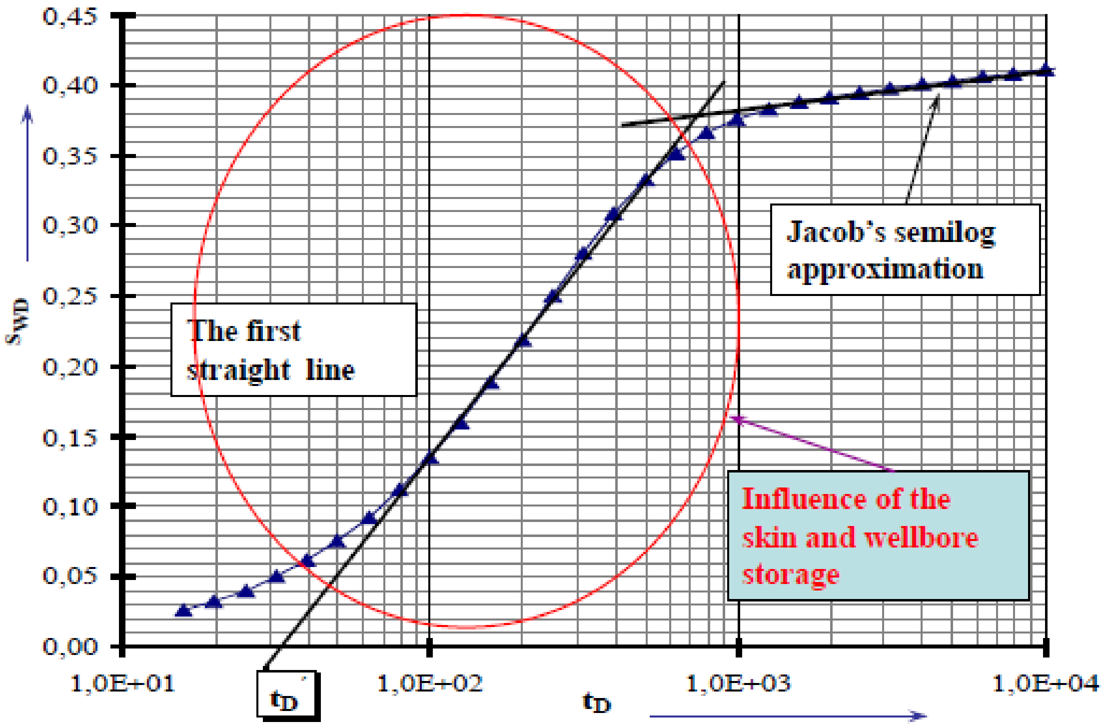
Theis

$$W(u) = -\gamma - \ln u + u - \frac{u^2}{2.2!} + \frac{u^3}{3.3!} - \frac{u^4}{4.4!} + \ldots\ldots\ldots\ldots$$

$$s(r,t) = \frac{Q}{4\pi T} W(u)$$

$$s(r,t) = \frac{Q}{4\pi T} \ln \frac{2,246Tt}{r^2 S}$$

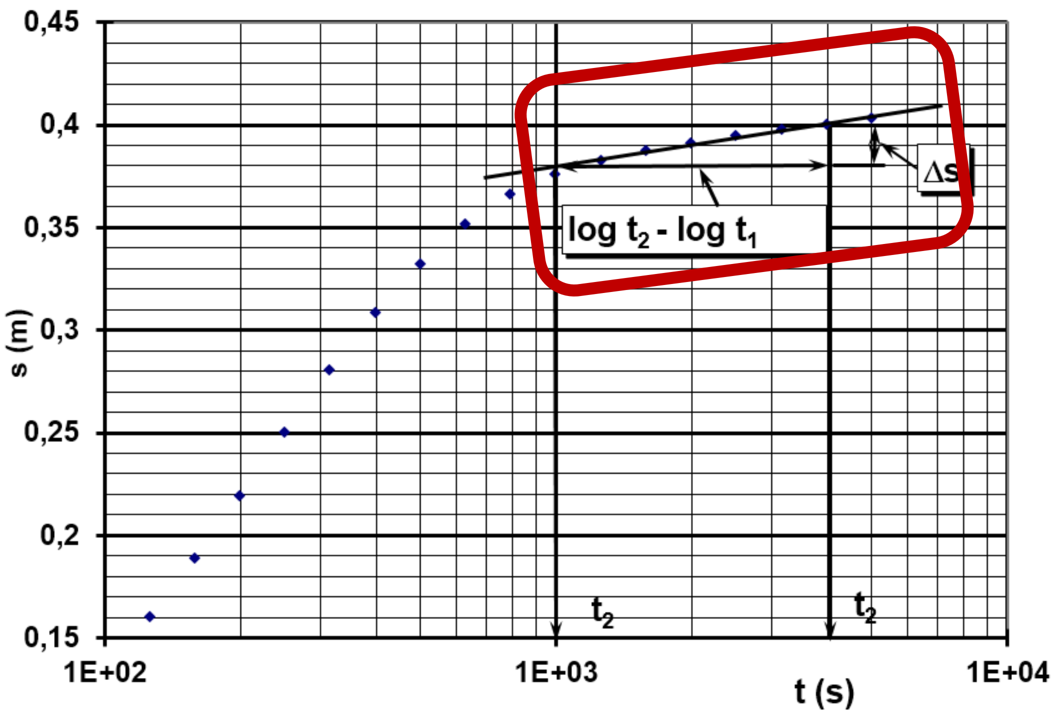
$$s(r,t) = \frac{2.3Q}{4\pi T} \log_{10}\left(\frac{2.25Tt}{r^2 S}\right) = \frac{0.183Q}{T} \log_{10}\left(\frac{2.25Tt}{r^2 S}\right)$$



Cooper – Jacob – semilog method

$$s(r,t) = \frac{Q}{4\pi T}\ln\frac{2{,}246Tt}{r^2S}$$

$$\boldsymbol{s(r,t) = \frac{0,183Q}{T}\log\frac{2,246Tt}{r^2S}}$$



Transmissivity, T:

$$\boldsymbol{\Delta s} = s_2 - s_1 = \frac{0{,}183Q}{T}\log\frac{2{,}246Tt_2}{r^2S} - \frac{0{,}183Q}{T}\log\frac{2{,}246Tt_1}{r^2S}$$

$$\boldsymbol{i} = (s_2 - s_1)/(\log t_2 - \log t_1) \quad \Rightarrow \quad \boldsymbol{T = \frac{0,183Q}{i}}$$

For: $(\log t_2 - \log t_1) = 1 \quad \Rightarrow \quad \boldsymbol{T = \frac{0,183Q}{\Delta s}}$

Cooper – Jacob semilog method

$$s(r,t) = \frac{0,183Q}{T} \log \frac{2,246Tt}{r^2 S}$$

for t_0 ……. $s_{OW} = 0$

Storativity – from observation well

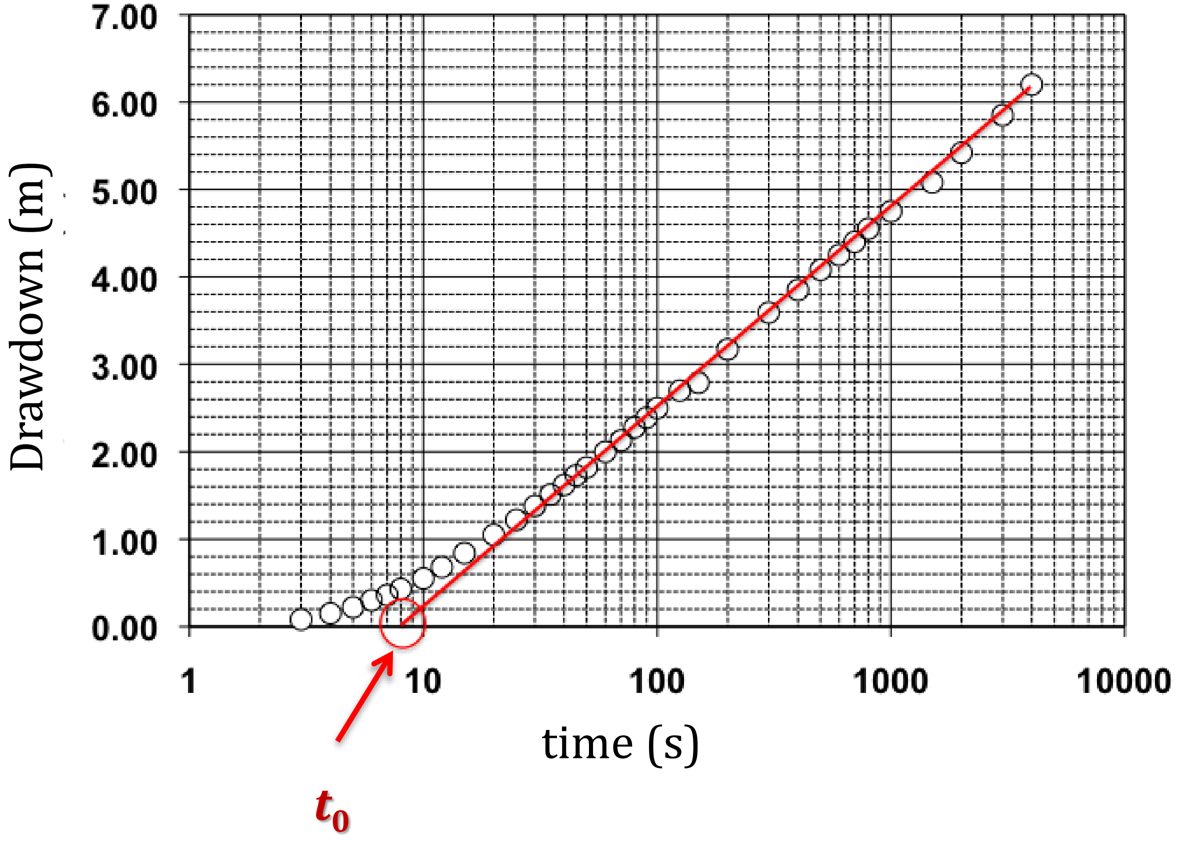
$$0 = s_{OW}(r_{OW}, t_0) = \frac{0,183\, Q}{T} \, . \, log \frac{2,246\, T\, t_0}{r_{OW}^2 S}$$

≠0 =0

$$0 = s_{OW}(r_{OW}, t) = \; log \frac{2,246\, T\, t_0}{r_{OW}^2 \boldsymbol{S}}$$

$$1 = \frac{2,246\, T\, t_0}{r_p^2 S}$$

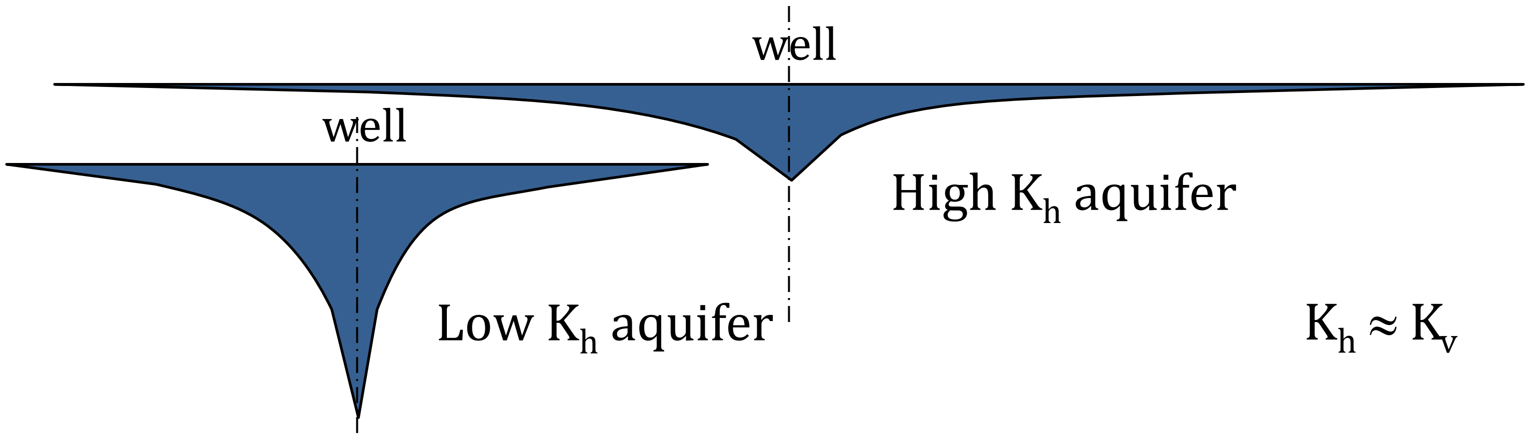
Pumping test-observation well



and **storativity S**: →

$$\boldsymbol{S} = \frac{2,246\, T\, t_0}{r_p^2}$$

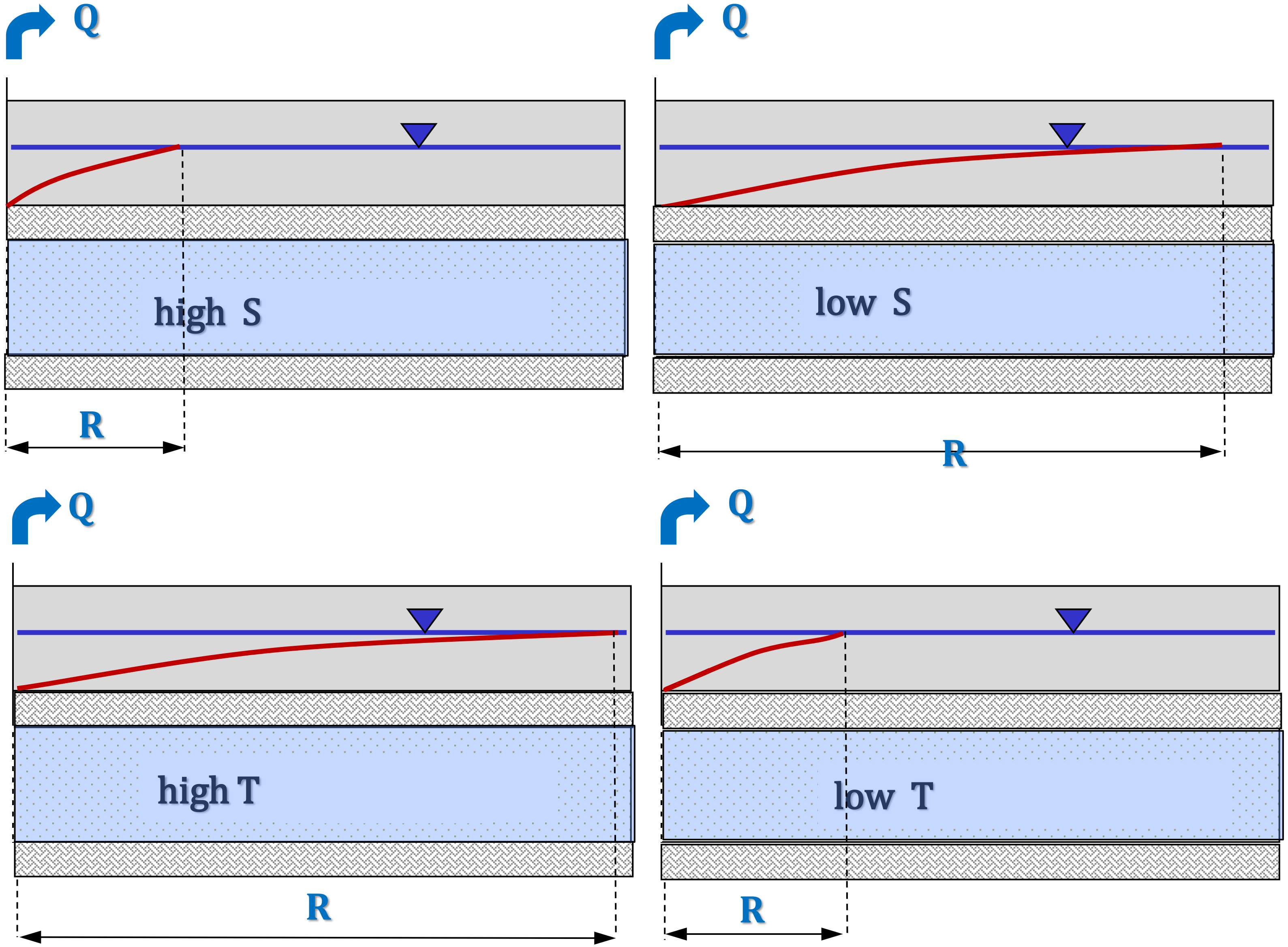
CONE OF DEPRESSION



- A zone of low pressure is created centred on the pumping well
- Drawdown is a maximum at the well and reduces radially
- Head gradient decreases away from the well and the pattern resembles an inverted cone called the cone of depression
- The cone expands over time until the inflows (from various boundaries) match the well extraction
- The shape of the equilibrium cone is controlled by hydraulic conductivity

steeper gradients occur in low K material

DIFFERENT T, S YIELDS DIFFERENT GRADIENT AT WELL BORE



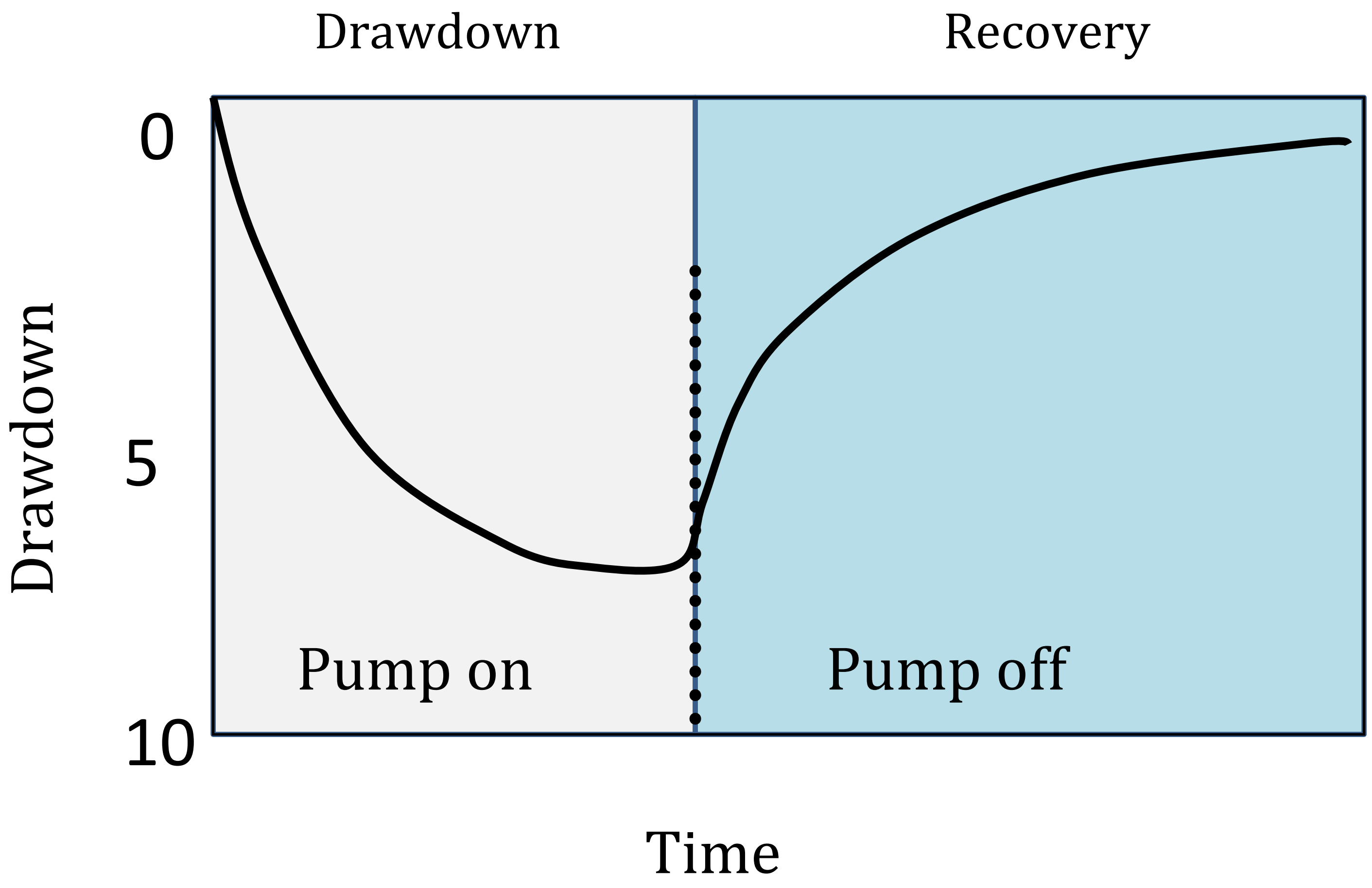
RECOVERY (BUILD-UP) TEST

RECOVERY DATA

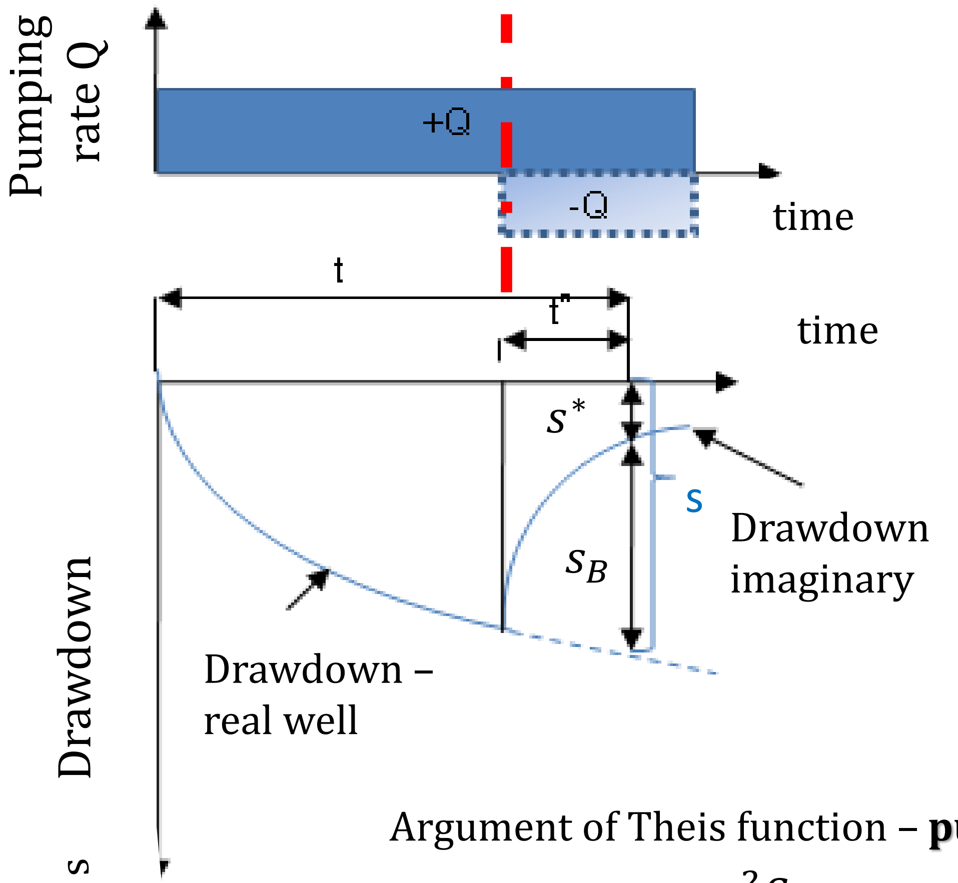
- When **pumping is halted**, water levels rise towards their pre-pumping levels.
- The rate of recovery provides a second method for **calculating aquifer characteristics.**
- **Monitoring recovery heads** is an important part of the well-testing process.
- **Observation well data** (from multiple wells) is preferable to that gathered from pumped wells.
- ***Pumped well recovery records are less useful*** but can be used in a more limited way to provide information on aquifer properties.

RECOVERY DATA

(after pumping ceases)



RECOVERY (BUILD-UP CURVE)



Drawdown for build-up $\Rightarrow$ $s^* = s + s_B$

$$s^* = \frac{+Q}{4\,\pi T} W(u) + \frac{-Q}{4\,\pi T} W(u_B)$$

Drawdown for pumping

$$s = \frac{+Q}{4\,\pi T} W(u)$$

Drawdown for imaginary – build-up

$$s_B = \frac{-Q}{4\,\pi T} W(u_B)$$

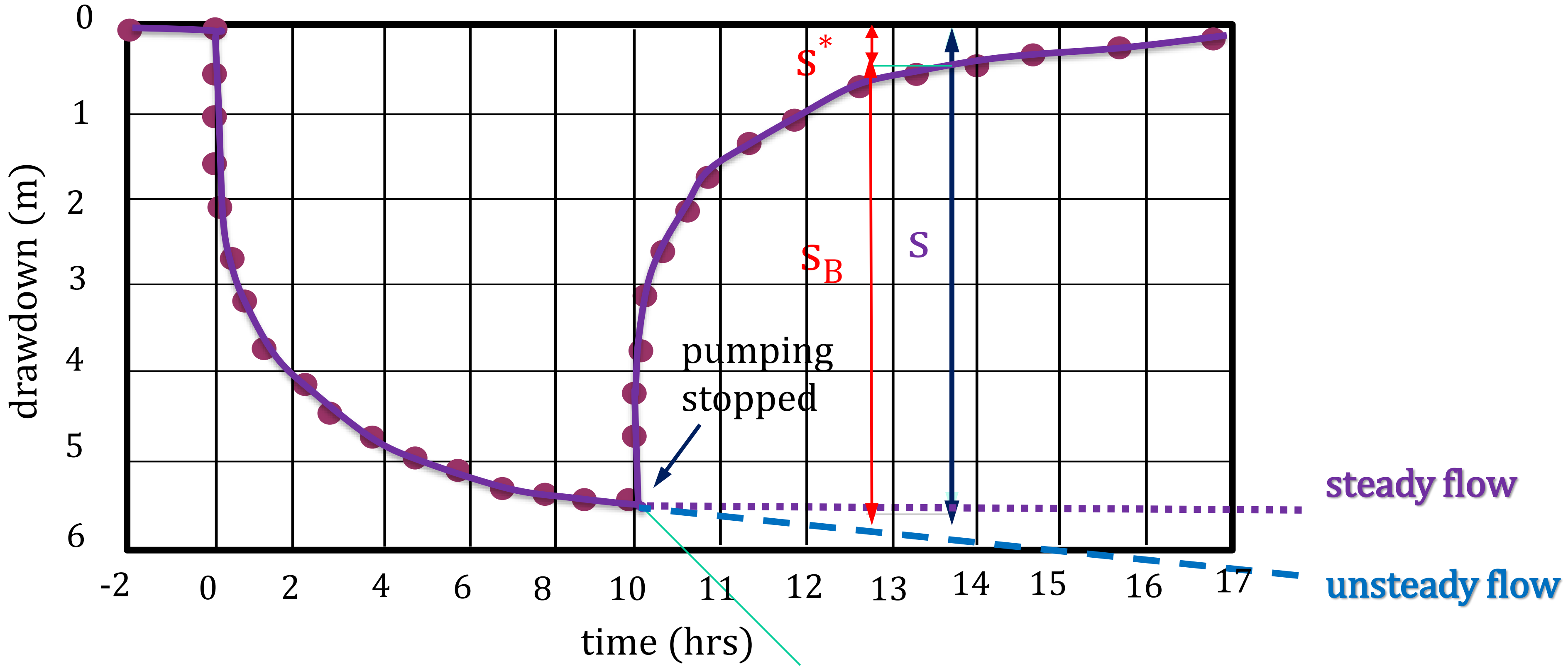
Argument of Theis function – **pumping**

$$u = \frac{r^2 S}{4\,T\,t}$$

Argument of Theis function – **build-up**

$$u_B = \frac{r^2 S}{4\,T\,t^*}$$

RECOVERY(BUILD-UP) CURVE



RESIDUAL DRAWDOWN AND RECOVERY

SUPERPOSITION

- The **total drawdown** for $t > t_r$ is:

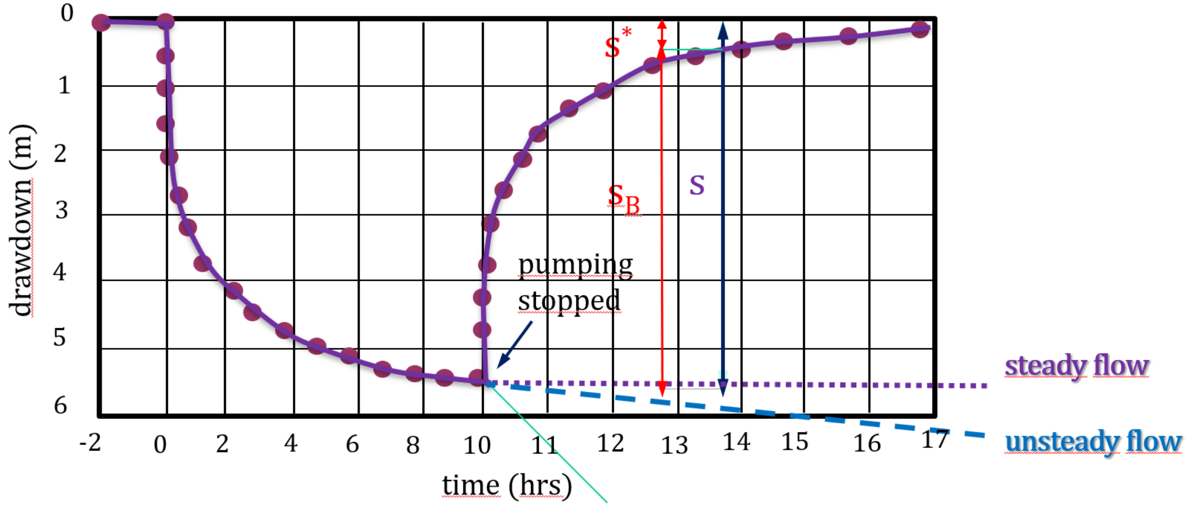
$$s^* = s - s_B = \frac{Q}{4\pi T}(W(u) - W(u^*))$$

- The **Cooper-Jacob approximation** can be applied giving:

$$s^* = s - s_B = \frac{Q}{4\pi T}(ln\frac{2.25Tt}{r^2S} - ln\frac{2.25Tt^*}{r^2S}$$

- Simplification gives **the residual drawdown** equation:

$$s^* = s - s_B = \frac{Q}{4\pi T}(ln\frac{t}{t^*})$$



- The equation predicting **the recovery** is:

$$s_B = \frac{-Q}{4\pi T}(ln\frac{2.25Tt^*}{r^2S})$$

For $t > t_r$, the recovery s_r is the difference between the observed drawdown s* and the extrapolated pumping drawdown (s).

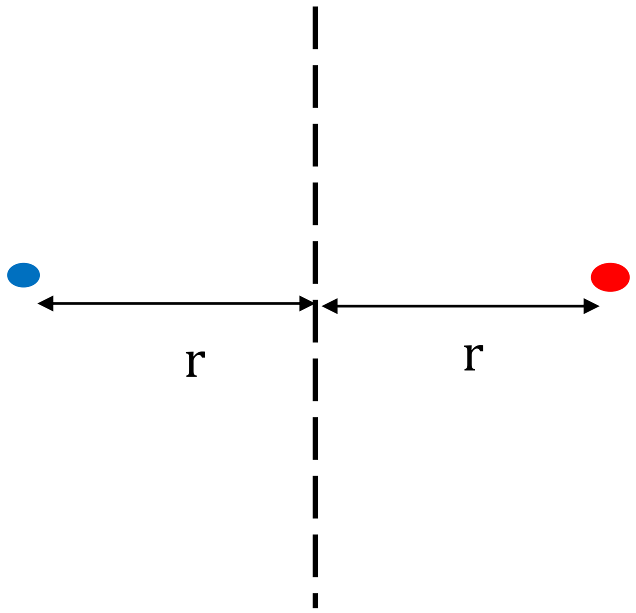
BOUNDED AQUIFERS

BOUNDED AQUIFERS

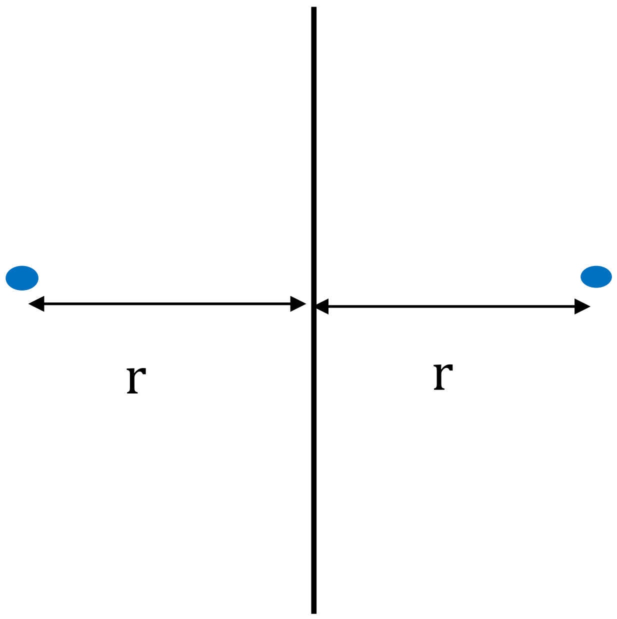
- **Superposition** was used to calculate well recovery by **adding the effects of** a **pumping** and **recharge** well starting at different times.
- Superposition can also be used **to simulate the effects of aquifer boundaries** by adding wells at different positions.
- For boundaries, the wells that create the same effect as a boundary are called **image wells.**
- This relatively simple application of superposition for analysis of aquifer boundaries was for described by **Ferris (1959)**

IMAGE WELLS

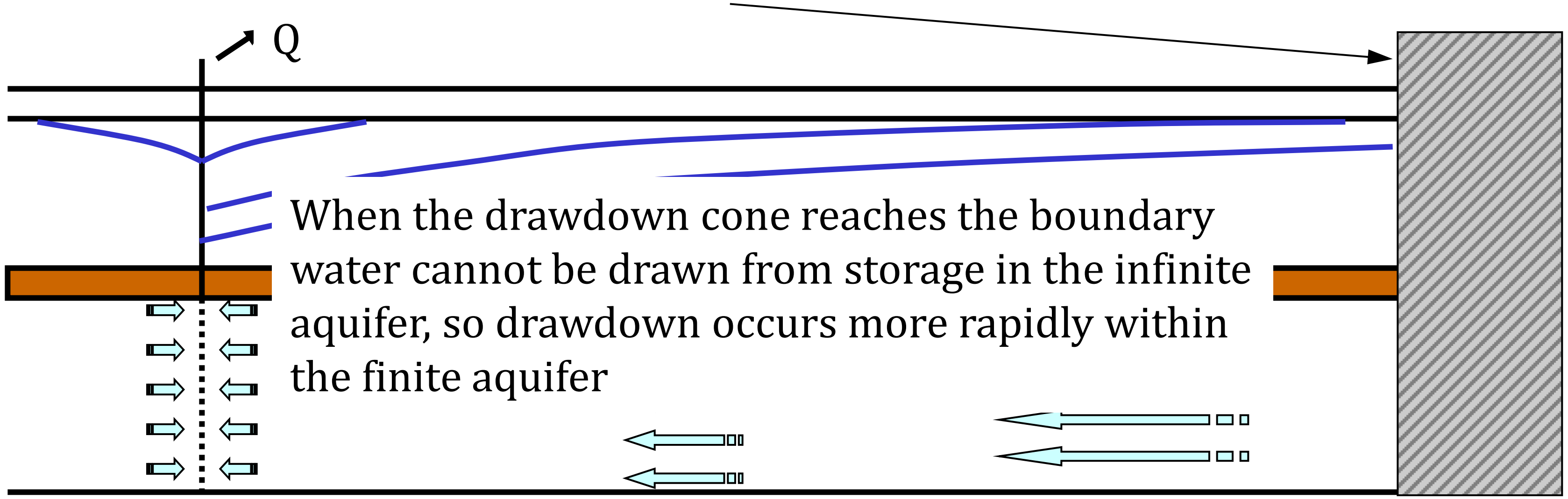
- **RECHARGE BOUNDARIES** at distance (r) are simulated by a recharge image well at an equal distance (r) across the boundary.



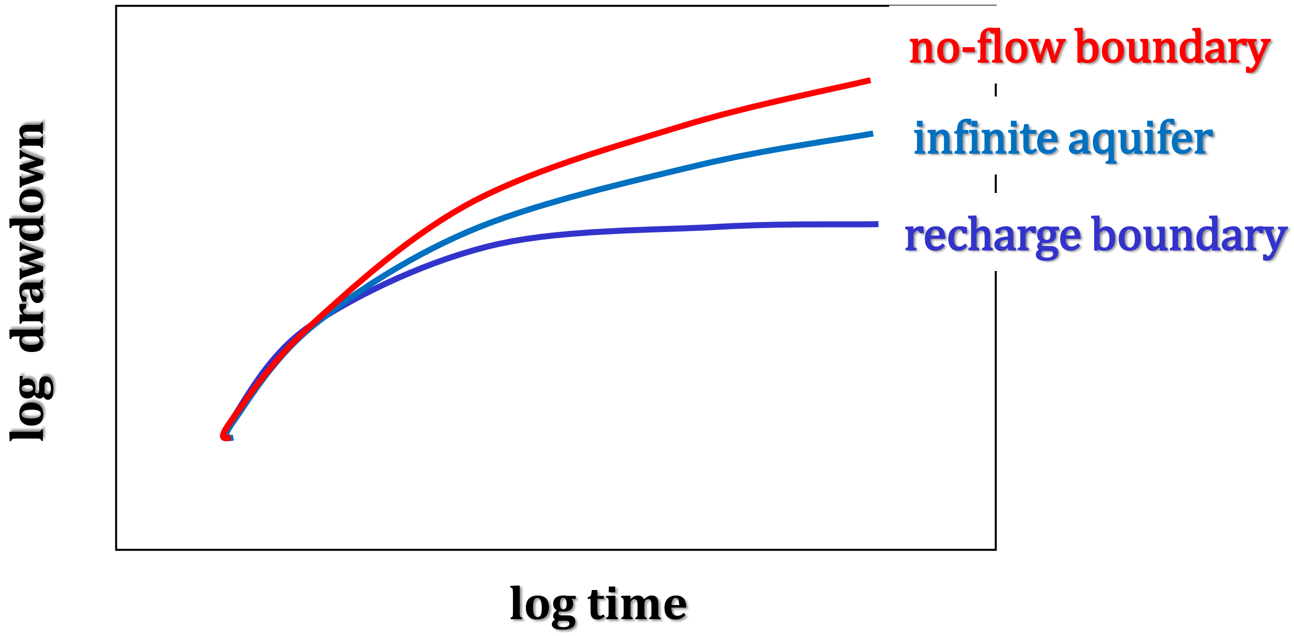
- **BARRIER BOUNDARIES** at distance (r) are simulated by a pumping image well at an equal distance (r) across the boundary.



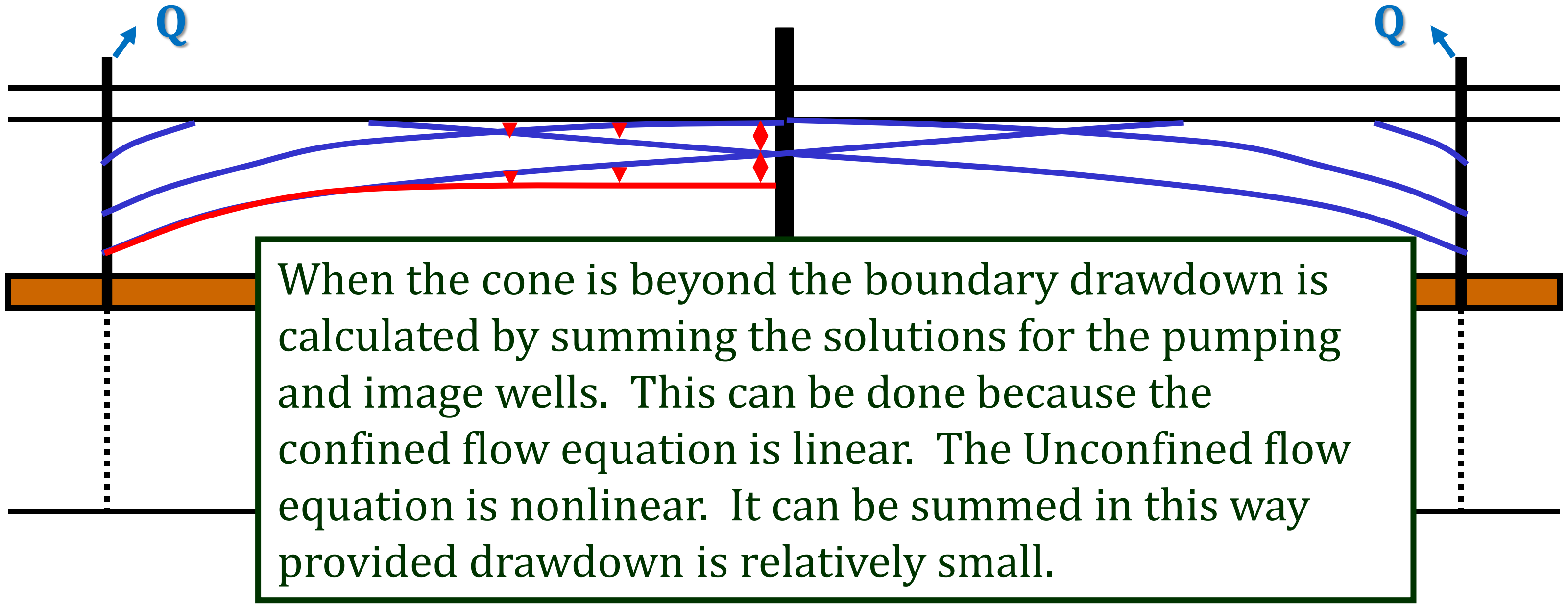
IMPERMEABLE or NO-FLOW BOUNDARY



When the drawdown cone reaches the boundary water cannot be drawn from storage in the infinite aquifer, so drawdown occurs more rapidly within the finite aquifer



IMPERMEABLE OR NO-FLOW BOUNDARY



When the cone is beyond the boundary drawdown is calculated by summing the solutions for the pumping and image wells. This can be done because the confined flow equation is linear. The Unconfined flow equation is nonlinear. It can be summed in this way provided drawdown is relatively small.

Method of Images - can be used to predict drawdown by creating a mathematical no-flow boundary

NO-FLOW = NO GRADIENT

So if we place an **imaginary well**

of equal strength

at **equal distance across the boundary**

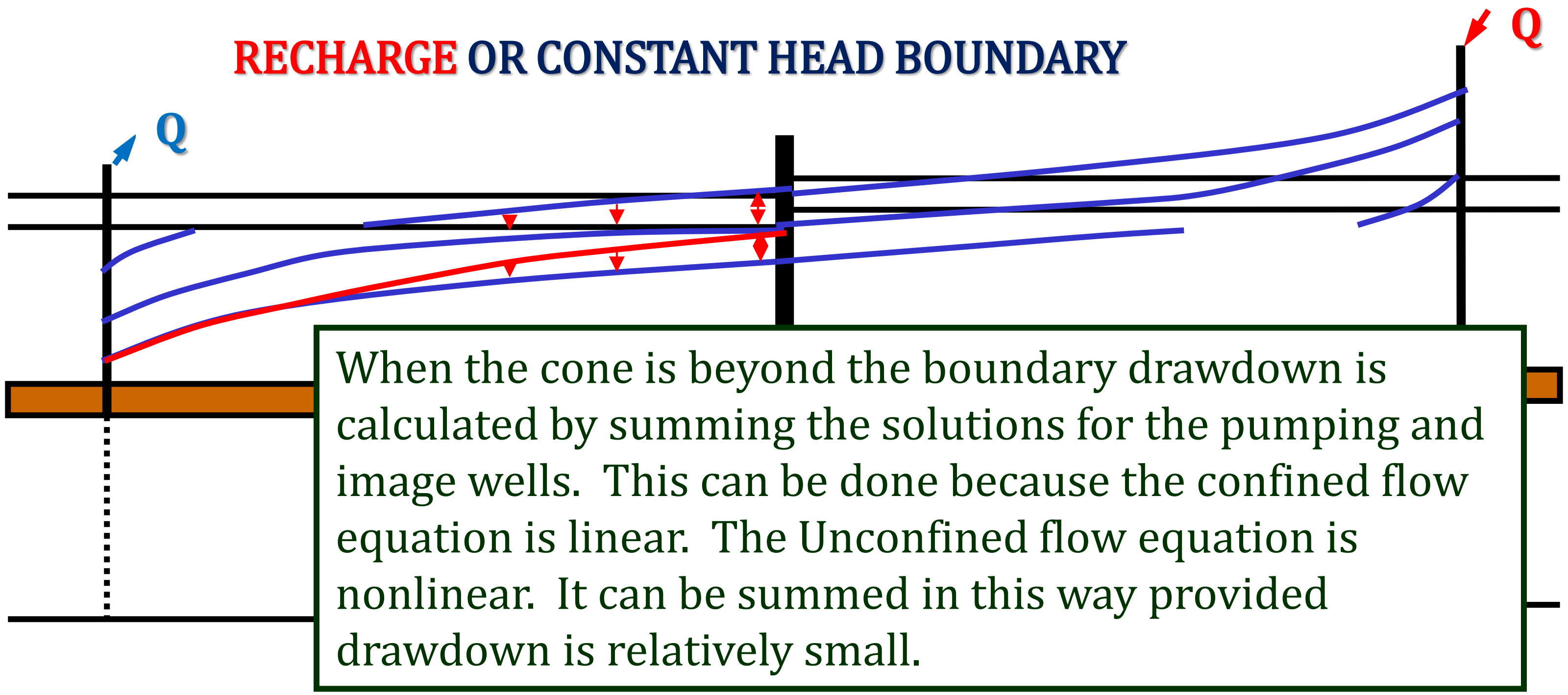
And **superpose the solutions**

We will have

equal drawdown, therefore equal head at the boundary, hence NO GRADIENT

Let's look at it

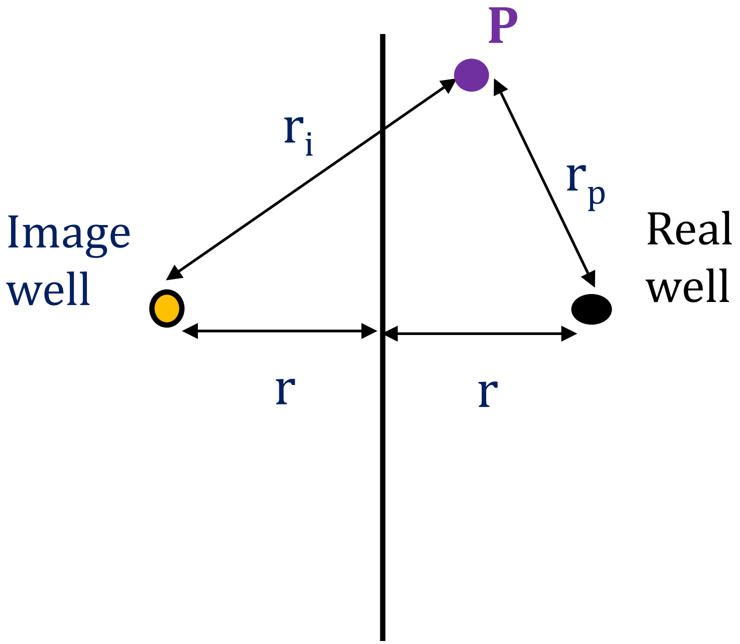
RECHARGE OR CONSTANT HEAD BOUNDARY



When the cone is beyond the boundary drawdown is calculated by summing the solutions for the pumping and image wells. This can be done because the confined flow equation is linear. The Unconfined flow equation is nonlinear. It can be summed in this way provided drawdown is relatively small.

Method of Images - can be used to predict drawdown
by creating a mathematical constant head boundary
CONSTANT HEAD = NO CHANGE IN HEAD
So if we place an **imaginary well**
of **equal strength** but opposite sign
at **equal distance across the boundary**
And **superpose the solutions**
We will have
equal but opposite drawdown, therefore NO HEAD CHANGE
Let's look at it

GENERAL SOLUTION



The general solution for adding image wells to a real pumping well can be written:

$$s_P = s_R \pm s_i = \frac{Q}{4\pi T}\left(W(u_R) \pm W(u_i)\right)$$

where

$$u_R = \frac{r_P^2 S}{4Tt} \qquad u_i = \frac{r_i^2 S}{4Tt}$$

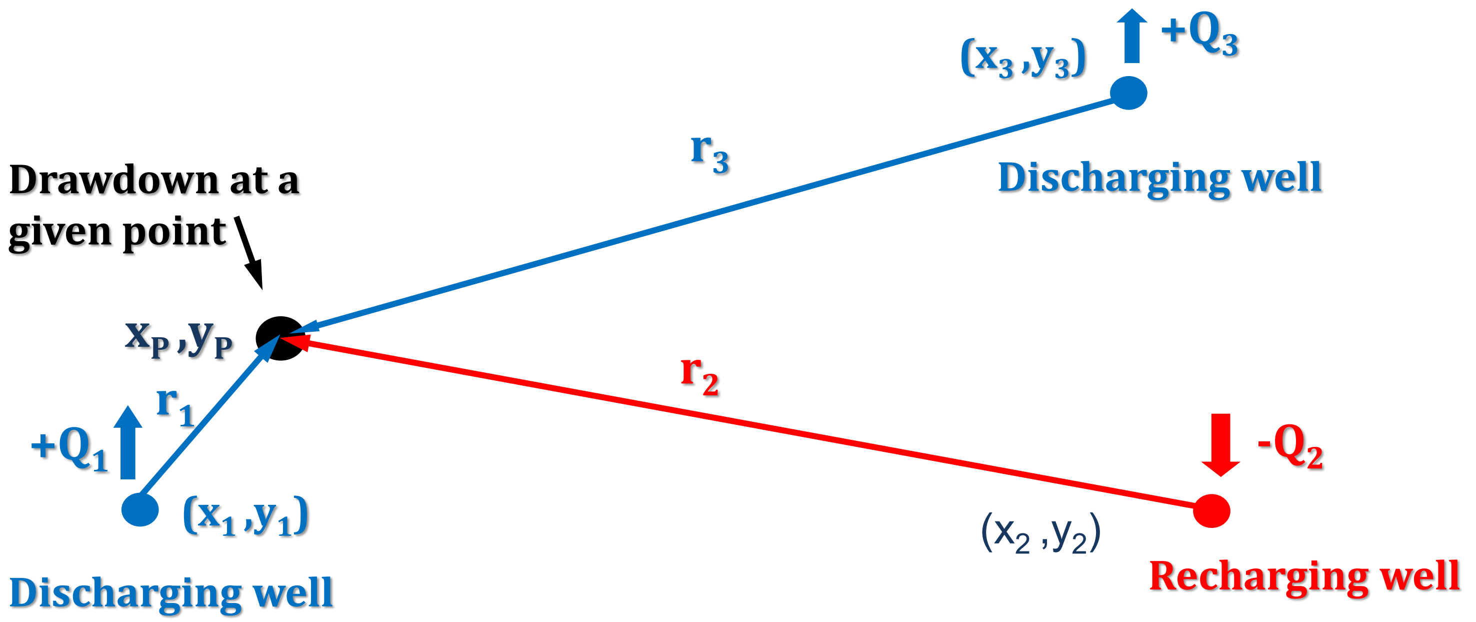
and r_p,r_i are the distances from the pumping and image wells respectively.

- For a barrier boundary, for all points on the boundary $r_p = r_i$ and the drawdown is doubled.
- For a recharge boundary, for all points on the boundary $r_p = r_i$ and the drawdown is zero.

MULTIPLE WELLS

$$s = \frac{Q_1}{4\pi T}W(u_1) - \frac{Q_2}{4\pi T}W(u_2) + \frac{Q_3}{4\pi T}W(u_3)$$

where $u_i = \frac{r_i^2 S}{4Tt_i}$ $\quad i = 1, 2, \ldots.$



$$r_1 = \sqrt{(x_p - x_1)^2 + (y_p - y_1)^2}$$

Sum of $s_1\ (Q_1, r_1)$, $s_2\ (-\ Q_2, r_2)$ and $s_3\ (Q_3, r_3)$ is $s_p\ (x_p, y_p)$

“REAL WELL“ – SKIN EFFECT

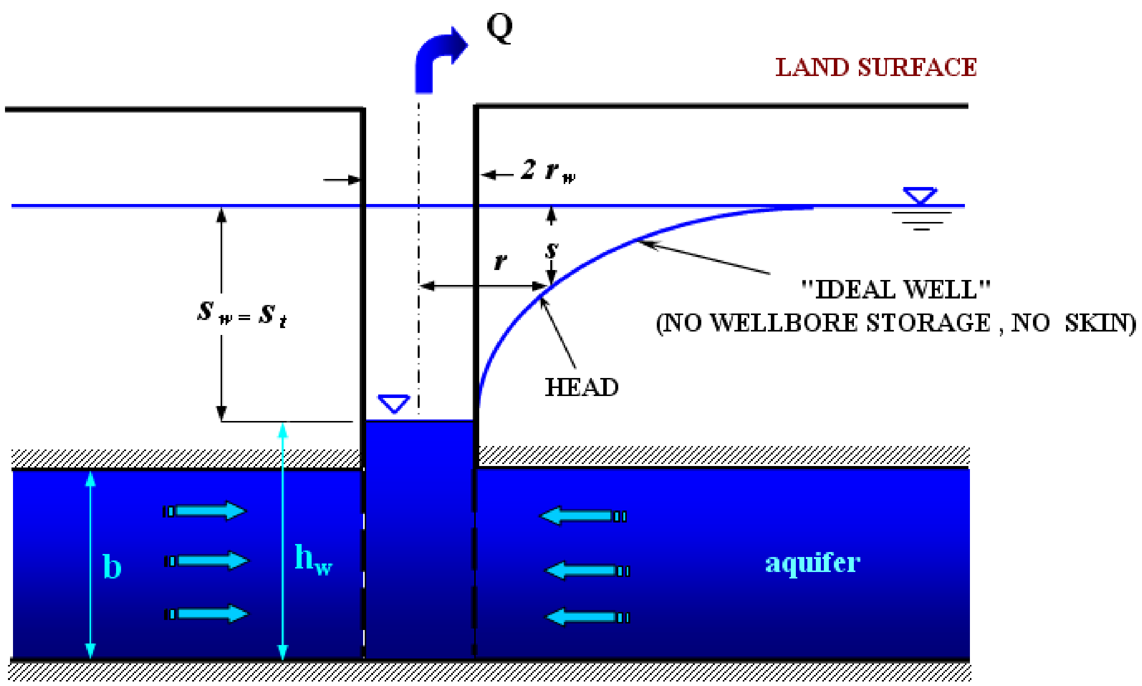
- **Skin, W**, refers to a region near the wellbore of improved or reduced permeability compared to the bulk formation permeability.

REASON FOR POSITIVE SKIN

- **Overbalanced drilling** (filtrate loss)
- **Damaged perforations**
- **Gravel pack**
- Unfiltered completion fluid
- **Partial completion**
- **Fines migration** after long term production
- **Non-darcy flow**
- Condensate banking (acts like **turbulence**)

“IDEAL WELL“

- **no additional resistance** at a well
- the well radius, r_w **is infinitesimally small**



Drawdown around a production well (ideal well)

The partial differential *equation* **describing radial flow to a well fully penetrating confined aquifer is (in cylindrical coordinates)**

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t}$$

Where **s** is drawdown; ***r*** is radial distance from well; ***S*** is storativity; ***T*** is transmisivity

$$s(r,t) = \frac{Q}{4\pi T} W(u)$$

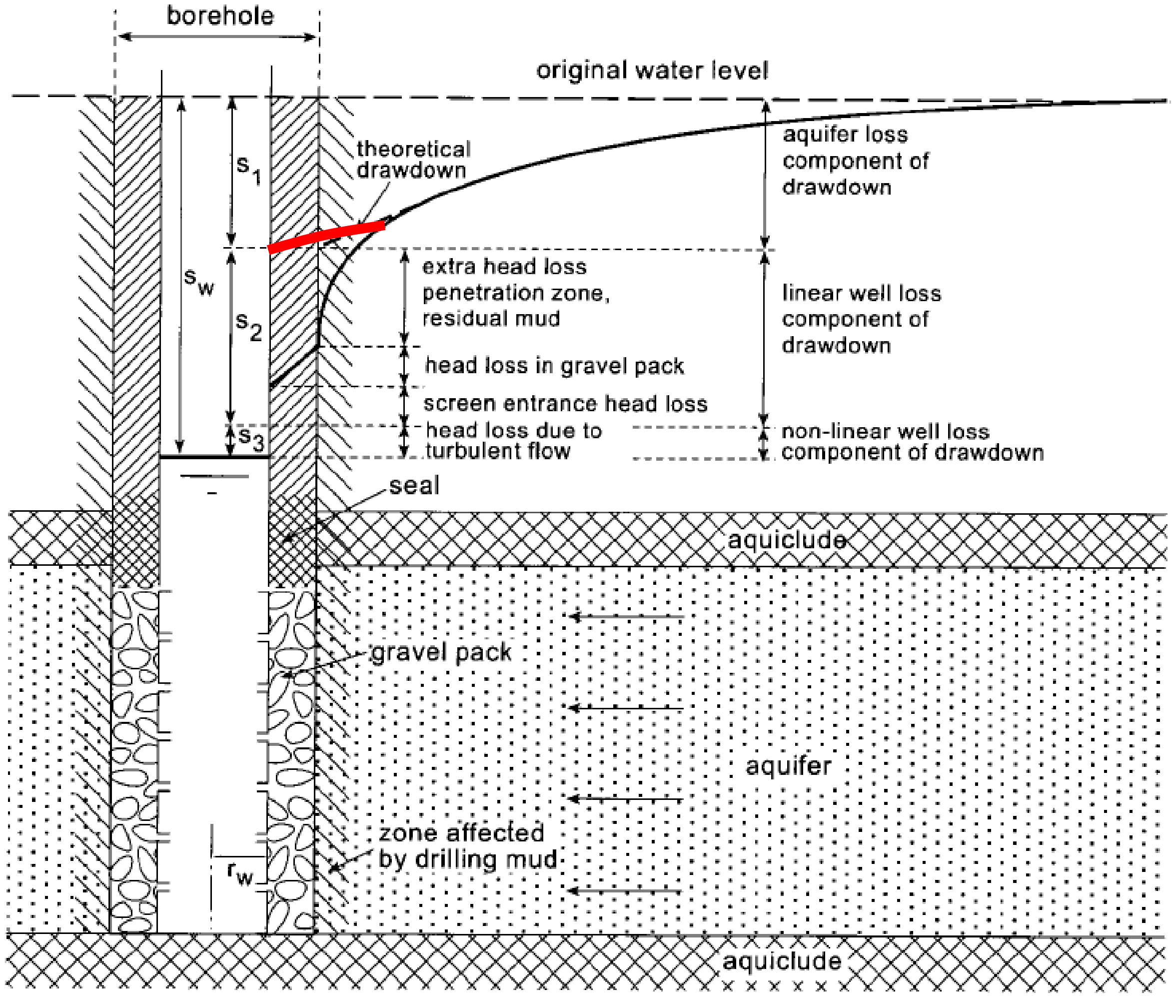
$$u = \frac{r^2 S}{4Tt}$$

DRAWDOWN AT THE REAL WELL

- Drawdown in a pumped well consists of two components:

- **Aquifer losses**
  - Head losses that occur in the aquifer where the flow is laminar
  - Time-dependent
  - Vary linearly with the well discharge

- **Well losses**
  - Aquifer damage during drilling and completion
  - Turbulent friction losses adjacent to well, in the well and pipe



REAL WELL (skin effect)

As a water well ages, the rate at which water may be pumped (commonly referred to as the well yield, flow or performance) tends to decrease,

More often, reduced well yield over time can be related to changes in the water well itself including:

- **Incrustation from mineral deposits** (Fe, Mn)
- **Bio-fouling** by the growth of microorganisms
- **Physical plugging of "aquifer"** (the saturated layer of sand, gravel, or rock through which water is transmitted by sediment
- Sand pumping
- Well screen or casing **corrosion**
- Pump damage

A submersible pump being pulled from a well exhibiting iron oxide, iron bacteria and biofilm.

“REAL WELL“

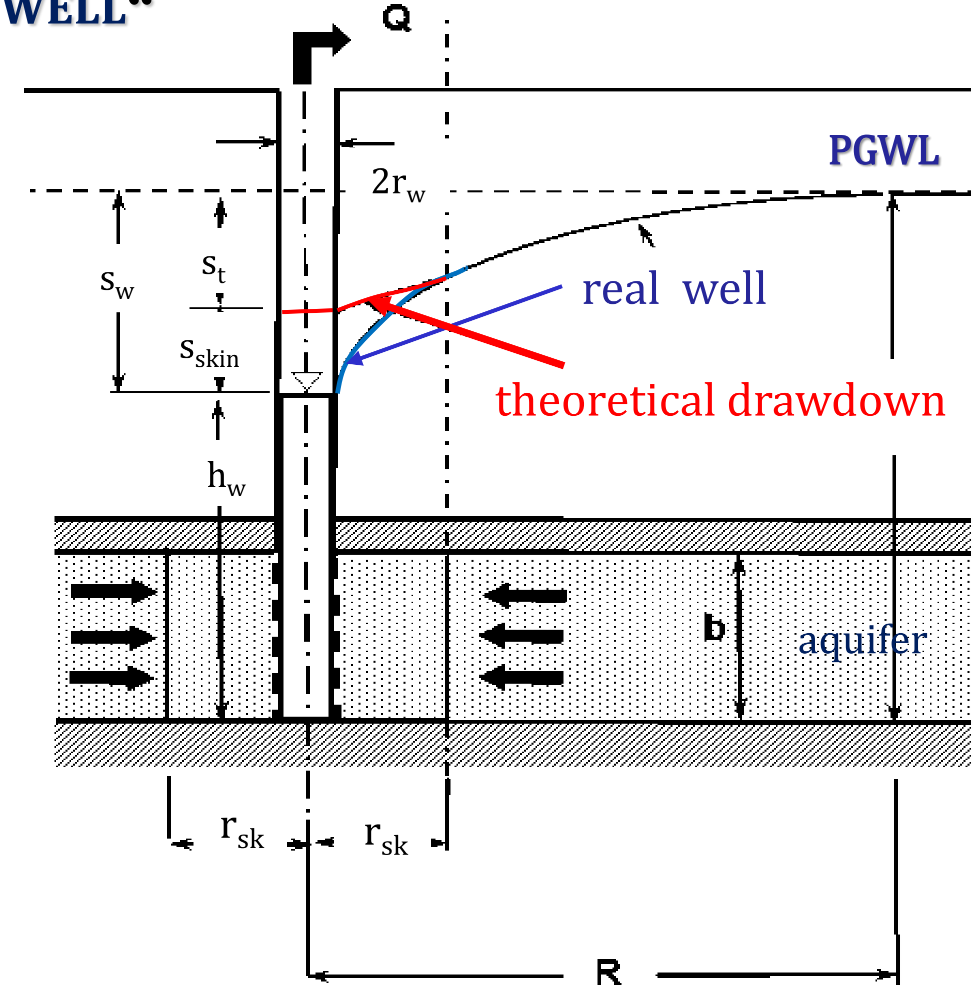
Major changes in any of the following well characteristics is an indication that your well or pump is in need of attention:

- **Decreased pumping rate**
- **Decreased water level**
- **Decreased specific capacity**
- **Increased sand or sediment content in the water**
- **Decreased total well depth**

The two most common methods to rehabilitate (clean) a water well are:

- **chemical**s to dissolve the incrusting materials from the well
- **physical**ly cleaning the well

"REAL WELL"



Drawdown around a production well with skin effect and wellbore storage (real well)

$$s_{skin} = s_d + s_{pp} + s_{inc} + s_{turb} + s_O$$

$$s_w = s_t + s_{skin}$$

TOTAL SKIN (s_{skin})

Total skin is a summation of the following skin components:

- **Skin due to damage** (s_d)
- **Skin due to partial penetration** (s_{pp}) for a partially penetrated well only
- **Skin due to inclination** (s_{inc})
- **Skin due to turbulence** (s_{turb}) or **non-Darcy flow** (for gas wells only)

The value of (s_{skin})

“REAL WELL“

$$\frac{\partial^2 s}{\partial r^2} + \frac{1}{r}\frac{\partial s}{\partial r} = \frac{S}{T}\frac{\partial s}{\partial t}$$

A) THE SKIN EFFECT

The additional resistance is due to hydromechanical, chemical, and biological factors that occur during drilling or completion operations, and during the exploitation of a well. This additional resistance causes an **additional“ drawdown** at a “real” well (s_{skin}).
The drawdown at the “real” well (with skin and wellbore storage

$$s_w = s_t + s_{skin}$$

← van Everdingen, 1953

s_t is drawdown at an “ideal” well , and s_{skin} is additional drawdown at a well caused by additional resistance.
Equation (1) indicates that the drawdown at a “real“ well differs from drawdown at an “ideal“ one by an additive amount

$$s_{skin} = \frac{Q}{2\pi T} W$$

where Q is pumping rate, T is transmissivity, and W is skin factor.

"REAL WELL"

ASSUMPTIONS

- confined aquifer
- pumping rate Q = const.
- Darcy´s law is valid
- all flow is radial to well
- well is fully penetrating
- flow is horizontal
- potentiometric surface steady prior to pumping
- homogeneous, isotropic, infinite areal extent
- pumping well fully penetrates and receives water from the entire thickness of the aquifer
- transmissivity is constant in space and time
- storativity is constant in space and time
- **well has finite diameter, d**
- water removed from storage is discharged instantaneously
- **additional resistances (skin effect) ≠0**

“REAL WELL“

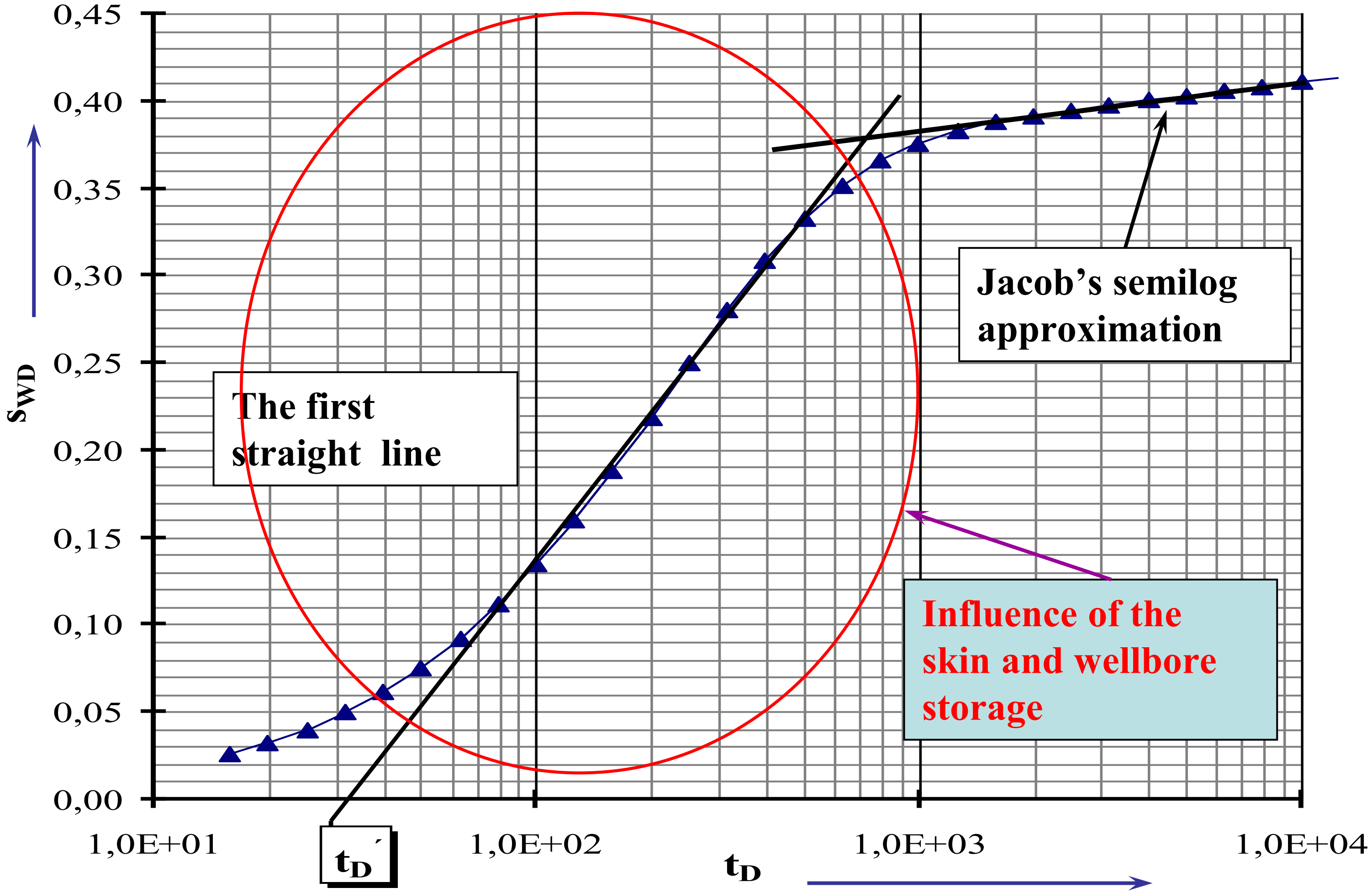
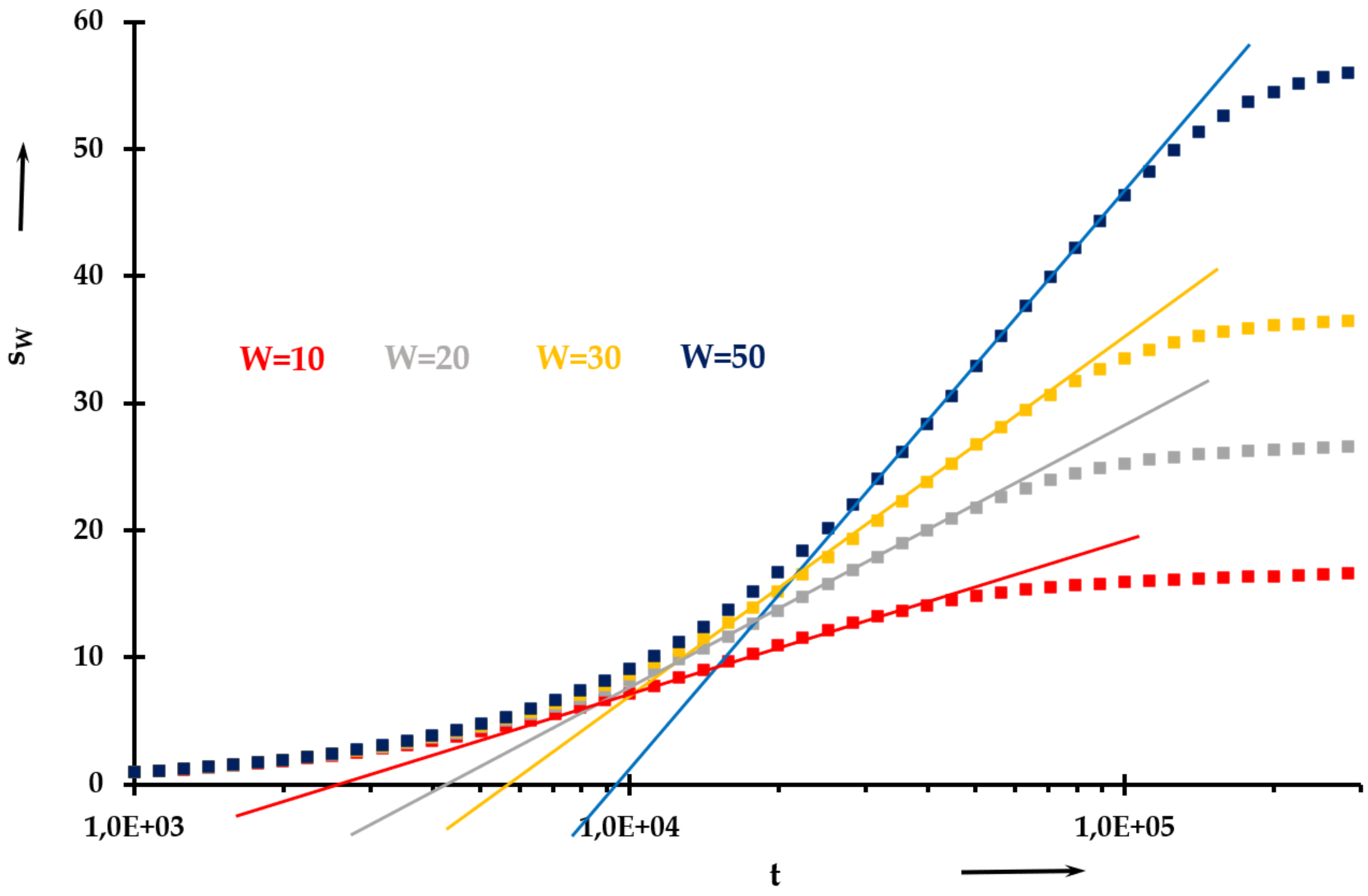


Fig. Graph s_{WD} *vs. log* t_D for a well with wellbore storage and skin (C_D = 100; W = 10)

"REAL WELL"

Pumping tests – graph for W=10, 20, 30, 50



SKIN FACTOR-W

- **Steady flow** $$s_w = \frac{Q}{2\pi T}\left(ln\frac{R}{r_W} + W\right)$$

- **Unsteady flow:**

a) Theis solution: $$s_w = \frac{Q}{4\pi T}(W(u) + 2W)$$

b) For *1/u > 100* (**Cooper-Jacob semilog. method**)

$$s_w = \frac{Q}{4\pi T}\left(ln\frac{2.246Tt}{r_w^2 S} + 2W\right) \Rightarrow W = \frac{2\pi\ T s_V}{Q} - \frac{1}{2}\left(\ln\ t + \ln\ \frac{T}{r_V^2 S} + 0,8091\right)$$

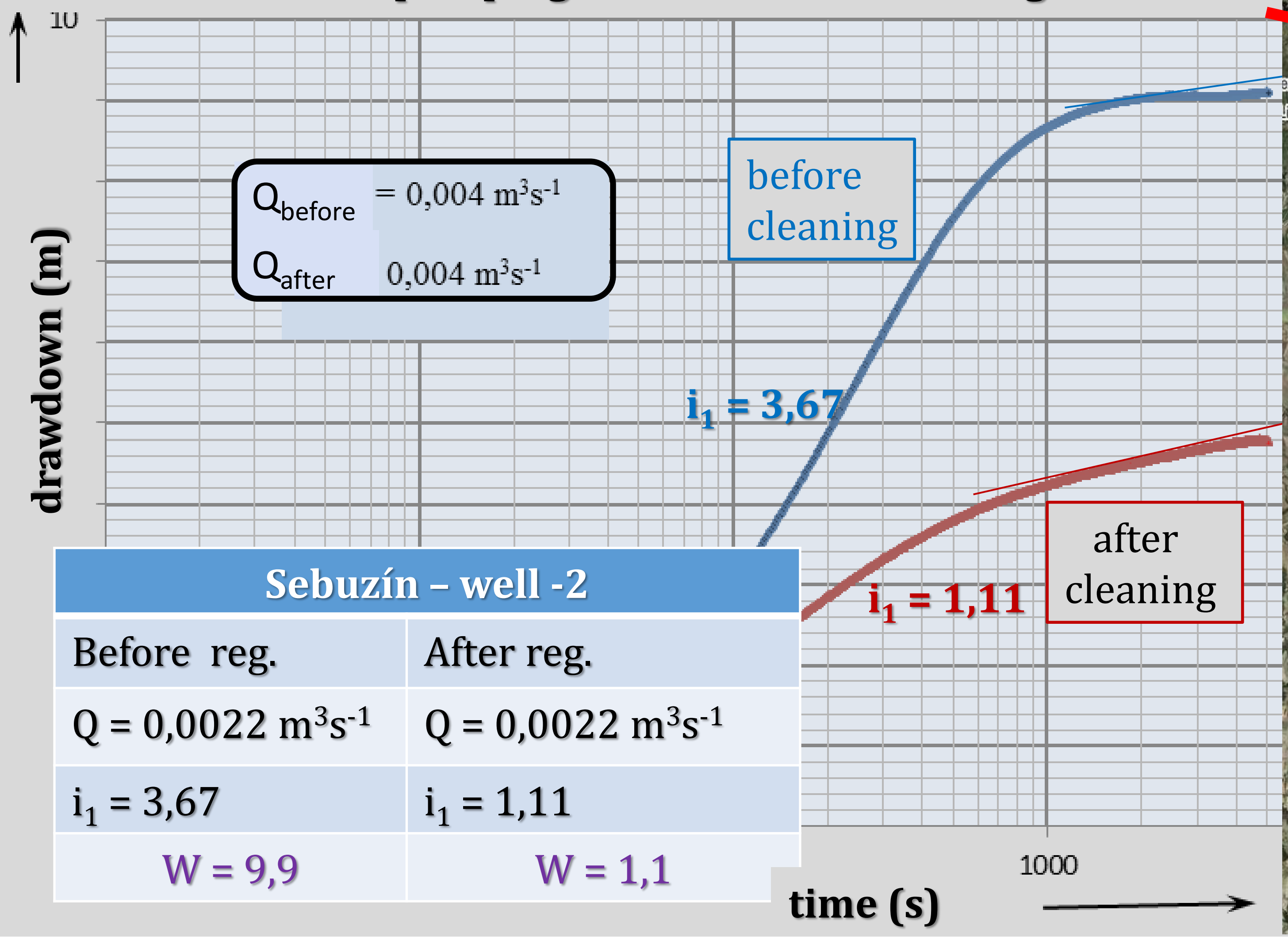
For drawdown $\mathbf{s_1}$ (time $\mathbf{t_1}$) and $\mathbf{s_2}$ (time $\mathbf{t_2}$)

$$s_2 - s_1 = \Delta s = \frac{0.183Q}{T}\left(log\frac{2.246T}{r_w^2\ S} + \log t_2 + 2W - log\frac{2.246T}{r_w^2\ S} - \log t_1 - 2W\right)$$

and

$$\Delta s = \frac{0.183Q}{T}\left(log\frac{t_2}{t_1}\right) \Rightarrow$$ Transmisivity, **T**

Sebuzín – well- 2 – pumping test before and after regenerartion



Sebuzín – well -2	
Before reg.	After reg.
Q = 0,0022 m^3s^{-1}	Q = 0,0022 m^3s^{-1}
i_1 = 3,67	i_1 = 1,11
W = 9,9	W = 1,1